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Walras' Law in Overlapping Generations Economies*

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ABSTRACT

Aiyagari (1992) shows that equilibria are nonoptimal for an overlapping generations economy if and only if Walras' Law fails. We demonstrate that this failure represents an earnings opportunity exploitable by unsecured debt issue. When unsecured debt is issued, Walras' Law holds.

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1. The Issue

The significant implication of Walras' Law in economies with finitely many agents and good types is that, in value terms, an excess supply cannot exist for some subset of goods without an excess demand existing for some other subset of goods. Aiyagari (1992) defines the failure of Walras' Law as a situation in which this implication of Walras' Law does not hold. Using a version of the Samuelson (1958) overlapping generations (OG) economy, Aiyagari's basic and interesting result is to show that "a competitive equilibrium is nonoptimal if and only if the above implication of Walras' Law fails in its neighborhood."

Shouldn't we be skeptical, however, of a model in which positively valued excess supplies can occur in equilibrium? After all, where do the excess supplies go? Nonsatiated consumers would not simply throw the excesses away.

This letter shows that the positively valued excess supplies which Aiyagari connects with Pareto inefficiency represent an earnings opportunity that can be exploited through the issuance of unsecured debt. When unsecured debt is issued, Walras' Law does not fail in the sense described by Aiyagari. Nevertheless, Pareto efficiency is still not guaranteed unless, for example, the debt issue opportunity is optimally exploited by an earnings-driven agent.

2. An OG Economy With No Unsecured Debt Issue

Consider a pure exchange OG economy that begins in period 1 and extends into the infinite future. One perishable consumable resource exists, which is distinguished in period t as "good t." A single two-period lived young consumer is born at the beginning of each period t, endowed with $w^1 > 0$ units of good t and $w^2 > 0$ units of good $t+1$. His utility function $U(c_t^1, c_{t+1}^2)$ over consumption profiles (c_t^1, c_{t+1}^2) is assumed to be twice continuously differentiable, strictly quasi-concave, and strictly increasing, and to satisfy $U(c_t^1, 0) = U(0, c_{t+1}^2) = U(0,0)$ and $MRS(w^1, w^2) = U_1(w^1, w^2)/U_2(w^1, w^2) < 1$. It is also assumed that consumer preferences satisfy gross substitutability, implying that the optimal savings level of each young consumer is an increasing function of the rate at which he can exchange good in youth for good in old age.

The population of the economy in the initial period 1 consists of two agents: one generation 1 young consumer; and one generation 0 old consumer with a positive endowment w^2 who prefers more consumption to less and who dies at the end of period 1.

Intertemporal trades are facilitated by a price system $\mathbf{p} = (p_1, p_2, \ldots)$, where p_t denotes the price of good t in terms of a unit of account. Given this price system, the lifetime utility maximization problem faced by each generation t young consumer is to maximize $U(c_t^1, c_{t+1}^2)$ with respect to (c_t^1, c_{t+1}^2) subject to the budget and nonnegativity constraints

$$p_t c_t^1 + p_{t+1} c_{t+1}^2 \;=\; p_t w^1 + p_{t+1} w^2 \; ; \tag{1}$$

$$c_t^1 \geq 0, \quad c_{t+1}^2 \geq 0 \; . \tag{2}$$

Given the stated restrictions on consumer preferences, any solution to this utility maximization problem must satisfy

$$MRS(c_t^1, c_{t+1}^2) \;=\; p_t / p_{t+1} \; . \tag{3}$$

Finally, the consumption level of the generation 0 consumer is given by

$$p_1 c_1^2 = p_1 w^2. \tag{4}$$

Let $\mathbf{c} = (c_1^0, (c_1^1, c_2^1), (c_2^1, c_3^2), \ldots)$ denote an *allocation* for the economy. A nonnegative allocation $\mathbf{c}$ is *feasible* if and only if the market for good t clears in each period $t \geq 1$, in the sense that

$$w^1 + w^2 \;\geq\; c_t^1 + c_t^2 \; , \quad t \geq 1 \; . \tag{5}$$

Following Aiyagari (1992, Section 2.1), an *equilibrium* for the economy is an allocation $\mathbf{c} \geq 0$ and a price system $\mathbf{p} > 0$ that satisfy conditions (1) through (5).

For this model, there are an infinite number of equilibrium allocations and Walras' Law fails almost universally. To illustrate, consider only the stationary equilibria for which $p_t = (1/\rho)^t$ for some constant ρ satisfying $MRS(w^1, w^2) \leq \rho \leq 1$. The (gross) rate of return p_t/p_{t+1} then takes on the constant value ρ for all $t \geq 1$, which implies from (3) that each generation t young consumer consumes the same consumption profile $(c_t^1, c_{t+1}^2) = (c^1, c^2)$. Moreover, the lifetime budget constraint (1) reduces to $\rho c^1 + c^2 = \rho w^1 + w^2$ for each $t \geq 1$, implying that

$$w^1 + w^2 - c^1 - c^2 \;=\; [1 - \rho][w^1 - c^1] \; . \tag{6}$$

Three cases will now be considered: $\rho = MRS(w^1, w^2)$; $\rho = 1$; and $MRS(w^1, w^2) < \rho < 1$.

