Cellular Automata: Basic Intro

Leigh Tesfatsion
Professor Emerita of Economics
Courtesy Research Professor of ECpE
Iowa State University, Ames, IA 50011
https://www2.econ.iastate.edu/tesfatsi

What are Cellular Automata (CA)?

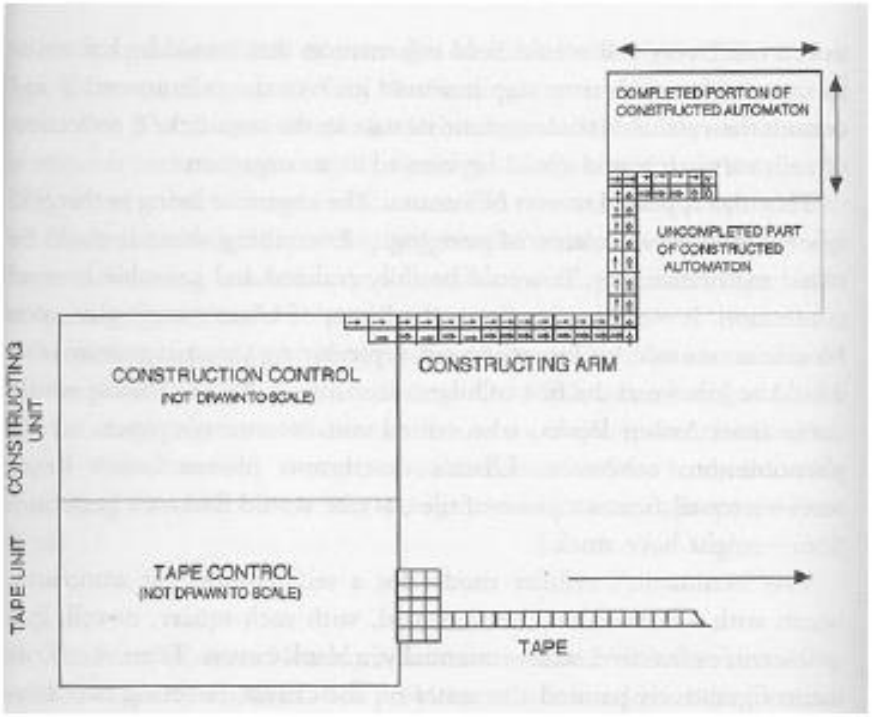
Illustrative Examples:

- John Conway's Game of Life
- Thomas Schelling's Segregation Model
- Per Bak's Sand Pile Model

Acknowledgement: The CA slides below are based in part on course lecture slides prepared by Prof. Bill Tomlinson (2004), Prof. Inman Harvey (2005), Prof. Stefan Rahmstorff (2005), and Prof. Lars-Erik Cederman (2005).

What are Cellular Automata?

Cellular automata are discrete dynamical systems that model complex behavior based on simple, local rules animating cells on a lattice

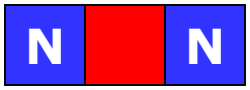


Invented by John von Neumann

Illustrations of Simple Cellular Automata

- A Cellular Automaton (CA) is a stylized universe.
- In the simplest forms of CA, space is represented by a uniform M-dimensional grid of cells (e.g., M=1,M=2), with each cell containing some data.
- Time advances in discrete steps and the laws of the "universe" are expressed through a rule (or “finite state machine” = FSM) dictating how, at each time step, each cell computes its new state given its old state and the states of its K closest neighbors (K = key parameter).
- Thus, each CA’s behavior is determined by a uniformly applied rule governing local unit behaviors.

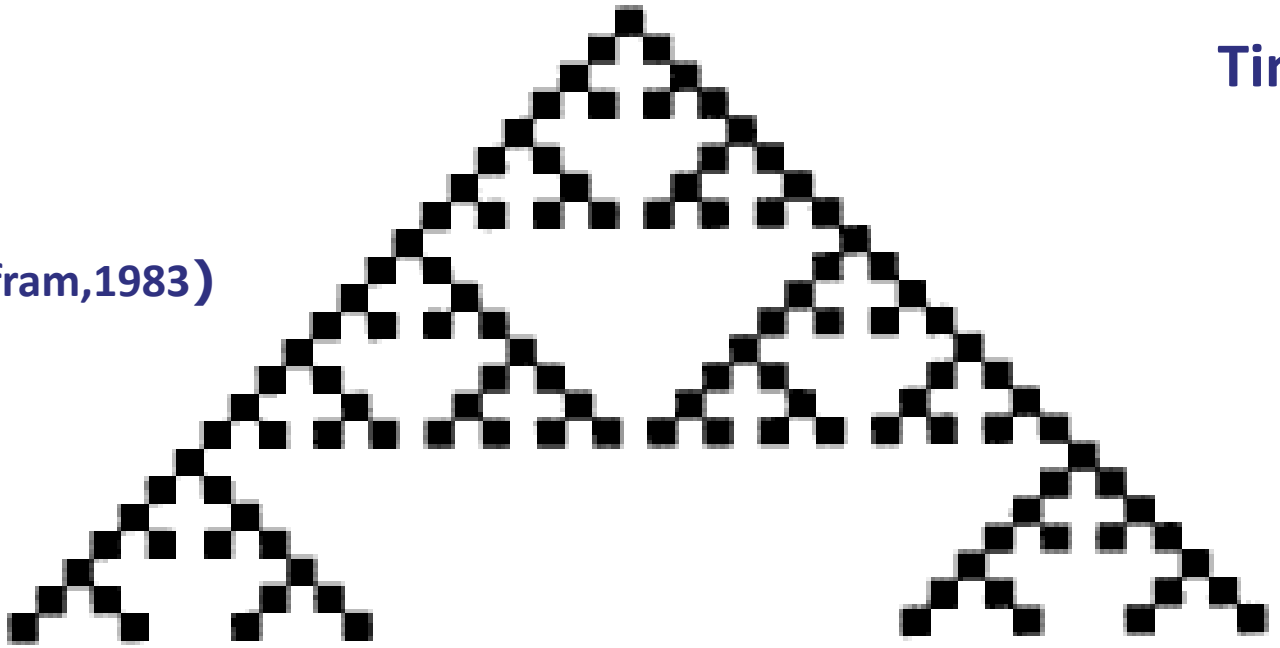
Examples of 1-Dimensional (1D) CA

- 1D CA: Each cell has at most two neighbors 

- Example: 1D CA operating through time under “Rule 90”

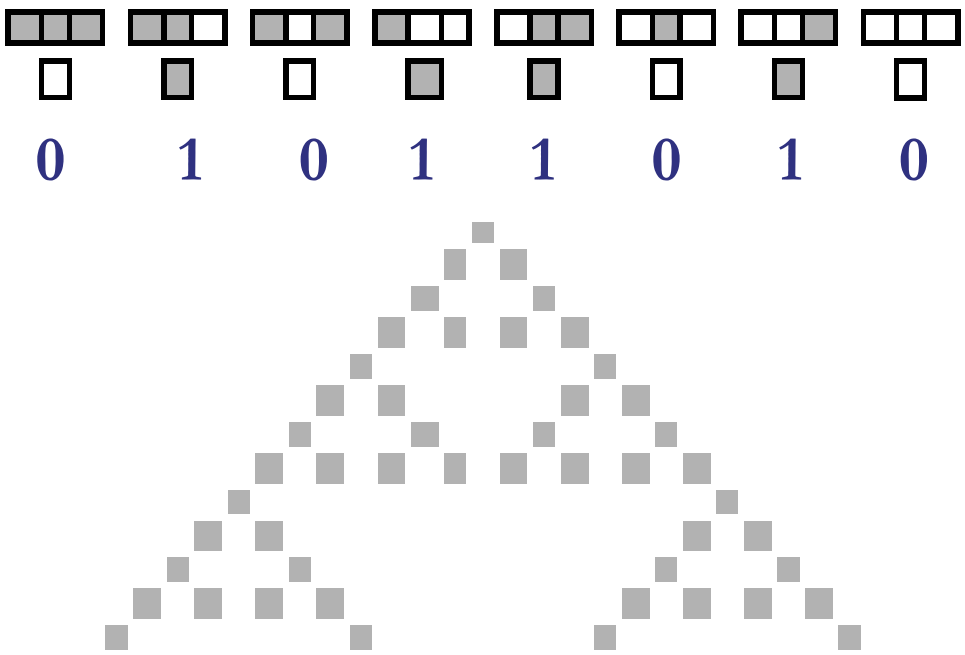
Time T=1

Time T=2



(Wolfram,1983)

