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Course Site for Econ 502 (M.S. Macro Theory)
https://www2.econ.iastate.edu/tesfatsi/Syl502.htm

Link for these notes:
https://www2.econ.iastate.edu/tesfatsi/nonwalra.pdf

NON-WALRASIAN EQUILIBRIUM: ILLUSTRATIVE EXAMPLES

1. Introduction

In previous lectures it was seen that market clearing constitutes an essential part of the definition of a Walrasian equilibrium. In particular, assuming a positive wage rate, the demand for labor must equal the supply of labor in any Walrasian equilibrium. But must markets clear in order for economies to be in equilibrium, in the sense of an unchanging situation (rest point)? In particular, can an economy become stuck at a point where positive "involuntary" unemployment persists? Or do market clearing situations (such as Walrasian equilibria) constitute "stable attractors" of decentralized market economies in the sense that, assuming flexible prices, and absent government intervention, such economies tend to converge over time to points where all markets clear?

This issue was at the heart of the debate between new classical and new Keynesian economists in the 1980s and 1990s; see King (1993) and Mankiw (1993). In general, new classical economists espoused the view that decentralized market economies are inherently stable, tending naturally towards an equilibrium state in the sense of Walras, whereas new Keynesian economists argued this was not the case. More precisely, new Keynesians argued that a decentralized market economy might or might not tend towards an equilibrium state depending on its supporting institutional structure and circumstances. Moreover, the meaning of "equilibrium state" did not presuppose any optimality or efficiency properties. Rather, roughly summarized, new Keynesians defined an economy to be in an *equilibrium* *state* if those with the *incentive* to change this state have no power to do so and those who have the *power* to change this state have no incentive to do so.

New Keynesians also stressed that, in general, a given economy can have multiple possible equilibrium states, some more socially desirable than others. A key factor affecting the ability of macroeconomies to coordinate on a socially desirable equilibrium state is whether agents are credibly able to signal their intended actions to each other.

More precisely, if other market participants fail to signal (communicate) to agent A in a credible (believable) way what actions they intend to take, how can agent A rationally take these intended actions into account in planning his own actions? And if agent A cannot take these intended actions into account in his planned actions, what guarantees that his actions will be coordinated with the actions of these other market participants? For example, if agent A is a producer, and consumers do not credibly signal to him today their intended future purchases of his goods, how can he take these intended purchases into account when planning today for future production? And if he cannot take these intended purchases into account, what ensures that future demand for his goods will equal future supply?

In summary, new Keynesians identified two basic types of signalling problems that can give rise to coordination problems among economic agents:

Problem 1: Incomplete Signalling

> In new classical models, price-taking consumers and producers in the initial period are assumed to choose complete lifetime consumption and production plans for current and future periods, and prices are assumed to be at levels where all of these consumption and production plans are consistent. Thus, all spot and futures markets clear. In contrast, real-world consumers typically choose consumption levels and savings (e.g. money holdings) today without yet having fully decided how their savings will be spent in the future on goods and services.

Consequently, producers do not receive complete signals about future consumption plans and must make production and investment decisions today that might prove to be wrong tomorrow.

Problem 2: Signalling Not Credible

Even if consumers signal to producers today their intentions to consume certain goods and services in the future, these signals will not be credible to the producers unless they are backed today by actual purchasing power.

Before the 2007-2009 financial crisis, mainstream macroeconomic theory seems to have been moving towards the adoption of an alternative paradigm, the *New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) Model*, that synthesizes aspects of new classical and new Keynesian thinking; see Colander (2006). As exemplified by the well-known model developed by Smets-Wouters (2003), NK-DSGE models typically permit some degree of heterogeneity among consumer-workers, and incorporate various types of market frictions and random shocks to the economy. However, rational expectations and complete market equilibrium are still generally assumed to hold at each point in time. Consequently, these models are subject to many of the same criticisms as new classical macroeconomic models in terms of ignoring fundamental coordination issues arising from incomplete and/or incredible signalling.

This section begins by discussing the meaning of involuntary unemployment and the concepts of notional versus effective demands and supplies. Several simple examples are then given that illustrate the argument that decentralized market economies are inherently subject to signalling problems that can result in persistent coordination failure – in particular, persistent involuntary unemployment. Economists adhering to this view argue that public institutions and policies should be carefully designed to prevent or at least alleviate these coordination problems.

2. Involuntary Unemployment - Redux

As discussed in previous packet readings, the definition for involuntary unemployment currently found in standard intermediate macro texts is roughly as follows: People are said to be *involuntarily unemployed* when they desire to work at the wage rate being paid to currently employed workers, they are as qualified to work as currently employed workers, and yet they cannot obtain employment. In his famous book *General Theory*, Keynes (1965 edition, p. 15) attempted to provide an operational test for this conception of involuntary unemployment.

> "Men are *involuntarily unemployed* if, in the event of a small rise in the price of wage goods relatively to the money wage, both the aggregate supply of labor willing to work at that wage and the aggregate demand for it at that wage would be greater than the existing amount of employment."

The conventional interpretation of Keynes' test for involuntary unemployment is that the current real wage rate is "too high" relative to the market clearing rate where demand equals supply, implying that firms succeed in satisfying their demand for labor at the current real wage rate but workers are not on their labor supply curves. Consequently, a small decrease in the real wage rate $w = W/P$ (i.e., a small rise in the price of wage goods P relative to the money wage W) would result in an increase both in the amount of labor demanded by firms and in the actual amount of labor supplied; see Fig. 1.

In the more recent macroeconomic theory literature, this practical "real-world" conception of involuntary unemployment has been generalized to include the consideration of other possible equilibria in which employment is higher than in the current situation. Consider, for example, the following definition paraphrased from Drazen (1980, p. 285):

> An individual is *involuntarily unemployed* if, at current market wages and prices, he would be willing to supply more labor but perceives he is facing a constraint

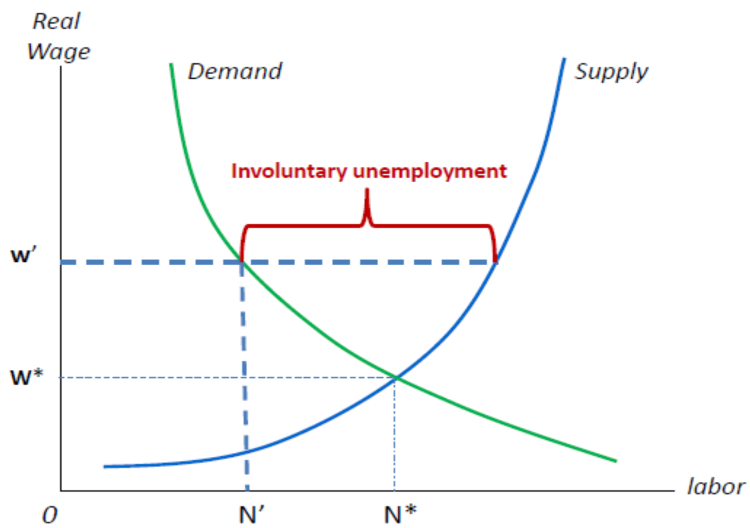

Figure 1: Keynesian involuntary unemployment: Excess labor supply at a real wage w' higher than the market-clearing real wage w^*

> that prevents him from doing so, and if, for the same initial endowments, tastes, and technology, there exists an alternative equilibrium where in fact he would be both willing and able to supply more labor.

