Research Statement

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My scholarship and this document are divided into two components: research in pure mathematics and research in education.

Research in pure mathematics

I am interested in geometric flows in general and the mean curvature flow (MCF) in particular. Mostly, I work on gluing constructions but I have also studied long-time behavior for MCF and variational methods for constructing self-similar surfaces.

For curves in the plane, the curvature at a point measures how fast the direction is changing. Sharp turns have high curvature; straightaways have zero curvature. To differentiate between left turns and right turns, the curvature comes with a sign and the convention is that it is positive for circles. If each point moves perpendicularly to the loop at a speed equal to the curvature, the resulting flow is called the *curve shortening flow* (CSF). It is the one-dimensional version of the MCF described below. To visualize the CSF, one can think of the loop as the edge of a thin layer of ice floating on water. Corners will round out instantly; skinny offshoots disappear fast; inlets fill in; and flat edges move slowly. If the ice layer is circular, it will shrink while remaining circular and disappear eventually. This last example is called a *self-similar* solution because its shape does not change under the flow. Because the CSF smoothes out irregularities fast, it is used in image processing to smooth the edges of a figure, after they are extracted from a pixel map.

For surfaces in the three-dimensional space, because so many paths go through a point, one considers an average. The quantity is called *mean curvature* and is positive on bulges, negative on divots, and close to zero on saddles (because of the combination of $\cup$ and $\cap$ curves). As before, we can move each point perpendicularly to the surface at a speed proportional to the mean curvature there and obtain the *mean curvature flow*. Spheres (cylinders) will shrink to a point (a line respectively) while keeping the same shape and are again self-similar solutions. The MCF can be used to model the interface between two liquid metals, after it is modified to conserve the volume of the enclosed liquid. Unlike what happens in lava lamps, there is no heat or gravity in the model. However, like globs in those lamps, some tearing can occur. For example, a large dumbbell with a long thin neck will separate into two and the splitting is called a *singularity*.

The MCF is part of a larger family of geometric flows, named such because they involve various forms of curvature. Notably, the Ricci flow was used to split 3-dimensional manifolds into their building blocks to solve the Poincaré conjecture, a famous long-standing problem in mathematics. The analysis required understanding all the possible behaviors of the flow near its singularities and how to continue the flow past them. In physics, the knowledge of singularity formation in MCF can explain the remarkable phenomenon of charged droplets bouncing off each other if the difference in charge is large enough. More generally, geometric flows are useful when the notion of curvature is relevant to a problem (relativity, string theory, etc.)

Understanding the behavior around singularities is crucial in working with geometric flows. Self-similar solutions, which are solutions that keep the same general shape, are particularly useful in describing what happens near those problematic events. In the example of the dumbbell, we would see the pinching of the neck take the shape of a thin cylinder, if we were to zoom in near the break point. Self-similar solutions are named according to the way they evolve under the flow: in the case of the MCF, they can be shrinking (sphere and cylinders), expanding, or translating. The availability of examples of self-similar solutions is central to the study of the MCF to extend it past its singularities.

I use "gluing" techniques to construct new examples of self-similar surfaces under MCF. The idea is to build a new solution to the relevant partial differential equation by taking two known solutions that intersect, removing a small neighborhood of the intersection, and grafting a suitably modified special surface, called a Scherk surface. A first rough pass provides an approximate solution; the complicated part is solving the perturbation problem for an exact solution. The procedure, called desingularization, is powerful but highly technical. The main complication comes from the presence of small eigenvalues for the linear operator, throwing interference into the scheme. This is comparable to performing a transplant with the patient on a vibrating table (and the way the patient is placed on the table changes the resonance).

One way to deal with the presence of these small eigenvalues is to impose symmetries on all the surfaces considered, and hopefully rule out some of the corresponding eigenfunctions. The method only works if the initial configuration has the required symmetries. For more general purposes, one has to invert the linear operator modulo eigenfunctions corresponding to small or vanishing eigenvalues, which amounts to theoretically ignoring the vibrations. This is done by adding or subtracting a linear combination of eigenfunctions to the inhomogeneous term in order to land in the space perpendicular to the kernel, where the operator is invertible. The key to a successful construction is to be able to generate these linear combinations by slight adjustments of the initial configuration. In other words, one needs some flexibility to adjust the limbs of the patient to cancel the vibrations of the table. Once the study of the linear case is completed, the nonlinear problem is solved with a fixed point theorem.

