Making rainbow matchings together

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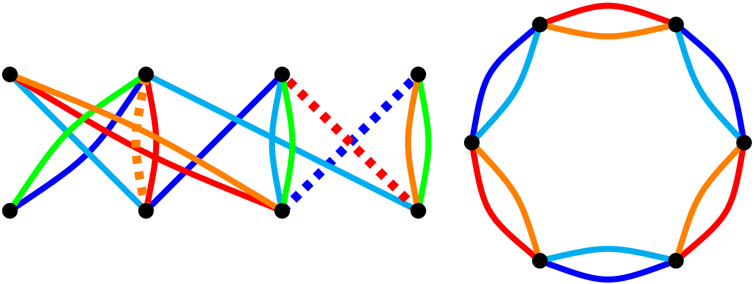
Technion

(joint work with Ron Aharoni, Jinha Kim and Minki Kim)

Rainbow Matchings

Theorem (Drisko)

$2n-1$ n-matchings in a bipartite graph span a rainbow n-matching.

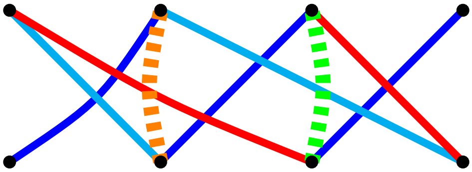


Drisko semi-greedy proof

M: k-matching. "M-AAP" $\equiv$"M-augmenting alternating path".

Lemma

$k+1$ M-AAPs span a rainbow M-AAP (in a bipartite graph).

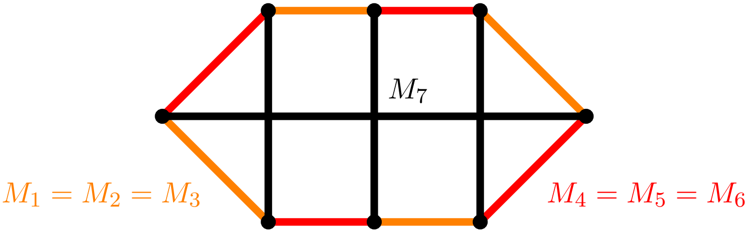


Nonbipartite Graphs

Theorem (ABCHS)

$3n-2$ n-matchings span a rainbow n-matching.

Conjecture (AB): $2n$ suffices!



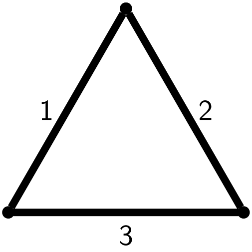
Theorem (Holmsen-Lee)

Any $3n-2$ nonempty edge sets, every 2 of which contain an n-matching, span a rainbow n-matching.

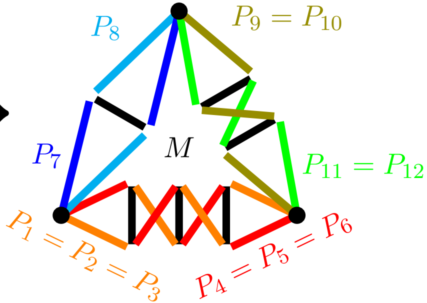
$2k$ M-AAPs without a rainbow M-AAP

Lemma

$2k+1$ M-AAPs have a rainbow M-AAP.



weight-6 graph



"6-badge"

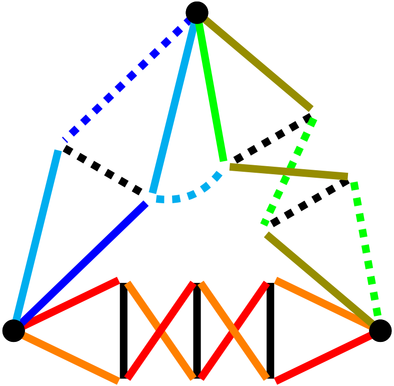
Theorem

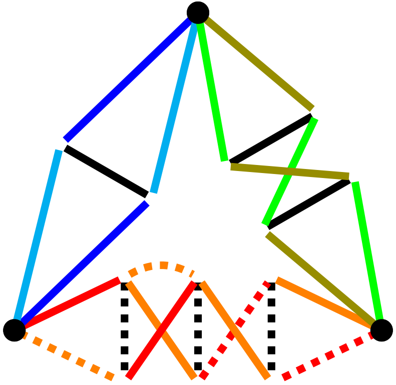
$2k$ M-AAPs either span a rainbow $M-$AAP or form a "k-badge".

Edge maximality of badges

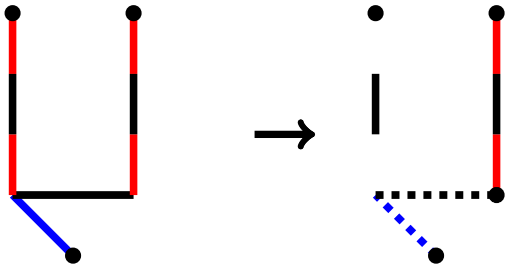
Lemma

Badges are edge-maximally M-AAP free.