A basic question that has been addressed by Keynesian economists since the *General Theory* is whether involuntary unemployment can rationally persist in the following sense: agents who perceive a benefit from reducing involuntary unemployment have no power to do so, and agents who do have power to reduce involuntary unemployment do not perceive any benefit from doing so.[1] An economy in such a situation is said to be in an *(involuntary) unemployment equilibrium*.

Economists adhering to the New Keynesian perspective typically accept unemployment equilibrium is a theoretical possibility, but disagreement persists concerning the exact reasons for its occurrence and the proper government response to such an occurrence. In contrast,

[1]Note this is a strengthening of the game-theoretic conception of a Nash equilibrium since it considers the power and incentives of coalitions of agents, not just individual agents. It is thus more in keeping with the concept of "core stability." See any basic book on game theory for a discussion of Nash equilibrium and core stability.

economists adhering to the classical perspective assume continuous labor market clearing. Indeed, Lucas (1983) has argued that all unemployment is voluntary in the following sense: every person who seeks a particular type of job is voluntarily engaging in a lottery with a calculable risk that the job either will not be found or, if found, will be lost through firing or lay-off after a period of time. Consequently, involuntary unemployment is not a well-defined concept, and macroeconomists would do better to focus instead on the level of *employment*.

In the following sections, a variety of examples are developed that purport to show how unemployment equilibria can arise and persist due to various types of signalling problems. In each of these examples, the involuntary unemployment in question takes the "other possible equilibrium" form along the lines of the definition presented above from Drazen (1980).

3. Non-Walrasian Equilibrium: A Schematic Illustration

A simple example of a non-Walrasian equilibrium will now be sketched that illustrates the potentially important role of signaling through effective supplies and demands in decentralized market economies. This example is a modification of a game model due to John Roberts (1987).

Consider the market economy depicted in Fig. 2. This economy includes: (a) two firms, F_1 and F_2, which produce consumable goods Q_1 and Q_2, respectively; (b) two consumers C_1 and C_2; and (c) a bank B holding a small amount of retained earnings from firm F_1.

Firm F_2, consumer C_1, and consumer C_2 have no retained earnings (savings) from previous periods. Consumer C_1 is endowed with skilled labor L_1 for the production of good Q_1, but C_1 only obtains utility from the consumption of good Q_2. Conversely, consumer C_2 is endowed with skilled labor L_2 for the production of good Q_2, but C_2 only obtains utility from the consumption of good Q_1. In a simple stylized fashion, these restrictions reflect the basic real-world fact that a firm's goods are generally bought by agents other than its own

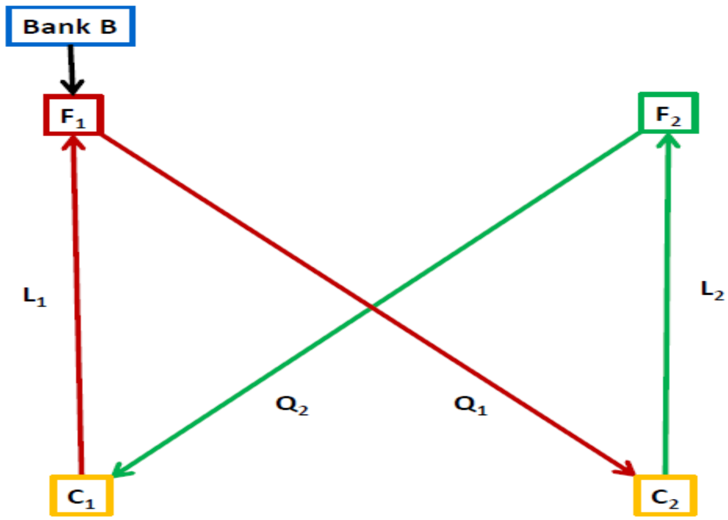

Figure 2: A simple market economy with multiple boom/bust outcomes depending on the prior expectations of the market participants

workers, so that its wage payments to its workers do not translate directly into a demand for its goods. Consequently, in contradiction to "Say's Law," supply does not necessarily create its own demand.

For simplicity, assume bank B only deals with firm F_1, and not with Firm F_2 or with any consumer, e.g., because the latter agents—being without acceptable collateral—are viewed as high credit risks. In particular, it will be assumed that firm F_1 has the option of requesting a line of credit from the bank B to finance the hiring of labor [i.e., to finance a "wage fund"] for production of Q_1, which requires more expenditure than permitted by firm F_1's retained earnings alone. Note the important role this assumption plays in the scenarios, below.

Scenario 1: Full Employment Boom

Firm F_1 anticipates good times ahead—in particular, firm F_1 anticipates that consumer C_2 will demand Q_1 from F_1—and the bank B concurs in this optimistic expectation. The bank extends credit to firm F_1 that allows firm F_1 to hire L_1 from C_1 to produce Q_1. This gives C_1 the income he needs to make effective his demand for Q_2 from firm F_2. Firm F_2 then hires L_2 from consumer C_2 to produce Q_2, which gives C_2 the income he needs to make effective his demand for Q_1 from firm F_1. Firm F_1 is then able to pay back his loan from bank B.

Bottom Line: All labor resources are fully employed, effective demand equals effective supply in each market, and the optimistic expectations of the firm F_1 and the bank B are fulfilled.

Scenario 2: Unemployment Bust: Pessimistic Firm Expectations

Firm F_1 anticipates bad times ahead—in particular, firm F_1 anticipates that consumer C_2 will NOT demand any Q_1 from F_1. Whether the bank B concurs or not in these pessimistic expectations is irrelevant. Since firm F_1 does not plan to hire any labor L_1 from C_1, firm F_1 does not request a line of credit from the bank. Consequently, C_1 does not have the income he needs to make effective his demand for Q_2 from firm F_2. Firm F_2 thus does not hire any L_2 from consumer C_2 to produce Q_2, implying that C_2 then does not have the income he needs to make effective his demand for Q_1 from firm F_1.

Bottom Line: The labor resources L_1 and L_2 are unemployed, effective demand equals effective supply in each market (here all effective demands and supplies are simply zero), and firm expectations are fulfilled. [Question: Is this unemployment "involuntary"?]