“Rule 90” = One of 2^8 Elementary 1D CA

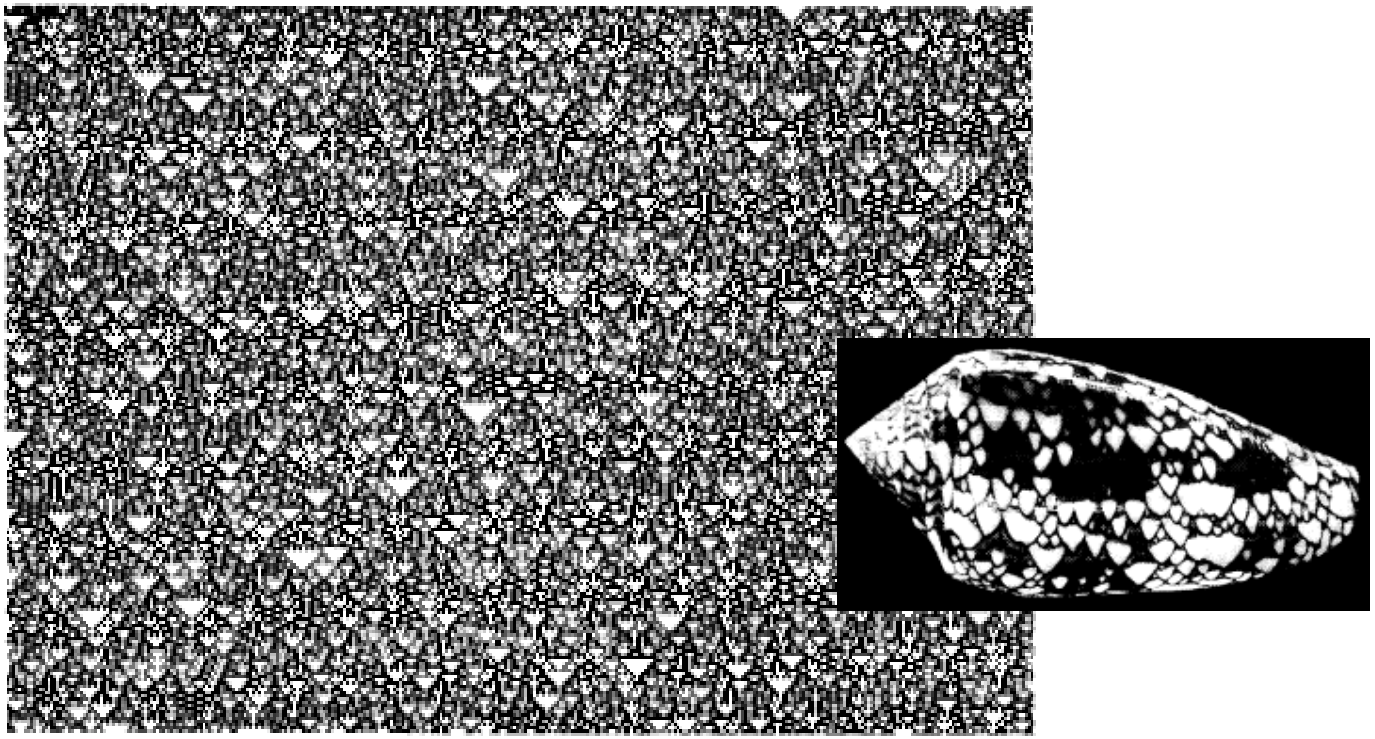


90 =(01011010)_base 2 = $0*2^7+1*2^6+0*2^5+1*2^4+1*2^3+0*2^2+1*2^1+0*2^0$

https://mathworld.wolfram.com/ElementaryCellularAutomaton.html

1D CA in Nature?

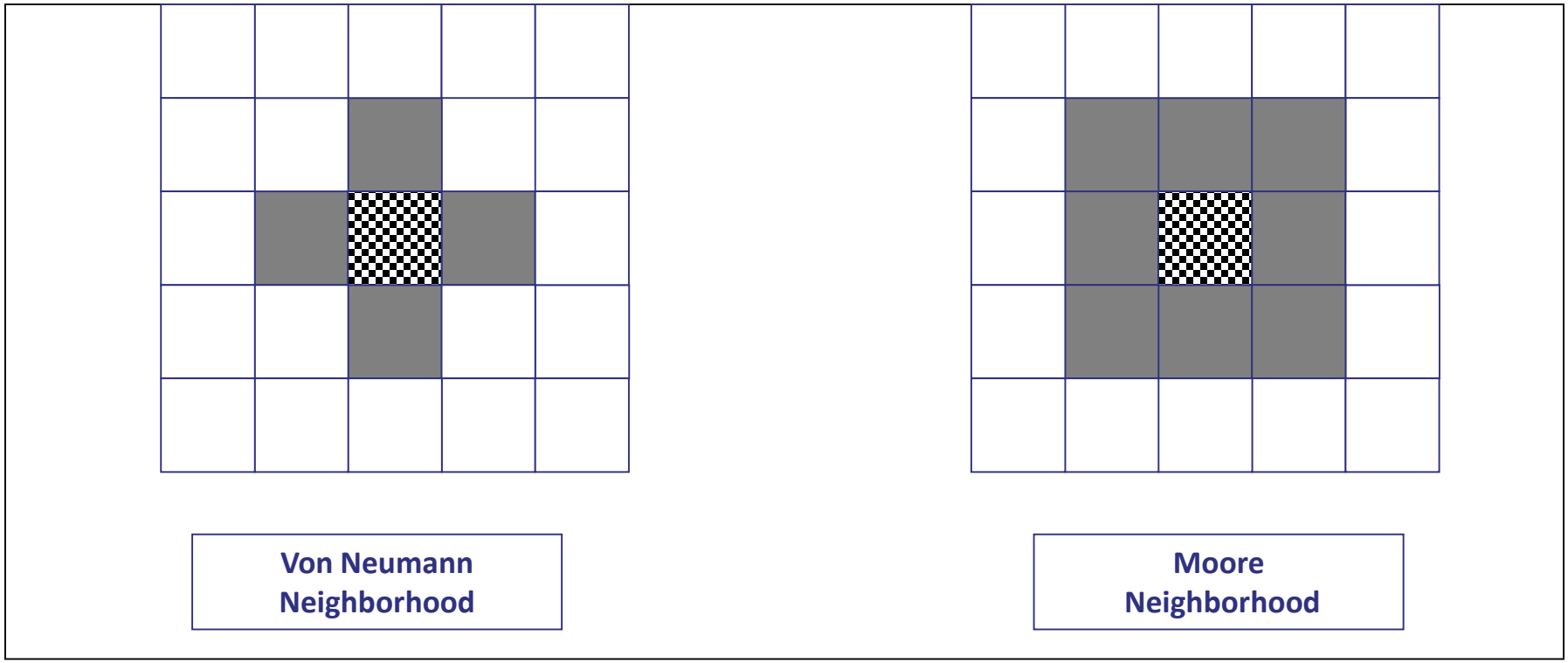
1D CA shell patterns (Wolfram, 1983)



Examples of 2D CA

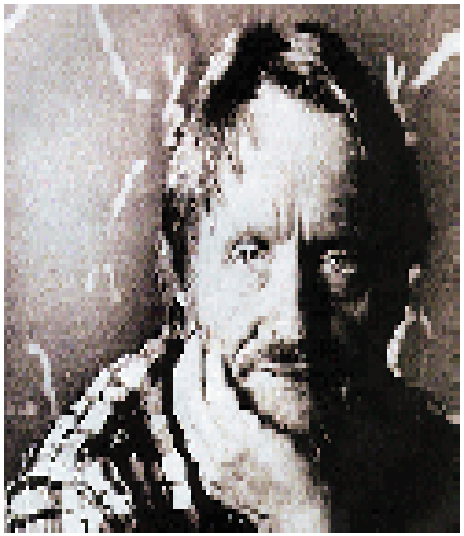
- 2D image entirely replaced each time step.