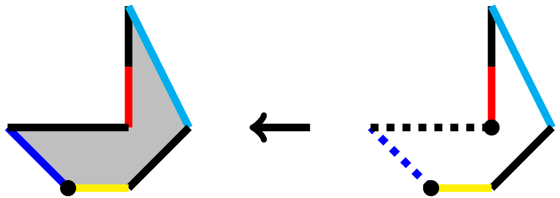




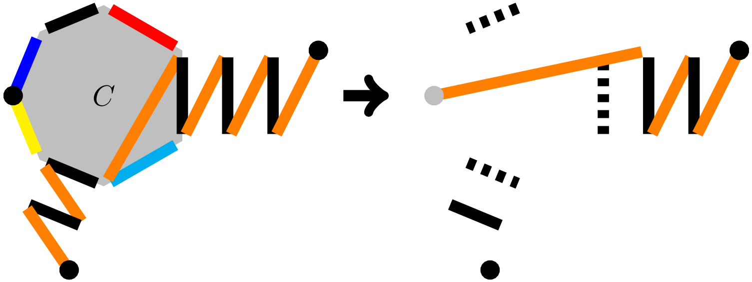
Find a rainbow odd alternating cycle



$|M| = k$, $2k$ AAPs $\qquad |M'| = k-1$, $2k-1$ AAPs

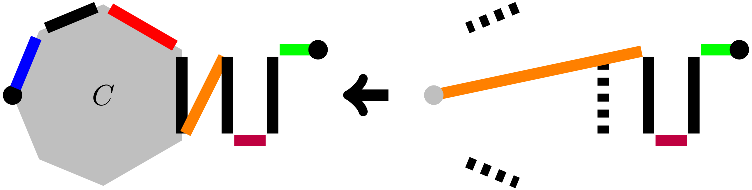


Contracting C

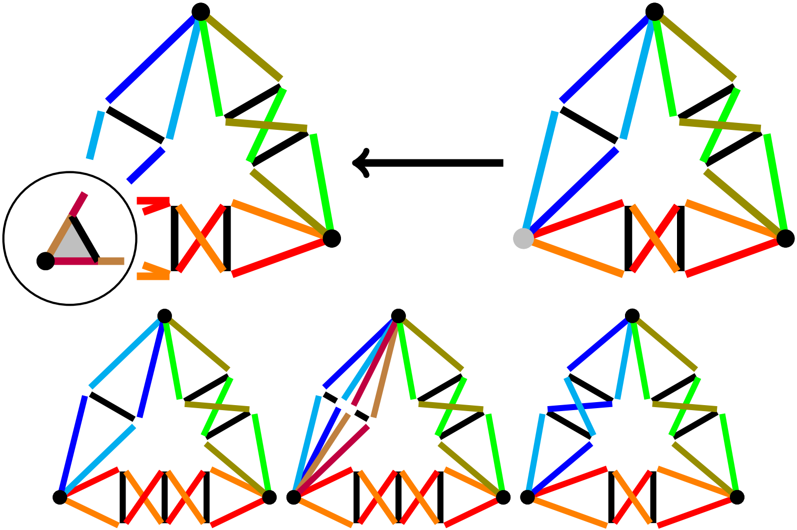


$|M| = k$, $2k$ AAPs

$|M'| = k-3$, $2k-4$ AAPs



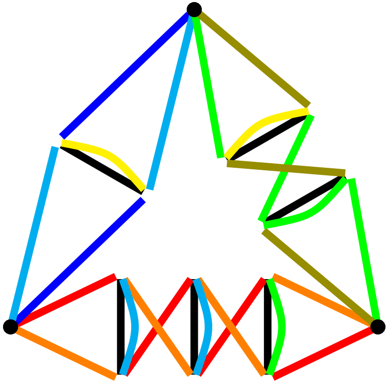
Badge extension



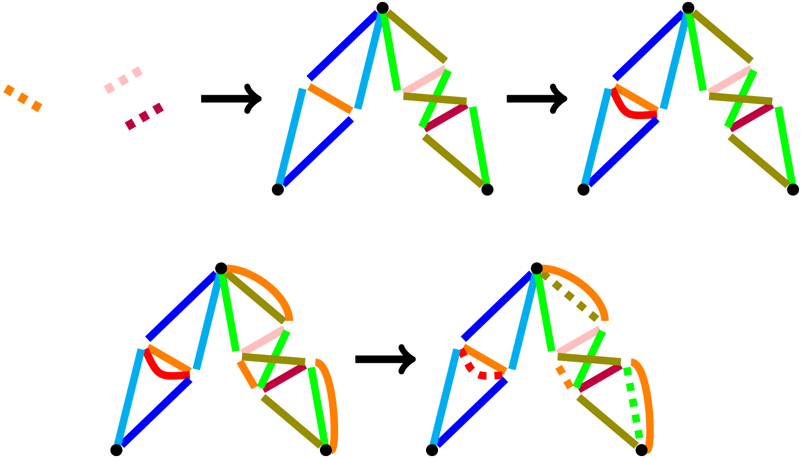
Cooperative badges

Theorem

Let $E_1, \ldots, E_{2k+1}$ be edge sets, any pair $E_i \cup E_j$ of which contains an M-AAP. If they span no rainbow M-AAP then they form a badge.



Holmsen-Lee semi-greedy proof



Further

Theorem

$3n-4+t$ edge sets, any t of which contain an n-matching, span a rainbow n-matching.

Conjecture

$2n$ nonempty edge sets, any 2 of which contain an n-matching, span a rainbow n-matching.

Conjecture (stronger)

Let $E_1, \dots, E_{2n}$ be edge sets such that, for every $I \subset [2n]$ of size $\leq n$, $\cup_{i \in I} E_i$ contains a matching of size $|I|$. Then they span a rainbow n-matching.

Topologise proofs?