Scenario 3: Unemployment Bust: Pessimistic Bank Expectations (Credit Crunch)

The bank B anticipates bad times ahead, hence the bank will not offer firm F_1 a line of credit even if firm F_1 requests it. The scenario then plays out exactly as for scenario 2.

The interesting aspect of this simple sketch of an economy with multiple equilibria is that technology, preferences, and endowments are the *same* in each equilibrium. Only the expectations of firm F_1 or the bank B differ across the equilibria. On the one hand, given optimistic expectations *on the part of both the bank B and firm F_1*, a "full employment equilibrium" is achieved. On the other hand, given pessimistic expectations *either* on the part of firm F_1 *or* on the part of the bank, the result is an "unemployment equilibrium."

Note, in particular, that to ensure the full employment boom scenario, it is not simply enough that firm F_1 has access to a line of credit, and hence purchasing power for hiring labor. Firm F_1 must also be *optimistic* that the good it produces with this labor will be sold.

4. Clower's Concept of Effective Demands and Supplies

In a now-classic paper, Robert Clower (1965) sets out another possible scenario for unemployment equilibrium. Clower focuses on the difficulties faced by consumers when they attempt to signal their demands for goods and services to other agents in the market. He argues that the only demands that can be credibly signalled are those backed by actual purchasing power. That is, a consumer can only make effective his demands for consumption goods if these demands are backed by income or credit *in hand*. Moreover, in order for coordination of demands and supplies to be achieved, consumers must signal now their intentions to buy goods and services in future periods as well, where these intentions are backed by purchasing power in hand. If consumers instead simply retain their savings in the form of money or bonds without signalling the planned use of these savings for future consumption purchases, how will firms be able to plan ahead to meet the future demand for their goods and services?

Clower argues, however, that real-world consumers are often unable to make effective their current and future demands for goods and services. For example, due to imperfections in capital markets, consumers may be unable to borrow against future income to finance their desired purchases today. Consequently, in contrast to Walrasian models in which consumers operating in perfect capital markets simultaneously determine their lifetime demands for goods and services along with their planned lifetime labor supplies (hence incomes), consumers may in fact face liquidity constraints. This can prevent the attainment of a Walrasian equilibrium because the demands that arise under Walrasian assumptions cannot actually be credibly signalled to other market participants.

More precisely, consider a consumer residing in an economy in some period T, and let $s(T)$ denote the state of the consumer at the beginning of period T consisting of a description of her tastes and resource endowments. Following Clower (1965), the consumer's demand and supply functions for period T are said to be *notional* if the following conditions hold:

(a) The demand and supply functions are determined taking only (expected) prices and dividends as given, together with $s(T)$.

(b) Given (expected) prices and dividends, the consumer's planned total purchases are less than or equal in value to her planned and/or anticipated income receipts.

(c) The consumer believes she can buy or sell all she wants to at the given (expected) prices, subject to her budget constraint in (b).

In contrast, again following Clower (1965), the consumer's demand and supply functions for period T are said to be *effective* if they are backed by actual purchasing power — that is, by income or credit determined prior to the construction of these demand and supply functions — so that the latter functions are constrained by quantity (real income) and not just by relative prices and dividend payments.

To illustrate the difference between notional and effective demands and supplies in more concrete terms, suppose the economy has only two goods, a commodity good C and leisure Le, and that the time period T under consideration is one day. Then $\bar{L} = 24$ is the maximum amount of labor (measured in person-hours) that the consumer can supply in period T, based purely on physical feasibility considerations. Let P denote the nominal price of the commodity good C in period T, and let $w = W/P$ and $d = D/P$ denote the real wage rate (the price of leisure) and the real dividend payment that the consumer expects to receive in period T.

Suppose, first, that the consumer chooses a commodity good demand c^d and a leisure demand le^d for period T to maximize her utility $U(c^d, le^d)$ subject to a budget constraint, taking prices and dividends as given. More precisely, suppose she solves a "Walrasian" utility maximization problem of the form

$$\max_{c^d, le^d} U(c^d, le^d) \tag{1}$$

subject to the budget and nonnegativity constraints

$$\begin{aligned} c^d + w le^d \ &\leq \ w\bar{L} + d \ ; \\ 0 \leq c^d \ ; \\ 0 \leq le^d \leq \bar{L} \ . \end{aligned}$$

The solution for problem (1) yields the consumer's *notional* demand functions for C and Le. By construction, these demand functions take the form

$$c^{d*} = c^d(w, d, s(T)) \ ; \tag{2}$$

$$le^{d*} = le^d(w, d, s(T)) \ , \tag{3}$$

where the state $s(T) = (U(\cdot), \bar{L})$ of the consumer at the beginning of period T comprises her tastes $U(\cdot)$ and her labor endowment $\bar{L}$. Note that, apart from private state information, these demand functions depend only on relative prices and dividend payments.

After figuring out her notional demand functions for C and Le, suppose the consumer suddenly learns from her prospective employer that the maximum amount of labor she will actually be allowed to supply in period T is an amount $\bar{l}$ that is strictly less than her notional labor supply $\bar{L} - le^{d*}$, where le^{d*} is determined as in (3). Thus, the consumer is forced to re-optimize by solving the revised utility maximization problem

$$\max_{c^d, le^d} U(c^d, le^d) \tag{4}$$

subject to the budget, quantity, and nonnegativity constraints

$$\begin{aligned} c^d \ \le \ w[\bar{L} - le^d] + d \ \le \ w\bar{l} + d \ \ ; \\ 0 \le c^d \ ; \\ le^d \ \le \bar{L} \ . \end{aligned}$$

The solution to problem (4) yields the consumer's *effective* demand functions for C and Le. These effective demand functions take the quantity-constrained form

$$c^{Ed} = c^{Ed}(\bar{l},\ w,\ d,\ s(T)) \ ; \tag{5}$$

$$le^{Ed} = le^{Ed}(\bar{l},\ w,\ d,\ s(T)) \ , \tag{6}$$

where again $s(T) = (U(\cdot), \bar{L})$. Letting $\bar{y} = w\bar{l} + d$ denote the consumer's maximum possible real income in period T, note that an alternative way to express these effective demand functions is as follows:

$$c^{Ed} = c^{Ed}(\bar{y},\ w,\ d,\ s(T)) \ ; \tag{7}$$

$$le^{Ed} = le^{Ed}(\bar{y},\ w,\ d,\ s(T)) \ . \tag{8}$$

The effective consumption demand function (7) is similar in form to the Keynesian absolute income consumption function, $c = c(y, ...)$. In contrast, the notional consumption demand function (2) depicts consumption demand as a function only of relative prices and dividend payments, not income. Clower (1965) stresses this point, arguing that Keynes' must have had in mind that consumers face some type of quantity-constrained optimization problem of the form (4) rather than a simultaneous Walrasian optimization problem of the form (1).