The year before arriving at ISU, I successfully constructed self-translating surfaces under MCF by gluing a so-called grim reaper cylinder and a plane under simplifying symmetry conditions [M2]. Since then, I have generalized the approach so it can be performed with a finite family of grim reaper cylinders in general position [M4] or infinitely many of them, provided there exists some periodicity [M6]. In 2015, J. Dávila, M. del Piño and I proved the existence of the first examples of self-translating surfaces with finite genus by desingularizing the intersection of a rotationally symmetric bowl soliton and a translating catenoid [M8]. This last construction is more delicate because the ends of catenoid converge to the bowl soliton at infinity, so there is not much room for adjustment.

The article [M5] is my most important scholarly work to date. There, I tackled the more complex problem of finding self-shrinking surfaces by desingularizing the intersection of a sphere and a plane through its center. The equation for this problem was significantly more difficult because it contained a coefficient involving the position, which went to infinity on the unbounded planar piece. Roughly speaking, any small perturbation near the gluing site has a large impact as we move away. As a result, the self-shrinking surfaces obtained do not converge to the original plane at infinity but grow linearly. A very careful analysis of the behavior of solutions at infinity and a definition of the correctly weighted Hölder spaces were necessary.

Because all these constructions are very involved, I have looked for other ways of finding self-shrinking surfaces. In [M10], G. Drugan and I obtained a new proof of the existence of the rotationally symmetric "shrinking doughnut", which was the first non classical example of self-shrinking surfaces found in 1989 by S. Angenent. Whereas Angenent's method consists in finding a closed curved among geodesics on a weighted half-plane, we start with closed curves (not necessarily geodesics) from a continuous family and prove that one of them evolves to a simple closed geodesic as time progresses. The flow used is a curve shortening flow weighted by the inverse of the Gauss curvature so that length decreases. The advantage of this approach is that the evolution is well-known and the crux of the proof is in selecting an appropriate family of curves that are shorter than the known examples (plane, sphere, and cylinders) and enclosing a desirable area quantity. The strength of this approach is that numerical evidence could be used for this purpose in specific dimensions and the flow is readily adapted to different settings. Because it is a theoretical paper, we did provide a full theoretical proof.

The mean curvature flow and the classic heat flow are both parabolic equations and behave very similarly, but not always. In [M7], G. Drugan and I highlighted some of the differences by considering graphs of functions over the entire n-dimensional flat space $\mathbb{R}^n$, with $n \geq 2$. In our first example, the MCF stabilizes at a different height from the heat flow as time progresses. In our second example, the MCF oscillates indefinitely while the heat flow stabilizes. These results contrast with what is known in dimension 1, where solutions to the MCF converge to solutions to the heat flow as time goes to infinity.

Somewhat unrelated, the article [M9] on iterated Routh's triangles is a collaboration with probabilists. For some special deterministic cases, we prove a necessary and sufficient condition for convergence of the triangles to a point. For a random sequence of iterations, the triangles converge also and we show that the expected value of the limiting point is the centroid. We also give an interpretation of this classical geometry problem and iterative process as a 3-person job allocation procedure.

Future work in pure mathematics

Constructing self-shrinking surfaces is challenging, even when there are no unbounded pieces. I am currently working with S. Kleene to expand gluing techniques to the case of self-intersecting rotationally symmetric surfaces. A typical example would be a surface generated by a figure-eight-type curve rotated around an external axis (there is numerical evidence of such curves yielding self-shrinkers). In this work, we need nondegeneracy conditions for the linear operator on the shape as a whole, but we do not require it for each of the disconnected "teardrops" once the intersection is removed. It is the first study with this level of generality. Furthermore, the number of independent Jacobi fields and the degree of freedom in the construction show that our nondegeneracy condition cannot be relaxed further.

With J. Dávila and M. del Pino, I plan on constructing a low genus self-translating surface, which would be the analogue of the Costa-Hoffman-Meeks minimal surface. The idea is to cut off the ends of the minimal surface and glue in ends of translating surfaces instead. The study of small graphs over ends of rotationally symmetric translating ends was already performed in [M8]. The main difficulty is finding the right spot for the grafting of the new ends.