Possible Neighborhoods for 2D CA



Famous 2D CA Example:
The Game of Life

- First practical CA invented by John Conway in the late 1960s
- Later popularized by Martin Gardner



John Conway

The Game of Life …

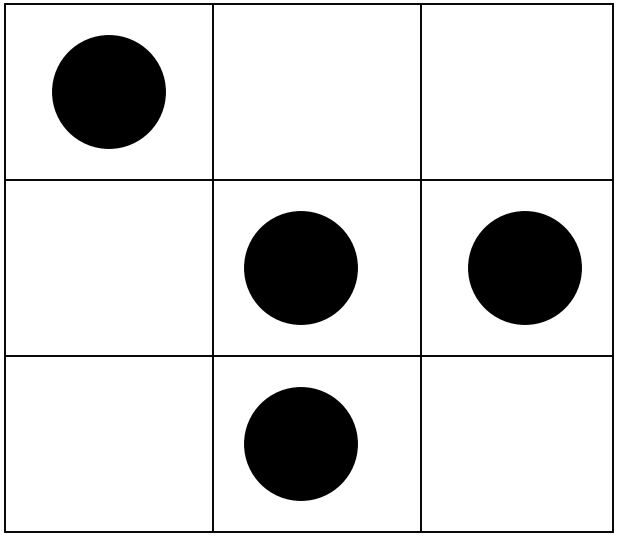
John Horton Conway's "Game of Life," a 2-D CA invented in late 1960s at the Univ. of Cambridge

Objective: To make a CA 'game' as unpredictable as possible using the simplest possible CA rule.

- 2D grid of squares on a (possibly infinite) plane.
- Each square can be alive (black) or dead (white)

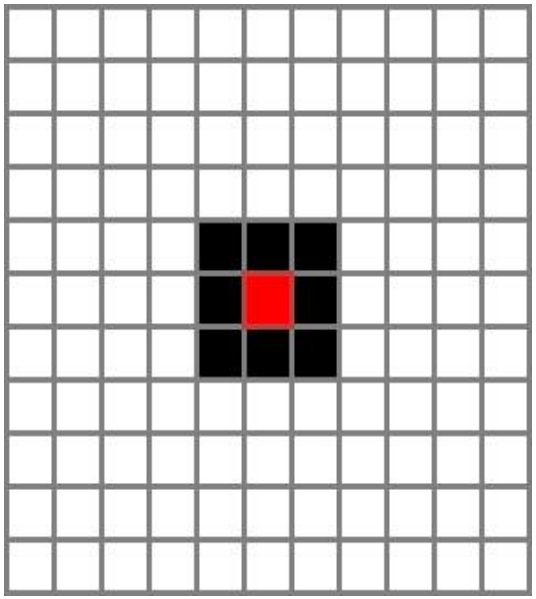
The Game of Life …

- Dead cells can come alive, and alive cells can die, depending on their neighbors



The Game of Life …

- Each cell has 8 alive or dead neighbors (pasted edges assumed), 4 adjacent orthogonally and 4 adjacent diagonally.
- So, the Game of Life assumes Moore Neighborhoods.



The “Rules of Life”

- If an alive (black) cell has fewer than 2 alive neighbors, it dies (turns white) -- ***loneliness***
- If an alive cell has more than 3 alive neighbors, it dies – ***overcrowding***
- If an alive cell has either 2 or 3 alive neighbors, it goes on living (stays black) -- ***happiness***
- If a dead cell has exactly 3 alive neighbors, it comes alive -- ***reproduction***. Otherwise, it stays dead.

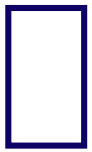
Visual Depiction of "Rules of Life"

And if the current number of your alive (black) neighbors is:

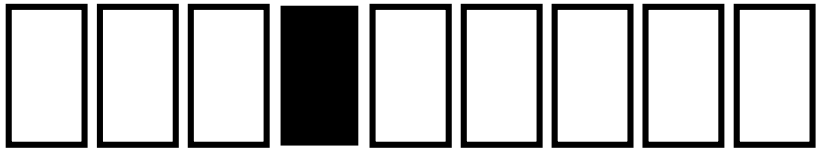
0 1 2 3 4 5 6 7 8

If your current state is:

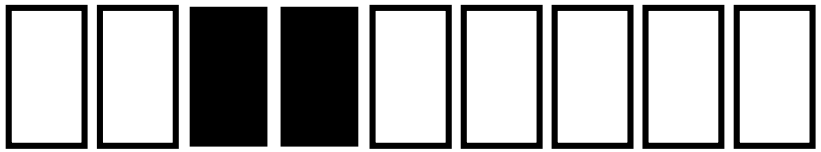
then your new state is:



White



Black

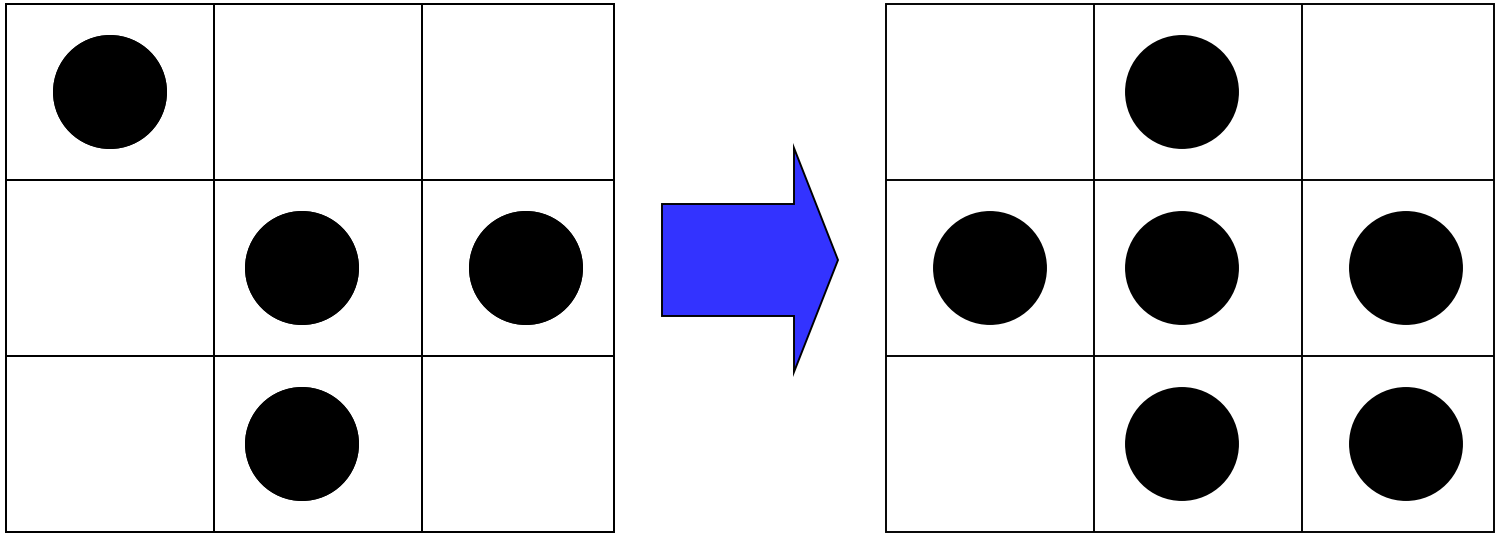


Example: **My current state is white, and I have 3 alive (black) neighbors, thus my new state is black: (white,3) → (black).**

How the Game of Life Proceeds

- The game proceeds in generations, one generation per time step T
- In the initial generation at T=1, a finite number of cells are alive
- In each successive generation, some cells come alive and some die according to the “Rules of Life.”