Clower's argument that liquidity and credit constraints can lead to persistent involuntary unemployment due to signalling problems has been criticized on the grounds that he exaggerates the effects of liquidity and credit constraints on consumers and producers as a whole. Econometric evidence supports the existence of some binding liquidity constraints, but the wide-spread availability of household credit cards and lines of credit to firms, even in recessionary times, undermines Clower's arguments. However, when Clower's insights about effective versus notional demands and supplies are combined with possible *expectational* problems, one obtains examples of unemployment equilibrium that appear to be more robust to these criticisms. Several of these examples are sketched below.

5. Non-Walrasian Equilibrium with Effective Market Clearing

A more fully articulated example will now be given of an economy that has multiple equilibria: (i) a Walrasian equilibrium, with market clearing defined in terms of notional demands and supplies; and (ii) a non-Walrasian equilibrium with effective (but not notional) market clearing that arises because of pessimistic real wage expectations on the part of the firm sector.

Suppose an economy in some period T has one produced consumption good, C. The economy consists of two private agents: (i) a utility-maximizing consumer with an exogenously given labor endowment $\bar{L}$ and a strictly increasing and strictly concave utility function $U(c, le)$ defined over consumption good amounts c and leisure amounts le; and (ii) a profit-maximizing firm that produces consumption good amounts via a strictly increasing and strictly concave production function $c = F(l)$ that uses labor services l as its only input. The state of the economy at the beginning of period T, consisting of a description of exogenously given tastes (preferences), technology, and resource endowments, thus takes the form

$$s^o \;=\; (U(\cdot), F(\cdot), \bar{L}) \;. \tag{9}$$

Consider, first, the completion of this economy in accordance with standard Walrasian modelling assumptions. A hallmark of Walrasian modelling is that all private agents are assumed to be price takers. Moreover, consumers are assumed to own shares in each firm, and firms are assumed to distribute all profits back to consumers in proportion to their share ownership. Since the economy under consideration has only one consumer and only one firm, it will be supposed that the consumer and firm are both price takers, that the consumer is the sole owner of the firm, and that the firm distributes all profits back to the consumer-owner.

Given these additional assumptions, the standard definition of a Walrasian equilibrium reduces for the economy at hand to three conceptually distinct types of requirements: (a) consumer optimization taking expected prices and dividends as given, and firm optimization taking expected prices as given; (b) fulfilled price and dividend expectations; and (c) market clearing.

Let c^d denote the consumer's demand for good C, and let l^s denote her supply of labor. Also, let $w^e \equiv [W/P]^e$ denote the real wage the consumer expects to receive per unit of her labor supplied, and let $d^e \equiv [D/P]^e$ denote the real dividend she expects to receive from her ownership of the firm. [The symbols W, P, and D denote the wage rate, price of good C, and dividend measured in terms of some unit of account – e.g., fiat money – and the qualifier "real" denotes measurement in terms of good C.] The utility maximization problem faced by the consumer then takes the form

$$\max U(c^d, \bar{L} - l^s) \tag{10}$$

with respect to c^d and l^s subject to the budget and physical feasibility constraints

$$\begin{aligned} c^d &\leq w^e l^s + d^e \ ; \\ 0 &\leq c^d \ ; \\ 0 &\leq l^s \leq \bar{L} \ . \end{aligned}$$

By construction, the solution to problem (10) gives the consumer's *notional* demands and supplies for good C and labor l, of the form

$$c^d = c^d(w^e, d^e; s^o) \ ; \tag{11}$$

$$l^s \quad = \quad l^s(w^e, d^e; s^o) \ . \tag{12}$$

Let c^s denote the firm's supply of good C and let l^d denote the firm's demand for labor. For simplicity, suppose the firm expects the same real wage as the consumer. Then the profit-maximization problem faced by the firm takes the form

$$\max \ [c^s - w^e l^d] \tag{13}$$

with respect to c^s and l^d subject to the technological and physical feasibility constraints

$$\begin{aligned} c^s &\leq F(l^d) \ ; \\ 0 &\leq c^s, l^d \ . \end{aligned}$$

By construction, the solution to problem (13) gives the firm's *notional* supplies and demands for good C and labor l, of the form

$$c^s \quad = \quad c^s(w^e; s^o) \ ; \tag{14}$$

$$l^d \quad = \quad l^d(w^e; s^o) \ . \tag{15}$$

By assumption, if the firm earns any positive profits in period T, these profits are distributed to the consumer-owner in the form of a dividend.

A precise definition can now be given for a Walrasian equilibrium for the economy at hand.

DEFINITION: For any given state of the economy $s^o = (U(\cdot), F(\cdot), \bar{L})$ in period T, a vector

$$e^* \ = \ (c^{d*}, l^{s*}, c^{s*}, l^{d*}, d^{e*}, w^{e*}, w^*) \tag{16}$$

with $w^* \geq 0$ is a **Walrasian equilibrium** for the economy if the following three conditions hold:

(a) [Consumer and Firm Optimization]: The consumer and firm are on their notional demand and supply curves. That is, e^* satisfies

$$\begin{aligned} c^{d*} &= c^d(w^{e*}, d^{e*}; s^o) \ ; \\ l^{s*} &= l^s(w^{e*}, d^{e*}; s^o) \ ; \\ c^{s*} &= c^s(w^{e*}; s^o) \ ; \\ l^{d*} &= l^d(w^{e*}; s^o) \ . \end{aligned}$$

(b) [Fulfilled Expectations]: All price and dividend expectations are fulfilled. That is, e^* satisfies

$$\begin{aligned} w^{e*} &= w^* \ ; \\ d^{e*} &= [c^{s*} - w^* l^{d*}] \ . \end{aligned}$$

(c) [Notional Market Clearing]: Notional supply is at least as great as notional demand in each market. That is,

$$\begin{aligned} c^{s*} &\geq c^{d*} \ ; \\ l^{s*} &\geq l^{d*} \ . \end{aligned}$$

REMARK: The consumer is everywhere nonsatiated because her utility function is assumed to be everywhere strictly increasing. Consequently, it can be shown that Walras' law (value of excess supply = 0) holds automatically for the economy at hand, given conditions (a), (b) and (c). To see this, consider the following. By nonsatiation, it follows directly from consumer optimization and fulfilled expectations that

$$c^{d*} \ = \ w^* l^{s*} + d^{e*} \ = \ w^* l^{s*} + [c^{s*} - w^* l^{d*}] \ , \tag{17}$$

or

$$0 \ = \ w^*[l^{s*} - l^{d*}] + [c^{s*} - c^{d*}] \ , \tag{18}$$

which is Walras' law (i.e., the value of excess supply equals zero). If $w^* > 0$, it follows from (18) that

$$l^{s*} = l^{d*} \ \text{ if and only if } \ c^{s*} = c^{d*} \ ; \tag{19}$$

i.e., given positive prices, if demand equals supply in all but one market, then demand must equal supply in the remaining market as well.