If $\rho = MRS(w^1, w^2)$, then $(c_t^1, c_{t+1}^2) = (w^1, w^2)$ for each $t \geq 1$, implying that the market clearing conditions (5) hold as equalities for $t \geq 2$. Since $p_1 > 0$, it follows from (4) that the generation 0 old consumer consumes his endowment (i.e., $c_1^2 = w^2$), hence market clearing also holds as an equality for $t = 1$. Consequently, no excess supply exists in this autarkic (no trade) equilibrium.

If $\rho = 1$, then (c_t^1, c_{t+1}^2) equals the golden rule consumption profile $(\bar{c}^1, \bar{c}^2)$ for each $t \geq 1$, and condition (6) implies that the market clearing conditions (5) hold as equalities for each $t \geq 2$. However, in period 1, the old consumer consumes his endowment (i.e., $c_1^2 = w^2$) while the restrictions on $U(\cdot)$ guarantee that the young consumer consumes less than his endowment (i.e., $c_1^1 = \bar{c}^1 < w^1$). Thus, there is an excess supply of good 1 in period 1 (i.e., $\bar{c}^1 + w^2 < w^1 + w^2$). Since $p_1 > 0$, this excess supply is positively valued, meaning Walras' Law fails for this equilibrium.

If $\rho = \hat{\rho}$, where $MRS(w^1, w^2) < \hat{\rho} < 1$, then $(c_t^1, c_{t+1}^2) = (\hat{c}^1, \hat{c}^2)$ for each $t \geq 1$, where $\hat{c}^1 < w^1$. Together with condition (6), this implies that an excess supply of good t exists in each period $t \geq 2$. In period 1, the old consumer consumes his endowment (i.e., $c_1^2 = w^2$) while the young consumer consumes less than his endowment (i.e., $c_1^1 = \hat{c}^1$). Thus, an excess supply of good 1 exists in period 1 (i.e., $\hat{c}^1 + w^2 < w^1 + w^2$). Since $p_t > 0$ for each $t \geq 1$, the excess supply present in each period $t \geq 1$ is positively valued, meaning Walras' Law fails for this equilibrium.

More generally, it can be shown that a positively valued excess supply exists in *every* equilibrium except the autarkic equilibrium. The results presented here for stationary equilibria are nevertheless sufficient to demonstrate the meaning of Aiyagari's assertion that Walras' Law fails for his OG model.

To obtain a clearer understanding of the failure of Walras' Law for the economy at hand, sum the budget constraints of all consumers through any generation $T \geq 1$ to obtain

$$0 \;=\; \sum_{t=1}^{T} p_t[w^1 + w^2 - c_t^1 - c_t^2] \;+\; p_{T+1}[w^2 - c_{T+1}^2] \; . \tag{7}$$

Combining (5) and (7), one sees that a positively valued excess supply can exist in net terms for goods $t \leq T$ if and only if $0 < p_{T+1}[c_{T+1}^2 - w^2]$. The latter term represents the value that the generation T young consumer effectively transfers from period T to period $T+1$, allowing his old-age consumption in period $T+1$ to exceed his period $T+1$ endowment.

In the present economy, trade is mediated by a price system. The precise form of the intermediation process is not explicitly articulated. Suppose an intermediary is explicitly introduced in the form of a central clearing house. The assets of the clearing house at the end of any period T would then consist of any excess supplies held after the completion of all trades over periods $t \leq T$. Offsetting these assets would be the liability $p_{T+1}[w^2 - c_{T+1}^2]$ obtained from condition (7). These observations suggest that the failure of Walras' Law is due to the implicit existence of an asset—specifically, a quantity of unsecured debt—that is not explicitly recognized in the structural equations for the economy.

3 An OG Economy With Unsecured Debt Issue

Suppose the economy described in Section 2 is now modified by allowing the generation 0 old consumer to issue an amount D_0 of unsecured debt, i.e., bonds (promises to pay) that are not backed by any real collateral. Let this unsecured debt be taken as the unit of account, so that p_t now denotes the number of units of unsecured debt necessary to buy one unit of good t, $t \geq 1$. Under these assumptions, the budget constraint of the generation 0 old consumer becomes