I am also interested in long-time behavior and stability of the mean curvature flow and will study the stability of the grim reaper cylinder. It is expected that initial surfaces that are compact graphs over a grim reaper cylinder will converge back to the cylinder as time progresses. However, with more general perturbations (bounded in L^1 or in L^2), the answer is less clear.

In a somewhat different direction, I recently started working on the optimization of Steklov and Wentzell eigenvalues over domains with P. Sacks and our graduate student L. Rodriguez-

Quinones. We are working on finding (normalized) punctured domains that maximize the first or second Steklov eigenvalue.

Research in education

I am interested in gender differences in enrollment in Science, Technology, Engineering and Mathematics (STEM) and the impact of introductory courses on student retention in STEM.

It is well known that fewer women choose careers in STEM than men. In 2015, women earned less than 20% of all Bachelor's degrees in physics, engineering, and computer sciences; the women's share of degrees was higher in Mathematics and Statistics, coming above 40% but it was still lower than men's [NSF2015]. This is a problem because the United States is in much need of skilled workers in these fields. A 2012 report to the President [PCAST2012] emphasizes that, in order to remain a leading nation in science and technology, the United States would need approximately one million more graduates in these fields in addition to the ones it produces at the current rate by 2022.

In my first semester at ISU, I joined a group of statisticians, mathematicians, and educators to understand better why female students were less likely to pursue STEM degrees when they start college or why they abandoned such a degree. For our research project, we collected college and high school academic data on 15,960 students who took an introductory mathematics or statistics course in Spring 2012, Fall 2012, or Spring 2013 regardless of their majors. In addition, these students were invited to take a 34-question survey on their attitudes towards mathematics at the beginning of each of these semesters (the response rate is 55%). We looked for possible gender differences in students' background, attitudes, and choice of major (STEM vs nonSTEM).

Because the population in our data was so varied, we first focused on students who were in their first semester at ISU ($n =$ 3,107), with a plan to analyze the rest of the data in the upcoming years. We found that, unlike what was in nationally reported data for students out of high school [Hyde et al. 2008], female students coming to ISU were mathematically less prepared than male students. They had statistically significant lower averages in number of math credits in high school (algebra, trigonometry, geometry, and calclulus), along with lower math ACT scores. However, their high school ranks were higher than their male classmates'. The results held even when we took into account all incoming students, not just the ones enrolled in our study.

To characterize students' academic background, researchers have traditionally looked at ACT (or SAT) scores and the courses taken in high school and they have typically considered these variables one at a time (see [Hill, Corbett, & St Rose, 2010] and the references therein). We did not adopt such an approach because it would put female students at a disadvantage from the start by emphasizing their lower ACT scores and ignoring the information from the high school ranks. In addition, identifying students with a similar background would be difficult. For example, are two students with the same ACT score but different math credits more similar than two students who took the same courses in high school but have different ACT scores? The novelty of our approach was the use of a cluster analysis to group students [E2, E5]. A cluster analysis creates natural groupings of students, in the sense that students in each group are more similar to each other than students in other groups. For this, we separated the students who did not have calculus in high school from the ones who did. The computer program searches the data for the closest two students where the distance is based on four variables: high school rank, ACT math score, ACT

English score, and number of science credits taken in high school, then links those two students. It then starts a new search to find the student closest to either an already linked group or another single student. In the former case, it links the student with the group; in the latter case, it links the two students. The program then repeats the search-link process until no student is left. Undoing the last 3 linkages would give us 4 clusters for example. Choosing the number of clusters is a trade-off between clarity and specificity of the interpretation of the data. We chose to have 4 clusters for the students who took calculus (Top, GoodCalc, Middle, and OverlookedCalc students) and 5 for the ones who did not (GoodNoCalc, Enthusiastic, OverlookedNoCalc, PoorTestTakers, and Struggling students). Clustering allows us to identify patterns between the variables. For example, the Overlooked students are good test takers but their potential is not reflected in their high school rank and the GoodNoCalc students are very good students who did not have calculus in high school for some reason.