Example …



Why the Game of Life is Addictive ...

- Various interesting patterns keep emerging

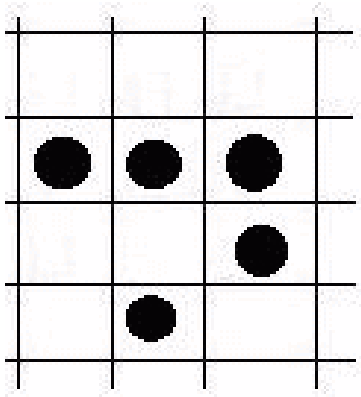
  ➔ “glider,” “pentomino,” “spaceship,” …

- Many Game of Life demos/applets are available on-line

Pattern Emergence?

At right, a **'Glider'**

On a clear background, after 4 time steps, Glider pattern will 'move' to the North East one square diagonally.



Glider

Each square does not actually 'move', but the pattern of black squares can be seen by an observer as a glider travelling across the background.

Pattern Emergence? …

- This pattern movement can be observed as ‘the movement of a glider’, even though no glider was mentioned in the rule.

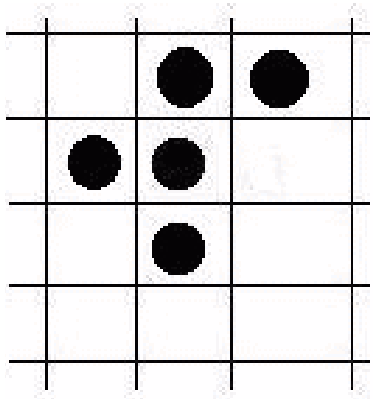
- 'Emergent' behaviour at a higher level of description, emerging from a simple low-level rule.

- Emergence = Emergence-in-eye-of-the-beholder? (Controversial definition?)

Pattern Emergence? …

At right, a 'pentomino'.

Simple starting state on a blank background => immense complexity over 1000 steps before it settles.



Pentomino

Demo/Applets for the Game of Life

- Demo/Applet: “What is the Game of Life?” by Paul Callahan

➔ Once the "pieces" are placed in the starting position, the rules determine everything that happens later. Nevertheless, Life is full of surprises! In most cases, it is impossible to look at a starting position (or *pattern*) and see what will happen in the future. The only way to find out is to follow the rules of the game.

- **Demo/applets are linked at:**
  https://www2.econ.iastate.edu/tesfatsi/abmdemo.htm

Game of Life: Some Implications

Basic Complex System Paradigm:

- ✓Many interacting units;
- ✓Parallel (distributed) actions;
- ✓Locally determined (bottom up) actions.

Complex global system behavior arising from ("emergent from") simple rules of unit behavior.

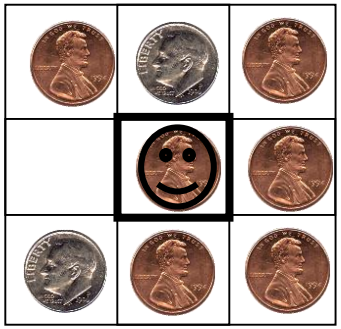
Emergent Patterns: Gliders, pentominos, blocks, traffic lights, blinkers, glider-guns, eaters, puffer-trains ...

Another Famous 2D CA Example: The Schelling Segregation Model

- An interesting and important puzzle:
  - after 1964 housing discrimination was illegal
  - since 1950 racial prejudice has declined
  - yet neighborhoods remain highly segregated
- T. C. Schelling (1978) hypothesized that segregation:
  - does not need to be imposed (top-down)
  - does not reflect preferences (bottom-up)
  - self-organizes through dynamic interaction
- Schelling was a co-recipient of the 2005 Nobel Memorial Prize in Economics. He is considered a “father” of agent-based modeling.

The Schelling Segregation Model

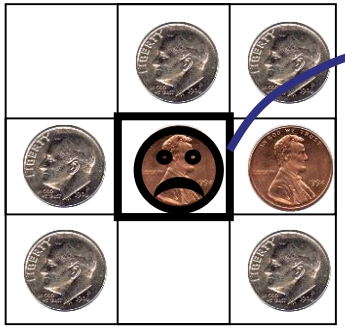
Micro-level rules of the game



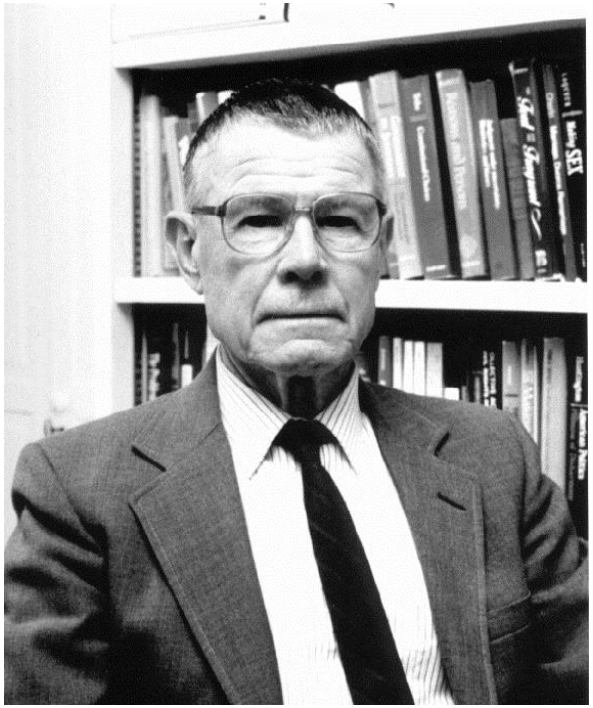
Stay if **more than one third** of your neighbors are "kin"

> 1/3

≤ 1/3



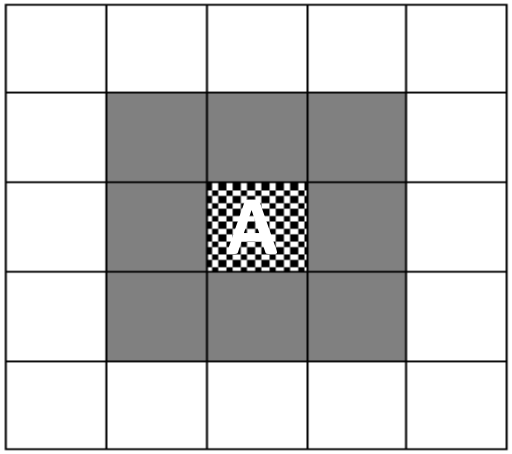
Move to random vacant location otherwise



Thomas C. Schelling

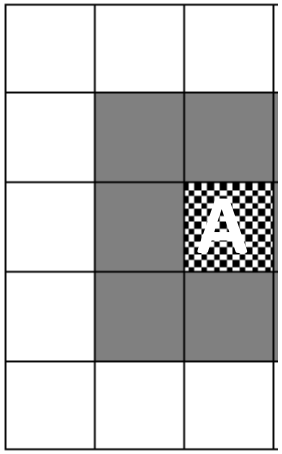
***Micromotives and Macrobehavior,* 1978**