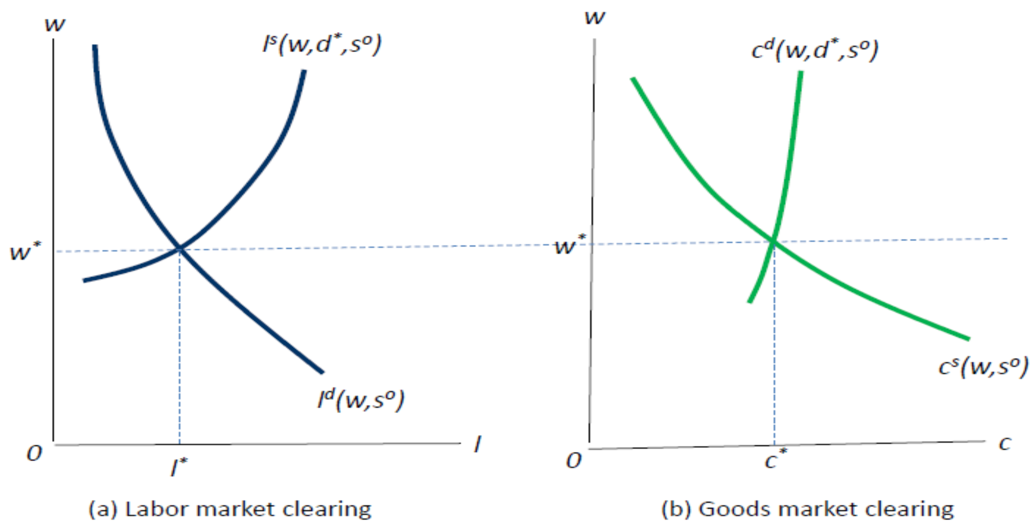

Figure 3: Walrasian equilibrium

To see these properties in graphical terms, consider Fig. 3. In this graphical depiction it is assumed that the consumer's dividend expectation d^e is fulfilled. This (correct) dividend expectation is denoted by d^*. It is also assumed that the equilibrium real wage w^* is positive. By (18), the equilibrium demands and supplies for labor and consumption good must then satisfy $l^{d*} = l^{s*}$ and $c^{d*} = c^{s*}$. These common equilibrium values for labor and consumption good are denoted in the figure by l^* and c^*, respectively. Finally, note that consumption good demand (supply) in Fig. 3 is depicted as an *upward* (*downward*) sloping function of the real wage $w = W/P$, implying as usual that consumption good demand (supply) is a *downward* (*upward*) sloping function of the consumption good price P, all else equal.

The concept of a Walrasian equilibrium is often considered to be a "benchmark of coordination success," in the sense that it shows how a price system might be used to achieve a Pareto-efficient market clearing allocation via the decentralized actions of individually optimizing private agents. As noted in Section 3, Clower (1965) criticizes this use of Walrasian equilibrium as a benchmark because it relies on *notional* market clearing.

Specifically, Clower argues that notional demands are not communicated ("signalled") very well in real-world decentralized market economies because private agents are often liquidity or credit constrained, meaning they cannot back their notional demands with purchasing power at the time these demands are made. In this case, Clower argues, the demands are not *credible* signals regarding purchasing intentions. These ideas of Clower have been criticized for placing an unrealistic emphasis on liquidity or credit constraints. It will now be shown, however, that notional demands might not be communicated very well even in the *absence* of liquidity or credit constraints, due purely to expectational problems.

For example, suppose the previous Walrasian economy is modified as follows. As before, the firm conditions its demands and supplies on expected prices; but now the firm happens to have the "wrong" expectation $\bar{w}^e$ for the real wage, where $\bar{w}^e$ is strictly greater than the Walrasian equilibrium real wage w^*. The firm also has an (unmodelled) line of credit that enables it to "make effective" (pay for) any notional labor demand it decides upon. Finally, the firm "moves first," in the sense that it announces its desired labor demand $l^d(\bar{w}^e, s^o)$ at the expected real wage $\bar{w}^e$ prior to any move by the consumer, where $l^d(\cdot)$ denotes the firm's (notional) labor demand curve defined earlier in (15). Since $\bar{w}^e > w^*$, this labor demand is strictly less than the firm's Walrasian equilibrium labor demand l^* at w^*. Also, by construction, the Walrasian equilibrium labor demand $l^d(w^*, s^o)$ coincides with the Walrasian equilibrium labor l^* supplied by the consumer, and hence is no greater than $\bar{L}$, the maximum amount of labor the consumer can feasibly supply. Thus,

$$l^d(\bar{w}^e, s^o) \ < \ l^d(w^*, s^o) \ \leq \ \bar{L} \ . \tag{20}$$

Suppose, for simplicity, that the real wage expected by the consumer coincides with the real wage $\bar{w}^e$ expected by the firm. The firm's labor demand now appears as a quantity constraint in the consumer's utility maximization problem. Expressed in relative prices, with the commodity good taken to be the numeraire good with real price 1, this problem takes the form

$$\max U(c^d, \bar{L} - l^s) \tag{21}$$

with respect to c^d and l^s subject to the budget, physical feasibility, *and quantity* constraints

$$\begin{aligned} c^d &\leq \bar{w}^e l^s + d^e \ ; \\ 0 &\leq c^d \ ; \\ 0 &\leq l^s \ \leq \ l^d(\bar{w}^e, s^o) \ . \end{aligned}$$

By construction, the solution to problem (21) gives the consumer's *effective* demands and supplies for good C and labor l. Letting $\bar{l}^{Ed} \equiv l^d(\bar{w}^e, s^o)$ denote the labor demand announced by the firm, and made effective by its line of credit, the consumer's effective demands and supplies take the form

$$c^{Ed} = c^{Ed}(\bar{l}^{Ed}, \bar{w}^e, d^e; s^o) \ ; \tag{22}$$

$$l^{Es} = l^{Es}(\bar{l}^{Ed}, \bar{w}^e, d^e; s^o) \ . \tag{23}$$

A definition for an "effective equilibrium" for the economy at hand will now be given.