We found that the difference in preparation between male and female students does not explain the gender difference in STEM enrollment. Within each cluster of students with similar academic background, STEM enrollment for female students was still lower than for male students proportionally. Our data also revealed a marked difference between Math-Intensive STEM majors (which require the first semester of engineering calculus) and non Math-Intensive STEM majors (all other STEM majors). Among students in a Math-Intensive STEM major, there was no longer any gender difference in number of math credits (algebra, trigonometry, geometry, and calculus) and the gap in ACT math scores narrowed. This result contrasted with the numbers for our entire population and suggests that, by measuring preparedness solely in terms of standardized test scores and courses taken in high school, only female students with a high enough score feel they can be successful. A more comprehensive view of the academic background that includes high school rank, however problematic this variable may be, would be more inclusive and help identify potential female STEM majors.

In [E4], we examined students' beliefs in their mathematical abilities rather than their academic background. To understand these beliefs and establish trends, we performed a factor analysis on the answers to the 34-question survey on attitudes mentioned earlier. Five factors emerged: interest, value, self-efficacy, outcome expectations, and effort. When we grouped students according to their majors (Arts and Humanities, Social Sciences, Math-Intensive STEM, and non Math-Intensive STEM), we found that within each of these four groups, there was no difference in attitude between male and female students. The enrollment was still highly biased (with a lot female students in Arts and Humanities and Social Sciences) but the male and female students in those majors had the same attitudes towards mathematics. Our two articles therefore hint at the fact that students self-select into their majors according to their attitudes rather than their academic background.

We have preliminary results on retention. Looking at incoming freshmen who took Calculus 1 in the Fall 2012, we found that female students tended to leave STEM in their first semester and male students usually waited until the second or third semester. Overall, proportionally as many female and male students stayed in STEM after three semesters (70%-74%). Once we included the five factors about attitude into our model, gender no longer played a significant role and the retention was correlated to students' attitude. A student's (lack of) outcome expectations was linked to dropping out of STEM in the first semester and a lack of interest to leaving STEM in the second semester.

I also led a separate research project to examine the effect of clickers on students' success in calculus in collaboration with H. Bolles, A. Jenkins, and E. Johnston [E1]. In this study, H. Bolles,

A. Jenkins, and E. Johnston each taught a large-enrollment class of Calculus I in Fall 2012 and again in Fall 2013. Two of the instructors used clickers during the Fall 2012 session but the third one did not. The clicker usage was reversed in Fall 2013. Instructors taught at the same times of the same days of the week in Fall 2013 as in Fall 2012. We found that students in clicker sections passed at a higher rate than in nonclicker sections. The effect was more pronounced for female students, although it was not statistically significant. Another noticeable impact, however, was that student attendance to the class sessions was higher in the clicker sections than in the nonclicker sections. Late into the semester, attendance continued to reach 80-85% of the enrollment for the class.

In the book chapter on "Peer grading on exams" [E3], I described and discussed a technique implemented by A. Bennett and I in a geometry course at Kansas State University (K-State). The goal was to teach students to grade proofs, which helped them critique their own work and become better writers. The central activity consisted of getting students to grade their own midterm exams and the exams of two classmates. A. Bennett was in charge of the course at K-State and taught it several times. I taught the course in my last year at K-State. At Iowa State University, I analyzed the data we collected and wrote the book chapter.

Future work in mathematics education

U. Genschel, A. Gansemer-Topf, G. Kremer and I will look into the issue of retention. For students in our population, we have their declared majors at the beginning and end of each semester so we can pinpoint the semester when they drop out of STEM. We finalized our cluster analysis only recently and will rerun the numbers on retention. For example, we want to determine if belonging to some particular cluster makes a student more likely to drop out of STEM. In addition, we are writing an NSF proposal for setting interviews to complement our current quantitative approach. The goal is to understand why female students from similar background choose different majors. We plan on interviewing female students within each cluster and compare the responses of the students in Math-Intensive STEM majors versus the ones from students in other majors.

We will also analyze the much more complicated data from transfer students. This work was postponed at first because of the difficulties of handling such a varied group of students. Some of them transferred in after a couple of classes from a community college; others have two years' worth of credits. To complicate things further, we have little to no information on their academic background (high school data or ACT scores). Nevertheless, we want to know how successful transfer students are, compared to students coming directly from high school.

We submitted a NSF proposal for a qualitative analysis. With the cluster analysis, we can identify which groups of students are more likely to go into STEM and which are not. We plan on interviewing male and female students who 1) are predicted to go into STEM and did 2) are predicted to go into STEM and did not 3) are not predicted to go into STEM but did. The goal is to understand how students chose their majors and what influenced their decisions.

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