$$c_1^2 = w^2 + \frac{D_0}{p_1} \ . \tag{8}$$

Young consumers in generations $t \geq 1$ are allowed to purchase debt and to resell it in their old age. Debt can also be sold short in youth, allowing young consumers to borrow. No other intermediation options are available. Under these assumptions, the lifetime utility maximization problem faced by the generation t young consumer is to maximize $U(c_t^1, c_{t+1}^2)$ with respect to (c_t^1, c_{t+1}^2, D_t) subject to the budget and nonnegativity constraints

$$c_t^1 \;=\; w^1 - \frac{D_t}{p_t} \ ; \tag{9}$$

$$c_{t+1}^2 \;=\; w^2 + \frac{D_t}{p_{t+1}} \ ; \tag{10}$$

$$c_t^1 \geq 0, \quad c_{t+1}^2 \geq 0 \ . \tag{11}$$

Let $\mathbf{D} = (D_0, D_1, D_2, \ldots)$ denote the sequence of unsecured debt holdings for the economy. An *equilibrium* for the economy is then a triplet $(\mathbf{c}, \mathbf{p}, \mathbf{D})$ consisting of an allocation $\mathbf{c} \geq 0$, a price system $\mathbf{p} > 0$, and a debt sequence $\mathbf{D}$ that satisfy conditions (3), (5), (8), (9), (10), and (11), together with the following market clearing condition for the unsecured debt:

$$D_{t-1} \;\geq\; D_t \ \ \text{for all} \ \ t \geq 1 \ . \tag{12}$$

The young age and old age budget constraints (9) and (10) together generate the lifetime budget constraint (1). Thus, the economy presented here differs in only two essential ways from the economy presented in Section 2. First, the generation 0 old consumer can here receive a wealth windfall from the issuance of unsecured debt, whereas no such windfall was previously possible. Second, the medium of exchange is here explicitly identified as unsecured debt, whereas the medium of exchange was not previously specified.

Although Walras' Law was shown to fail for the economy without unsecured debt, it cannot fail for the present economy. To understand this, combine the young age budget constraint for generation t with the old age budget constraint for generation $t-1$ to obtain

$$0 \; = \; p_t[w^1 + w^2 - c_t^1 - c_t^2] \; + \; [D_{t-1} - D_t] \; , \quad t \geq 1 \; . \tag{13}$$

Using (5), a positively valued excess supply of good t implies an excess demand for unsecured debt in period t, a violation of the market clearing conditions (12). Thus, no positively valued excess supply of any good $t \geq 1$ can exist in equilibrium, meaning Walras Law does not fail in the sense of Aiyagari (1992). Also, $[D_{t-1} - D_t] = 0$ must hold for each $t \geq 1$.

Summing the budget constraints of all consumers over generations $t \leq T$, one obtains

$$0 \; = \; \sum_{t=1}^{T} p_t[w^1 + w^2 - c_t^1 - c_t^2] \; + \; D_0 \; - \; p_{T+1}[c_{T+1}^2 - w^2] \; . \tag{14}$$

Since $p_t[w^1 + w^2 - c_t^1 - c_t^2] = 0$ and $D_t = D_{t-1}$ must hold for each $t \geq 1$, condition (14) implies $D_0 = D_T = p_{T+1}[c_{T+1}^2 - w^2]$. The asset value D_0 obtained in period 1 by the institution offering the initial debt issue is thus exactly offset by an associated liability D_T that is transferred forward into the infinite future. In particular, it is the ability to roll over debt into the infinite future that permits the initial issuer of unsecured debt to obtain a positive windfall return D_0. If the infinite time horizon were truncated at the end of period T, the term $p_{T+1}[c_{T+1}^2 - w^2]$ would not appear in condition (14) and $D_0 = 0$ would have to hold.

It is also of interest to note that *none* of the stationary equilibria for Aiyagari's economy is Pareto efficient; in each case the generation 0 old consumer can be made better off without making any other consumer worse off. While the introduction of unsecured debt restores Walras' Law in (13), it does not restore the First Welfare Theorem, nor does it reduce the model's multiplicity of equilibria to a single, determinate equilibrium. As is known from Gale (1973), Pareto efficiency for the present economy depends upon the real value of the period 1 unsecured debt. If the period 1 price p_1 is such that the initial real debt level is given by

$D_0/p_1 = [\bar{c}^2 - w^2]$, then the economy has a unique stationary Pareto efficient equilibrium allocation, and the rate of return in each period $t \geq 1$ is given by $\rho = 1$. All other feasible values for p_1 generate Pareto inefficient equilibria. The value of p_1 is thus indeterminate, meaning Pareto efficiency is not guaranteed.

4. Conclusion

We have illustrated that Walras' Law does not fail in the sense of Aiyagari (1992) when a fixed quantity of unsecured debt is explicitly introduced into the economy. Nevertheless, a Pareto efficient outcome is still not guaranteed. In Pingle and Tesfatsion (1991,1997) it is shown that introducing a private earnings-driven intermediary who is able to issue unsecured debt can ensure a Pareto efficient outcome.

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