DEFINITION: Given the state vector s^o and the expected real wage $\bar{w}^e \geq 0$, a vector

$$\bar{e} = (\bar{c}^{Ed}, \bar{l}^{Es}, \bar{c}^{Es}, \bar{l}^{Ed}, \bar{d}^e, \bar{w}) \tag{24}$$

is an **effective equilibrium** for the economy at hand if the following three conditions hold:

1. [**Consumer and Firm Optimization**]**:** The consumer and firm are on their effective

demand and supply curves. That is, $\bar{e}$ satisfies

$$\begin{aligned}
\bar{c}^{Ed} &= c^{Ed}(\bar{l}^{Ed}, \bar{w}^e, \bar{d}^e; s^o) \ ; \\
\bar{l}^{Es} &= l^{Es}(\bar{l}^{Ed}, \bar{w}^e, \bar{d}^e; s^o) \ ; \\
\bar{c}^{Es} &= c^s(\bar{w}^e; s^o) \ ; \\
\bar{l}^{Ed} &= l^d(\bar{w}^e; s^o) \ .
\end{aligned}$$

2. **[Fulfilled Expectations]:** All price and dividend expectations are fulfilled. That is, the actual real wage $\bar{w}$ and dividend $[\bar{c}^{Es} - \bar{w}\bar{l}^{Ed}]$ implied by $\bar{e}$ coincide with their expected values, in the sense that

$$\begin{aligned}
\bar{w} &= \bar{w}^e \ ; \\
[\bar{c}^{Es} - \bar{w}\bar{l}^{Ed}] &= \bar{d}^e \ .
\end{aligned}$$

3. **[Effective Market Clearing]:** Effective supply is at least as great as effective demand in each market. That is,

$$\begin{aligned}
\bar{c}^{Es} &\geq \bar{c}^{Ed} \ ; \\
\bar{l}^{Es} &\geq \bar{l}^{Ed} \ .
\end{aligned}$$

Fig. 4 depicts a possible effective equilibrium for the economy at hand with $\bar{w}^e > 0$, implying effective demand equals effective supply in each market. In this figure, $\bar{d}$ abbreviates the effective equilibrium dividend $[\bar{c}^{Es} - \bar{w}\bar{l}^{Ed}]$, and $\bar{c}$ and $\bar{l}$ abbreviate the effective equilibrium consumption good and labor service amounts, respectively. That is, $\bar{l} = \bar{l}^{Ed} = \bar{l}^{Es}$ and $\bar{c} = \bar{c}^{Ed} = \bar{c}^{Es}$. By construction, the firm's effective demand and supply curves coincide with its notional demand and supply curves; but the consumer's effective demand and supply curves differ from her notional demand and supply curves. Note that the Walrasian equilibrium e^* cannot be depicted using these notional demand and supply curves, for these curves are

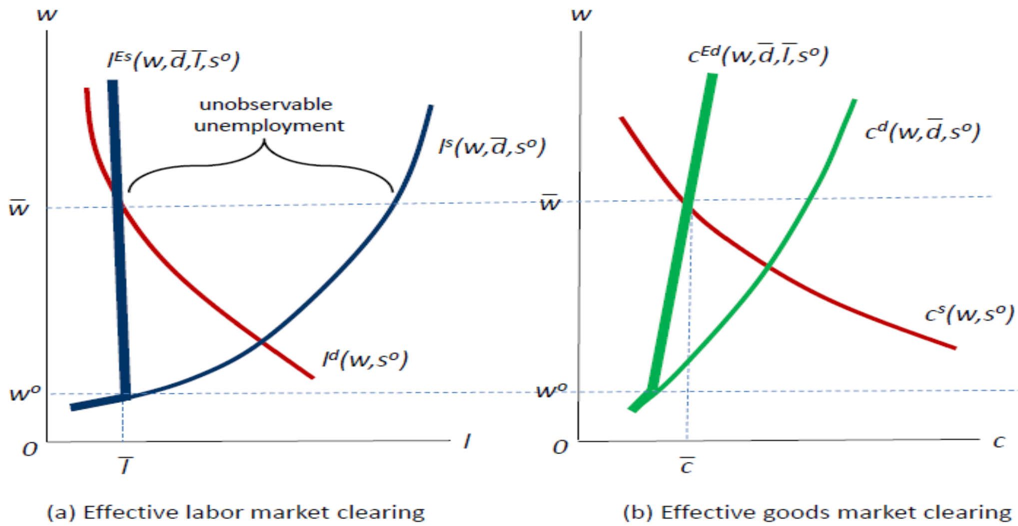

Figure 4: Effective equilibrium: A non-Walrasian equilibrium with effective market clearing

conditioned on the effective equilibrium dividend $\bar{d}$ rather than on the Walrasian equilibrium dividend d^*.

The labor service amount $\bar{l}$ is the labor demand announced by the firm in advance of any consumer decisions, which then appears in the consumer's utility maximization problem as a quantity constraint – in particular, an upper bound on the consumer's labor supply. The real wage level w^o is the level at which $\bar{l}$ becomes a binding quantity constraint for the consumer, in the sense that

$$l^s(w, \bar{d}, s^o) \ > \ \bar{l} \quad \text{if and only if} \quad w \ > \ w^o \ . \tag{25}$$

That is, in the absence of a constraint on its labor supply, the consumer would plan to supply more than $\bar{l}$ units of labor service at all real wage rates higher than w^o. Finally, note that involuntary unemployment exists at the depicted effective equilibrium in the sense of Drazen.

A substantial criticism of this concept of effective equilibrium is that the firm is modelled as a passive price taker rather than a price setter. In particular, the firm takes the expected real wage $\bar{w}^e = \bar{w}$ as given. What would happen if, instead, the firm were able *jointly* to set a real wage and a labor demand to maximize it's profits? Would the firm set a real wage *lower* than $\bar{w}$, hence reducing involuntary unemployment? In particular, would the firm necessarily set it's real wage at the Walrasian equilibrium level w^*?

Clower (1965), Leijonhufvud (1968), Chick (1983), and others stress that *unless* the firm *perceives* an excess demand for its good at the real wage $\bar{w}$, it will have no incentive to lower this real wage (i.e., to raise the real price of its output). They therefore seem to be arguing in terms of a traditional type of price adjustment equation of the form

$$dp/dt \;=\; h(c^d - c^s) \;, \quad h' > 0 \;, \; p \equiv P/W \;, \tag{26}$$

with one change: The consumption demands and supplies c^d and c^s that appear as arguments of $h(\cdot)$ in the price adjustment relation (26) are assumed to be *effective* rather than notional. Since effective demand equals effective supply in any effective equilibrium with positive prices, they argue that the firm will have no incentive to change the real wage in any such equilibrium.

Nevertheless, the price adjustment relation $h(\cdot)$ is not grounded in any explicit microfoundations; so it is unclear whether this heuristic argument would really stand up in a model where the profit maximization problem of the firm as a price *setter* is fully articulated. The "efficiency wage" theory addresses this issue. The theory represents a major component of the recent New Keynesian thrust to revitalize Keynesian economics by providing Keynesian theory with fully articulated microeconomic foundations. In particular, the theory attempts to explain how a profit-maximizing price-*setting* firm might come to set prices and wage rates at levels that result in persistent involuntary unemployment.