Counting “Neighbors“ for the Schelling Segregation Model



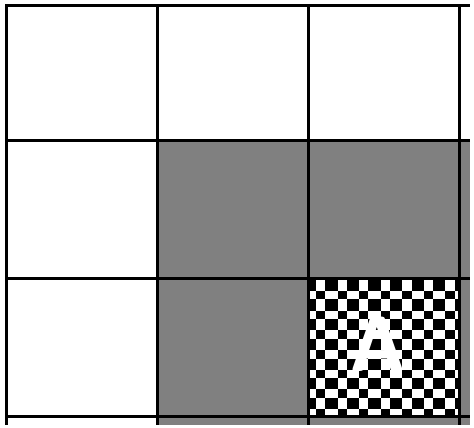
Interior agent A

up to 8 neighbors



Border agent A

up to 5 neighbors



Corner agent A

up to 3 neighbors

Illustrative "Happiness Rule" for the Schelling Segregation Model

- Each agent A is "happy" (no need to move) if **more than 1/3** of its neighbors are of the "same type".
- Second row of boxes, below, gives the minimum number of neighbors of an agent A that must be of the "same type" as A in order for A to be happy, given each possible total number of A's neighbors (from 0 to 8) indicated in the first row of boxes.

0	1	2	3	4	5	6	7	8
0	1	1	2	2	2	3	3	3

Total # Neighbors

Illustrative Starting Pattern for the Schelling Segregation Model

	o	x	o	x	o	x	
o	x	o	x	o	x	o	x
x	o	x	o	x	o	x	o
o	x	o	x	o	x	o	x
x	o	x	o	x	o	x	o
o	x	o	x	o	x	o	x
x	o	x	o	x	o	x	o
	x	o	x	o	x	o	

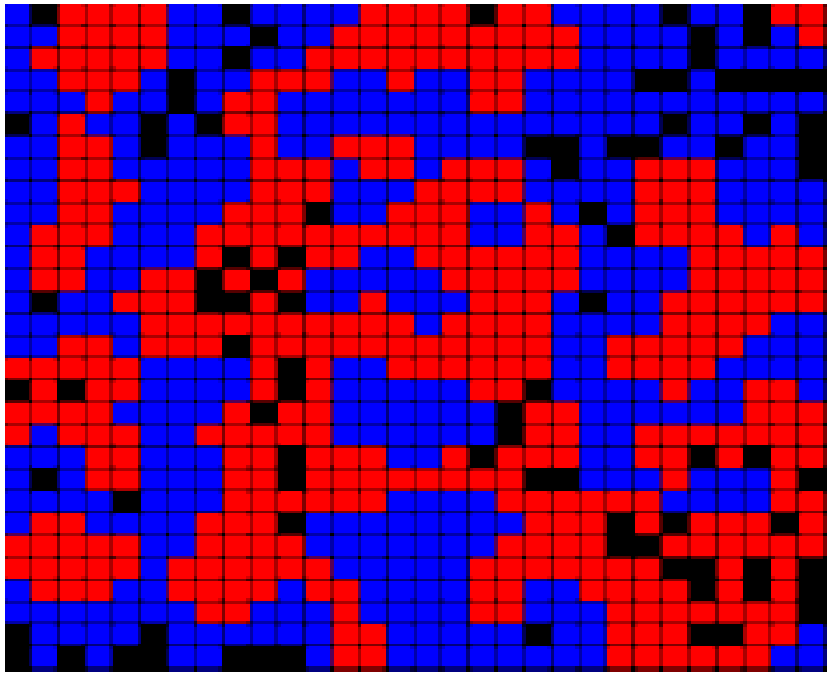
Now “Play the Game” !!

- Given the pattern on previous slide, everyone is happy and no one moves. Now ***remove 10*** randomly selected agents from the board.
- Starting from the top row, moving from left to right, row by row, ***check for unhappy agents***.
- Every time you encounter an unhappy agent, ***if possible move him*** to a “tolerable” vacant square where he is happy; ***otherwise remove him***.
- Keep going until there are no unhappy agents left on the board. What degree of segregation does the ***resulting pattern*** display?

Illustrative Segregation Pattern

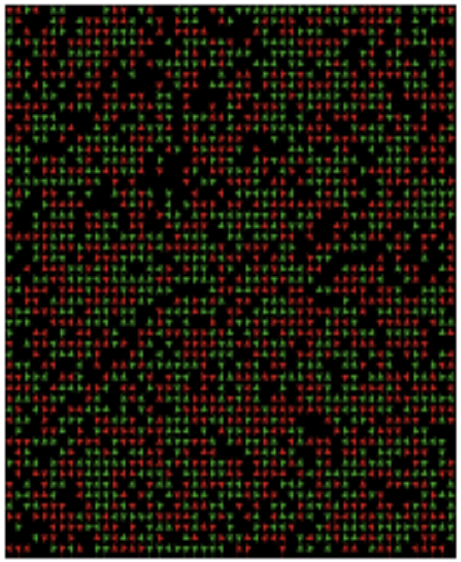
Blue Cells, **Red Cells** denote two distinct types of households

Black Cells denote “empty lots”

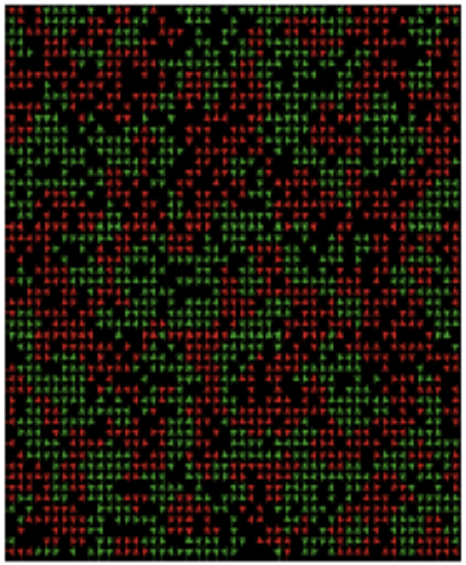


Illustrative Segregation Patterns with Different Levels of Tolerance for Un-Like Types

Two classes of households = Green Cells and Red Cells
Empty lots = Black Cells

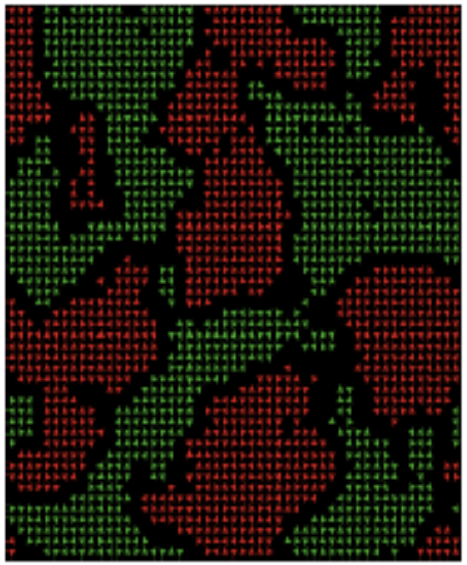


(a) Initial (random) pattern. The average share of racially similar neighbours is roughly 50%. With a tolerance level of 70% (40%), less than 20% (more than 80%) of the individuals are not satisfied.