6. Multiple Non-Walrasian Equilibria

As discussed at greater length in Tesfatsion (2017), within macroeconomics a *coordination failure* is commonly said to occur when mutual gains, potentially attainable from a feasible all-around change in agent behavior (strategies), are not realized because no *individual* agent has an incentive to deviate from his or her current behavior.[2] The final example described below demonstrates how coordination failure might occur when technological complementarities within a shared production process result in multiple non-Walrasian equilibria.[3]

Consider an economy with two consumption goods: leisure, and a physical commodity good. The economy is inhabited by N utility-maximizing agents, $N > 1$, with each agent permanently located at one of N separate sites. Each agent is endowed with L units of leisure (potential labor) and has a differentiable strictly increasing and strictly concave utility function $U(le, c)$, where le denotes the agent's consumption of leisure and c denotes the agent's consumption of the commodity good.

The commodity good is produced from labor in two stages. In the first stage of production, one distinct type of intermediate good is produced from labor at each site j, $j = 1, \ldots, N$, on a one-for-one basis – i.e., one supplied unit of work (sacrificed unit of leisure) yields one unit of intermediate good. Thus, the real wage w – the wage measured in units of intermediate good – is equal to one. All communication or observation between the N sites is impossible until *after* this first stage of production. Let I_j denote the amount of intermediate good produced at site j, $j = 1, \ldots, N$, in the first stage of production.

In the second stage of production, the commodity good is costlessly produced in accordance with the production relation

$$Q(I_1, \ldots, I_N) \ = \ N \cdot min\{I_1, \ldots, I_N\}. \tag{27}$$

The produced commodity good is then divided *equally* among all N agents. That is, the distribution rule is that each of the N agents receives an amount of good equal to

$$\frac{Q(I_1, \ldots, I_N)}{N} \ = \ min\{I_1, \ldots, I_N\} \ . \tag{28}$$

[2]More formally, a macroeconomic system is commonly said to exhibit *coordination failure* if and when it is in a Pareto-dominated Nash equilibrium.

[3]This example is a simplified version of the example developed by John Bryant (1983). The game structure underlying the Bryant example is known in the game theory literature as a *stag hunt game*; see Tesfatsion (2017). For related work on coordination games with complementarities, see Russell Cooper (1999).

The structure of this game is assumed to be common knowledge for all agents. That is, each agent knows the common leisure endowment L, the common utility function $U(le, c)$, the production relation (27), and the distribution rule (28). Moreover, each agent knows that each other agent knows these structural aspects, and each agent knows that each other agent knows that each other agent knows these structural aspects, etc.

A feasible vector $(I_1, ..., I_N)$ of intermediate good productions I_j is *Pareto efficient* for this economy if there exists no other feasible vector of intermediate good productions such that each of the N agents is at least as well off under the latter vector as under the former vector and at least one agent is strictly better off. How can this verbal definition of Pareto efficiency be translated into formal analytical terms?

For the economy at hand, a vector $V = (I_1, ..., I_N)$ of intermediate good productions is feasible if $0 \leq I_j \leq L$ for $j = 1, \ldots, N$. Let F denote the set of all feasible vectors V, and let $Q(V)$ denote the amount of commodity good produced using V in accordance with the production relation (27). By assumption, given V in F, each agent j consumes an amount $L - I_j$ of leisure and an amount $Q(V)/N$ of the commodity good. Thus, in analytical terms, a vector $V = (I_1, \ldots, I_N)$ in F is *Pareto efficient* if there exists no other vector $V' = (I'_1, \ldots, I'_N)$ in F such that

$$U(L - I'_j, Q(V')/N) \ \geq \ U(L - I_j, Q(V)/N), \quad j = 1, \ldots, N, \tag{29}$$

with strict inequality holding for at least one j.

A *Nash equilibrium* for this economy is any vector $V^E = (I^E_1, \ldots, I^E_N)$ in F such that, given V^E, no *individual* agent j perceives any feasible way to increase his utility by unilateral deviations from his current choice choice I^E_j. It will now be shown that the economy has infinitely many Nash equilibria that are *not* Pareto efficient.

If all agents choose the same level I of intermediate good, then the payoff to each agent is $U(L - I, I)$. Let I^* denote the value of I in $[0, L]$ that maximizes $U(L - I, I)$ over $[0, L]$. Thus, by construction, I^* is the value of I that maximizes the utility of the representative agent, assuming all agents choose the same I.[4] The uniqueness of I^* follows from the (derivable) fact that $U(L - I, I)$ is a strictly concave function of I.

Technical Note: Strict positivity of I^* requires an additional regularity condition: namely, that the U-indifference curve passing through the point $(L, 0)$ satisfies

$$U_1(L,0)/U_2(L,0) \;<\; 1 \;\;, \tag{30}$$

where $U_1 = \partial U/\partial le$ and $U_2 = \partial U/\partial c$ denote the positive first-order partial derivatives of $U(le, c)$. Condition (30) guarantees that $U(L - I, I)$ is a strictly increasing function of I evaluated at $I = 0$, implying that there exist positive I values that give a higher value for $U(L - I, I)$ than $I = 0$.

It will now be shown that the economy at hand has infinitely many Nash equilibria of the form $V' = (I', \ldots, I')$, $0 \le I' \le I^*$, only one of which is Pareto efficient.

CLAIM: Suppose condition (30) holds, implying that $I^* > 0$. Let I' be any given value in the interval $[0, I^*]$.

Part (a) The vector $V' = (I_1, \ldots, I_N)$ with $I_1 = I_2 = \ldots = I_N = I'$ is a Nash equilibrium. Consequently, there exist infinitely many Nash equilibrium for the economy at hand.

Part (b) The Nash equilibrium $V' = (I', \ldots, I')$ is Pareto efficient if $I' = I^*$ and Pareto inefficient if $0 \le I' < I^*$.

The formal proof of this claim is given in an appendix. An intuition understanding of this claim can be obtained by examining Fig. 5.

In what sense, if any, do the inefficient equilibria $V' = (I', \ldots, I')$ represent "unemployment" equilibria? At each such equilibrium V' with $I' < I^*$, each agent has *more* leisure $L - I'$

[4] Given the assumed continuity of the utility function $U(\cdot)$, the existence of such a value I^* in the compact interval $[0, L]$ is assured by a famous result known as the Weierstrass Theorem.