(b) Final pattern. The average share of racially similar neighbours is 72.1%. Everyone is satisfied

Final pattern, assuming a 70% tolerance level



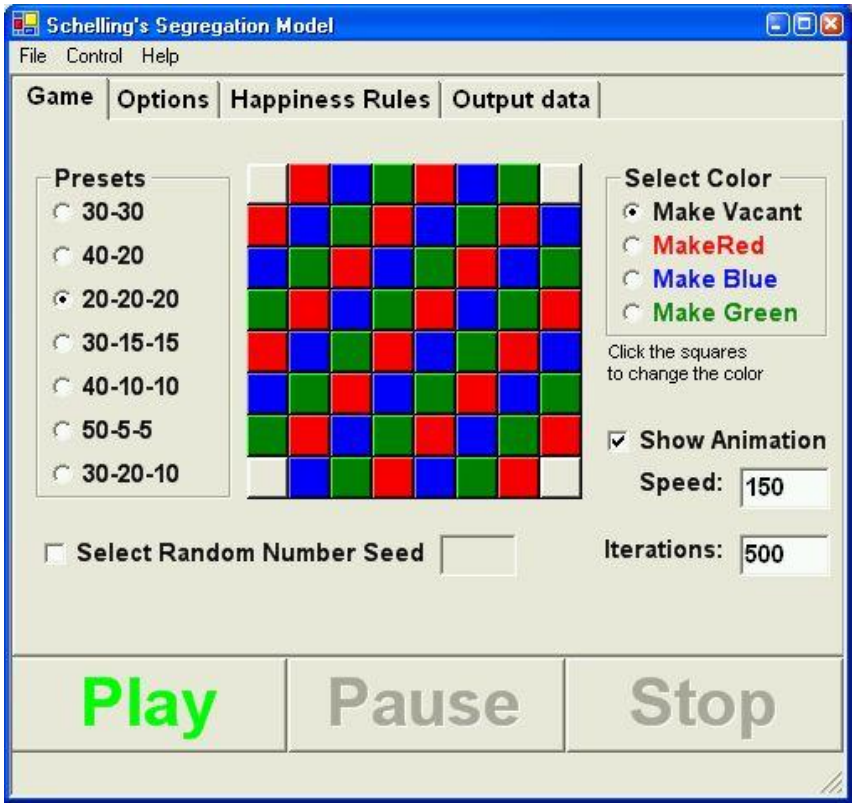
(c) Final pattern. The average share of racially similar neighbours is 99.7%. Everyone is satisfied

Final pattern, assuming a 40% tolerance level

Extended Schelling Segregation Demo

Basic Model by Thomas Schelling; Demo developed by Chris Cook

https://www2.econ.iastate.edu/tesfatsi/acedemos.htm

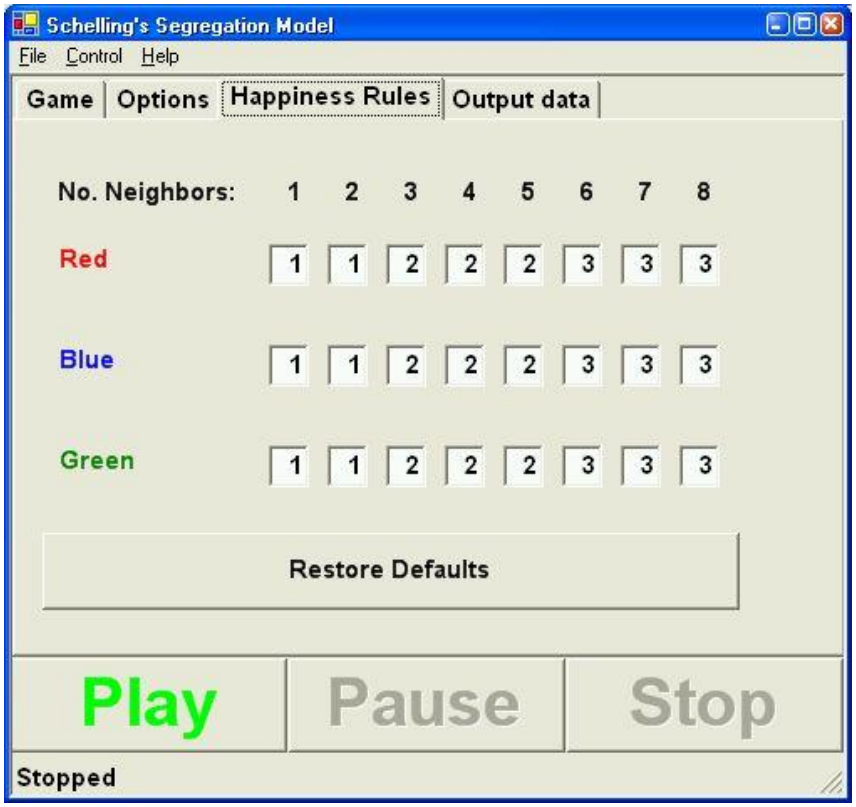


- Checkerboard city model
- Three classes of agents (red, blue, green) + vacant cells
- Agents satisfied with their location if "enough" of their neighbors are of their own type; otherwise, they move.

KEY FINDING: City can "tip" into a high segregation pattern even if agents have only mild preferences for living with agents of their own type!

Schelling Segregation Demo … Continued

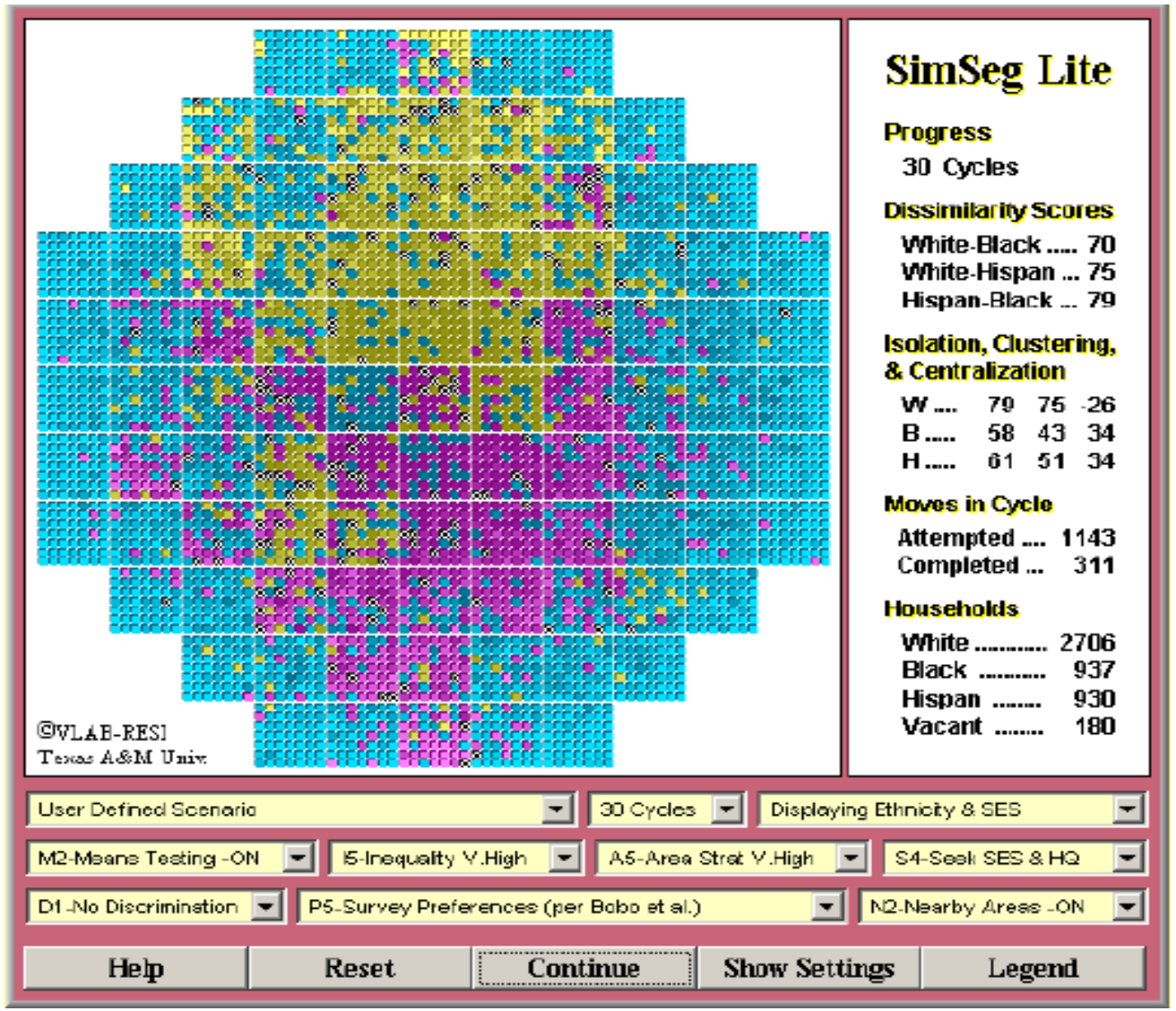
(Agent Happiness Rules)



- User specifies a happiness rule for each agent type
- Given n neighbors, how many of them must be the same type as me in order for me to be happy at my current location?
- Unhappy agents attempt to move to vacant spots at which they would be happy.
- Does this cause city to “tip” into a segregated pattern?

A More Elaborate Version of the Schelling Segregation Model

(Mark Fossett, Texas A&M, https://sociweb.tamu.edu/vlabresi/sslite3.htm)



Self-Organized Criticality and CAs

Per Bak et al. (1987) introduced the important concept of ***self-organized criticality (SOC)***.

Basic Idea: Systems are only able to sustain a limited amount of stress.

If stress exceeds ***locally*** a certain critical threshold, then…

- ➔ the system relaxes ***locally*** to an unstressed state
- ➔ the stress is ***distributed to a neighborhood*** of the original location ("chain reaction").

A Third Famous 2D CA: Per Bak's Sand Pile Model Illustrating SOC

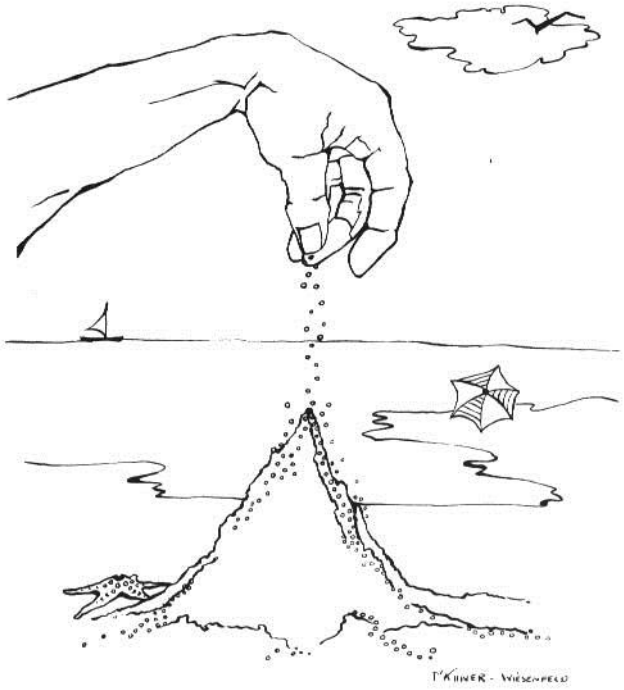
- Bak's most famous example of a system exhibiting Self-Organized Criticality (SOC) was a *sand pile.*

- The phenomenon of SOC was studied for sand piles both analytically, with 2D CA *sand pile models*, and experimentally with *real sand piles*.

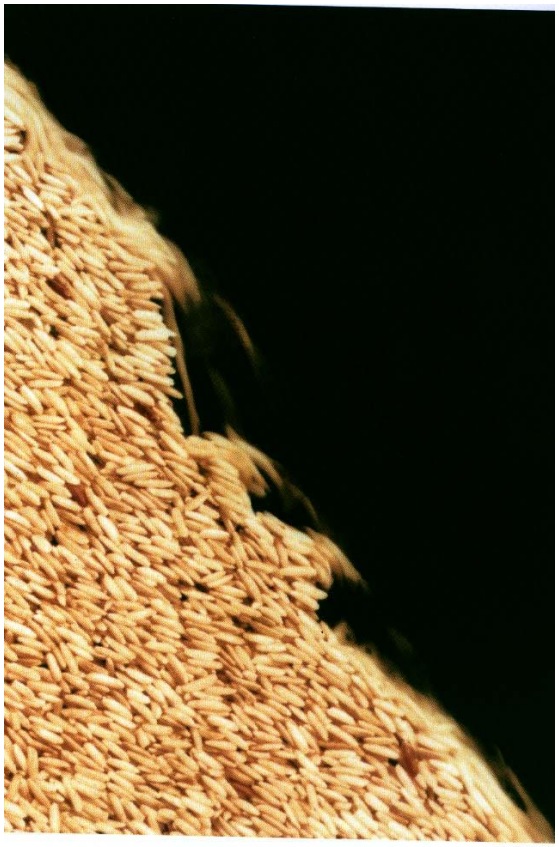
SOC and Sand Piles

- In the experimental case…
  - A sand pile is constructed by repeatedly dribbling grains of sand on an existing heap.
  - The experimenter then observes the size of the avalanches that are generated when the slope becomes unstable.

SOC and Sand Piles



Per Bak's Sand Pile



Power-law distributed avalanches in a rice pile

SOC and Sand Piles …

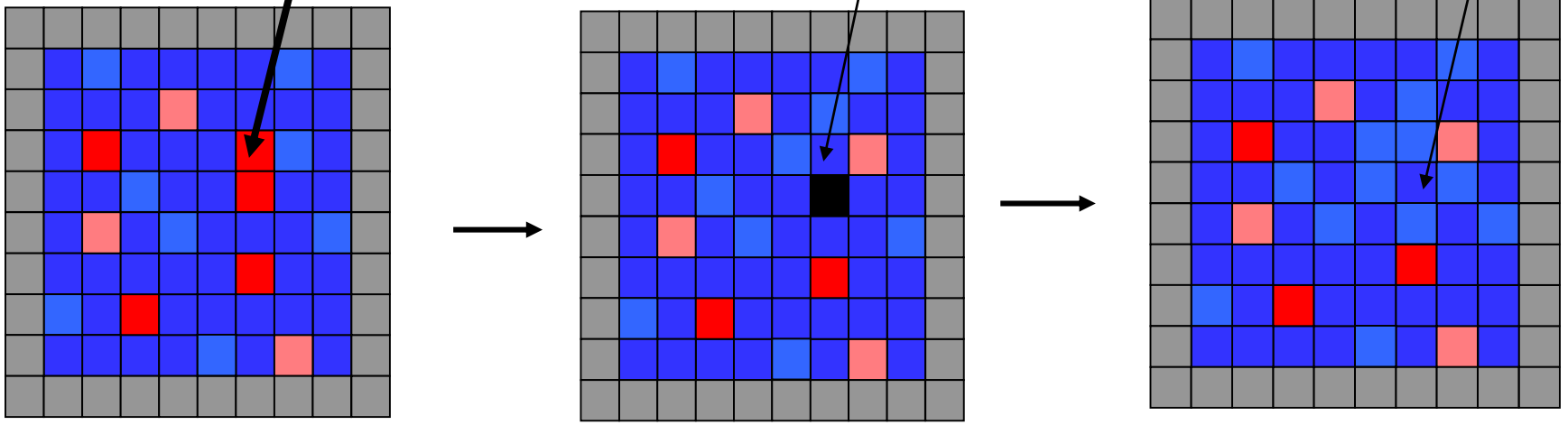
- **In the case where a 2D CA modeling is used …**
  - How should the “world” be represented?
  - How should “cells” be represented?
  - How should cell “rules of behaviour” be represented?

Nathan Winslow's 2D CA Approach (1997)

- Don't think of a grain of sand as the unit of analysis.
- Instead, let this unit be a **cell** = *a small square region on the table-top (world).*
- **G** = *state of a cell* = "average gradient" of the sand pile above this cell.
- **Cell Behavioral Rule**: Activate your neighbors by changing their G values, depending on your G value.

Example of Temporal Evolution in Sand Pile 2D CA

(Von Neumann Neighborhoods, Critical Value = 4)



G Value

Code 0 1 2 3 4 ■

Black=Critical
(Grains > 4)

Avalanche finished, Size = 8
(i.e., 8 Squares Change Color)

TYPICAL AVALANCHES

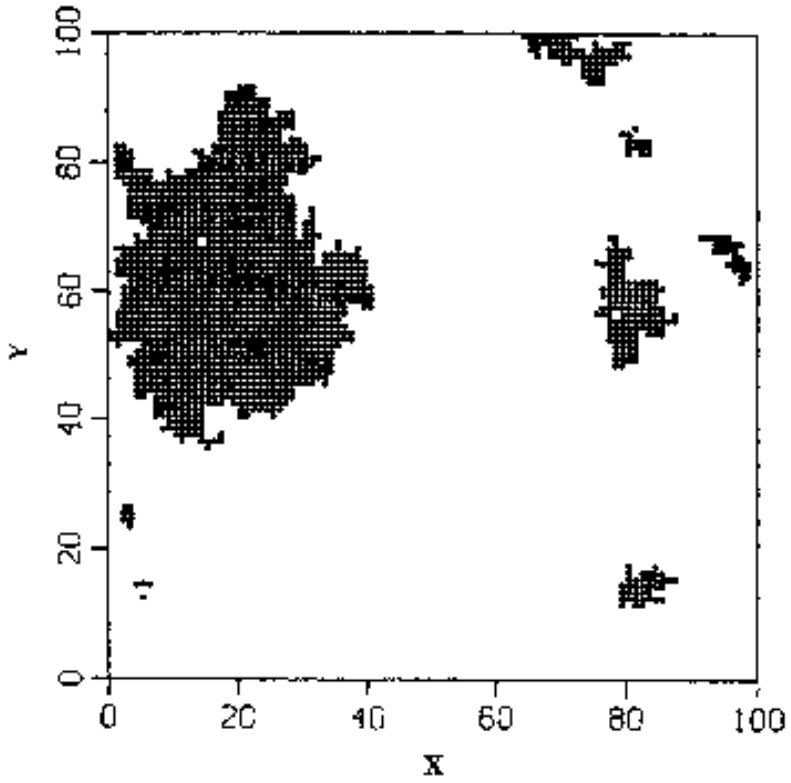
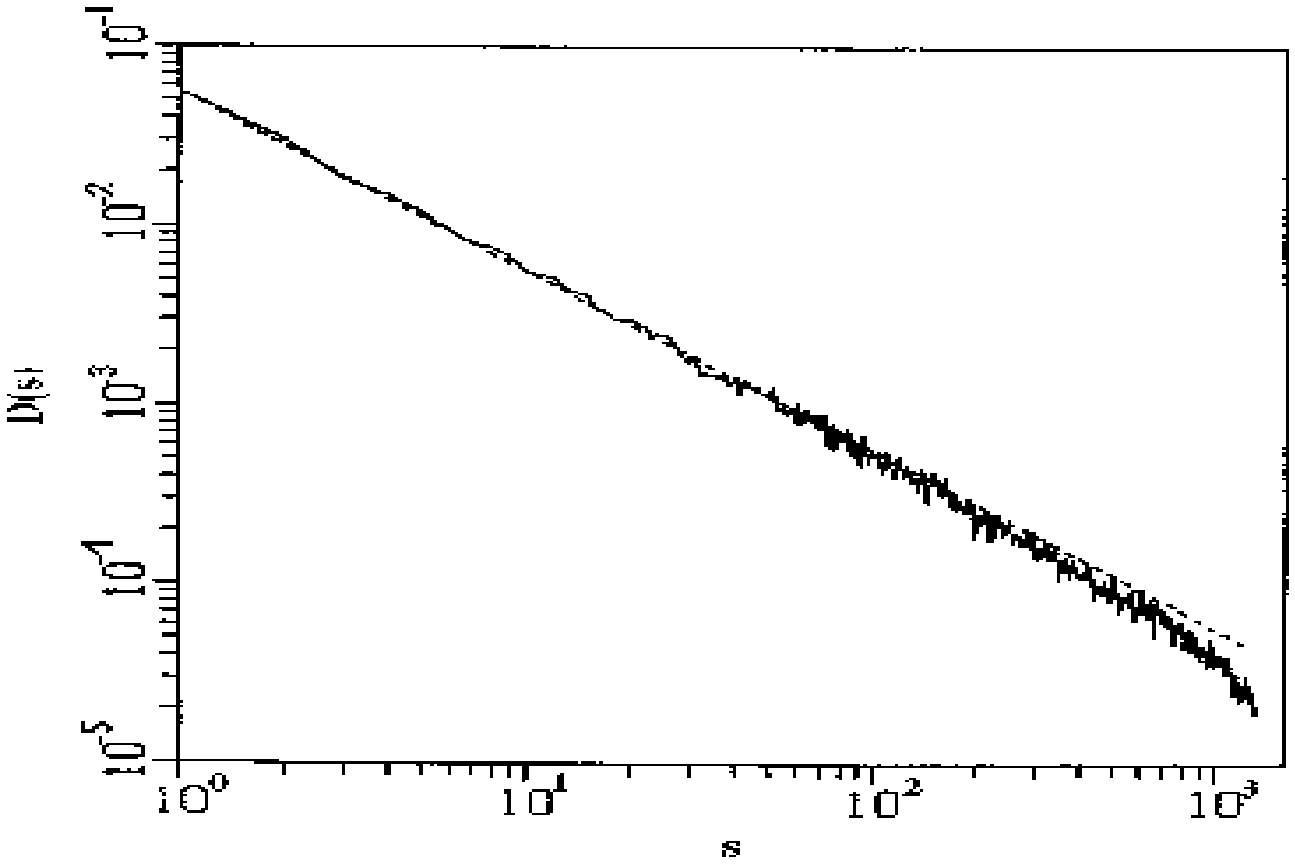


FIGURE 2

Aerial extent (domain) for several different avalanches in a SOC model. Each avalanche was triggered by the addition of a single grain. Avalanches have orders of magnitude difference in their sizes (Bak et al., 1987).

DOMAIN-FREQUENCY RELATION



Bak et al. (1987): Log-log plot of frequency of occurrence D(s) of avalanche of size s versus size s of avalanche for 200 avalanches. Avalanches exhibit a power law distribution ($D(s) \sim s^{-1}$).

Cellular Automata Classes

CAs can be divided into 4 classes (definitions by Stephen Wolfram)

- **Class 1 - point attractors.** CAs in this class eventually evolve to a homogeneous stationary arrangement, with every cell in the same state.

- **Class 2 - limit cycles.** CAs in this class form periodic structures that endlessly cycle through a fixed number of states.

- **Class 3 - chaotic.** CAs in this class form aperiodic random-like patterns that resemble static white noise on a bad T.V. channel, and are sensitive to initial conditions. ***NOTE:*** *All finite CAs eventually have to repeat themselves.*

- **Class 4 - structured.** CAs in this class (e.g., Game of Life) form complex patterns with localized structures that move through space over time. For finite CAs, the patterns eventually become homogeneous, as in Class 1, or periodic, as in Class 2.

Key References & Source Materials

- John Von Neumann (1951, 1966)
- Stanislaw Ulam (1950)
- John Conway (via Gardner, 1970)
- Stephen Wolfram (1982, 1983, 2002)
- William Poundstone, *The Recursive Universe*, Oxford U Press, 1985
- Primordial Soup kitchen, https://psoup.math.wisc.edu/kitchen.html
- https://www.math.com/students/wonders/life/life.html
- https://www.bitstorm.org/gameoflife/
- https://www.ibiblio.org/lifepatterns/