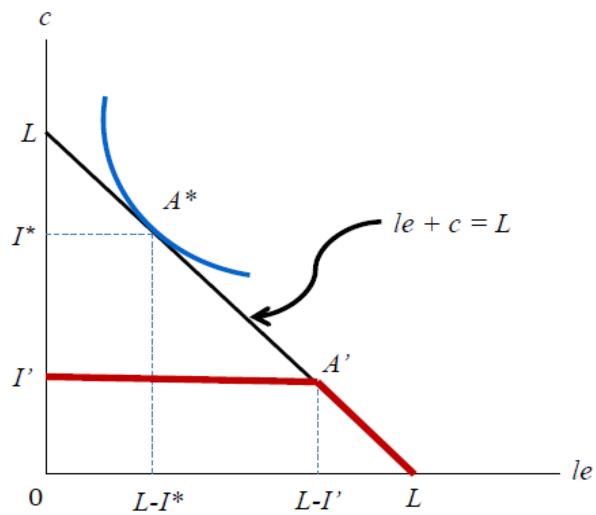

Figure 5: Kinked budget line arising from budget externalities

(hence is supplying *less* labor I') than at the Pareto efficient equilibrium $V^* = (I^*, \ldots, I^*)$. Moreover, all agents would be strictly better off if they all supplied the higher labor amount I^*. Consequently, the inefficient equilibrium V' might appropriately be termed an "unemployment" (or at least an "underemployment") equilibrium. On the other hand, given V', note that it is *not* true that any *individual* agent would want to supply more labor, ceteris paribus, for he perceives a "quantity constraint" in the form of an upper bound on the supply of his labor: namely, $I \leq I'$; see Fig. 5. More precisely, given V', each agent perceives that any labor he supplies in excess of I' would simply be wasted, in the sense that no payment (real wage) would be received for it. It is therefore suggestive to think of V^* as a "notional" equilibrium and V' as an "effective" equilibrium in the sense of Clower (1965).

Note that "wrong prices" are not the cause of coordination failure in this example. In particular, the unemployment or underemployment that can arise has nothing to do with real wages being set "too high" relative to productivity levels. Rather, coordination failure arises from the self-fulfilling nature of expectations: if everyone expects that at least one other person will choose an intermediate good level I no higher than I', then it is rational for each agent to choose an intermediate good level I no higher than I' since any higher level would be expected to have a zero marginal product. Consequently, it is lack of coordination among *expectations* that is the root cause of coordination failure, the failure of agents to coordinate their expectations on the "best" possible intermediate good level I^*.

Is $V' = (I', \ldots, I')$ in any sense a rational expectations equilibrium? Each agent has full knowledge of the *structure* of the economy. In addition, the rational expectations assumption that agents know the true probability distribution governing all exogenous variables holds trivially, because there are no exogenous random variables in the model. In a standard rational expectations model, one would then expect each agent – conditional on his own actions – to have subjective expectations that are in conformity with an objectively true probability distribution governing all endogenous variables.

This is *not* the case for the economy at hand, however. The difficulty is that knowledge of the model structure is *not* enough to derive such an objectively true probability distribution governing all endogenous variables, conditional on one's own actions, because there is an *endogenous* source of uncertainty within the model – namely, the behavior of other agents. Behavioral uncertainty is generally not characterizable in terms of objectively true probability assessments if each agent's optimal actions depend on the actions of other agents, i.e., if there is strategic interaction. Only by assuming that known decision functions govern the actions of all other agents can the usual rational expectations logic be applied. In short, the rational expectations hypothesis as commonly formulated is not designed for situations in which multiple strategically interacting agents are struggling to learn their "best" strategy choices on the basis of past experiences.

This reveals and highlights the stringency of a postulate commonly made in rational expectations macroeconomic models: namely, the *assumed homogeneity of agent behavior*, implicit in the use of a single "representative agent" with "representative expectations." This postulate assumes away any behavioral coordination problems across agents. Note that the assumption of homogeneous behavior is *stronger* than the assumption of structurally identical agents. Even though the agents in the economy at hand are structurally identical, in the sense that they have the same tastes, endowments, and technological opportunities, this does not *necessarily* mean that they will *behave* the same, i.e., choose the same intermediate good level I. The latter holds true in general only if, in addition to identical structures, all agents have identical expectations concerning the intended behavior of other agents. Since the agents in the economy at hand can have differing expectations, there is no guarantee that they will even end up at a Nash equilibrium (a common choice of I), let alone the unique Pareto-efficient Nash equilibrium (a common choice of I^*).

In the present two-stage production game, behavioral uncertainty arises from asymmetric information - agent j knows his own choice of I_j but not the choice of I_k for any other agent k. What rational basis do the agents have for forming expectations of other agents' behaviors? In particular, is it rational for each agent to expect that each other agent will choose I^*, implying that I^* is the only rational choice for him? Note that the failure of even *one* other agent to choose I^* implies that it is no longer rational for any other agent to choose I^*. Moreover, if an agent j decides to deviate from I^* because he expects that some other agent will deviate, note that he himself fulfills his own expectation.

In the game at hand, a form of stag-hunt game, there is a unique strategy (move) configuration V^* for the one-stage game that is both Pareto efficient and a Nash equilibrium. Given a one-stage game with a unique Pareto-efficient Nash equilibrium, many economic game theorists in past times – prior to the rise of experimental economics – supposed that repeated plays of the stage game would result in convergence towards this unique "rational" outcome.

However, laboratory experiments have not supported this supposition. For example, in numerous class experiments run for a repeated version of the two-stage production game analyzed in the present section, convergence towards the unique Pareto-efficient Nash equilibrium V^* has never been observed. Indeed, at least within the first four or five plays of the stage game, student subjects rarely even manage convergence towards a diagonal (Nash equilibrium) outcome.

APPENDIX: PROOF OF CLAIM

CLAIM: Suppose condition (30) holds, implying that $I^* > 0$. Let I' be any given value in the interval $[0, I^*]$.

Part (a) The vector $V' = (I_1, \ldots, I_N)$ with $I_1 = I_2 = \ldots = I_N = I'$ is a Nash equilibrium. Consequently, there exist infinitely many Nash equilibrium for the economy at hand.

Part (b) The Nash equilibrium $V' = (I', \ldots, I')$ is Pareto efficient if $I' = I^*$ and Pareto inefficient if $0 \leq I' < I^*$.

Proof of Part (a): Suppose that all agents except one, say agent N, have chosen an intermediate good production level I' lying in $[0, I^*]$. If agent N then chooses an intermediate good production level $I_N \leq I'$, it follows from the production relation (27) and the assumed equal distribution of commodity good that agent N will receive an amount $N \cdot I_N/N = I_N$ of the commodity good c, whereas for $I' < I_N \leq L$ he will receive $N \cdot I'/N = I'$. Consequently, agent N faces a kinked budget line with a kink at the point $A' = (L - I', I')$; see Figure 5.

By assumption, since $I' \leq I^*$, the point $A^* = (L - I^*, I^*)$ lies somewhere to the left of A' along the line $le + c = L$. Since $U(le, c)$ is strictly increasing and strictly concave, it follows that, given $I_j = I'$ for $j = 1, \ldots, N-1$, the utility of agent N is maximized along his kinked budget line at the point A'; i.e., his utility-maximizing choice of I_N is $I_N = I'$.

In summary, given that all other agents choose the common value I' in $[0, I^*]$, the best choice for agent N is to set $I_N = I'$. Thus, given the vector $V' = (I', \ldots, I')$ of intermediate productions, agent N has no incentive to deviate. The same argument holds by symmetry for all other agents j as well. It follows that V' is a Nash equilibrium.

Proof of Part (b): Suppose, first, that $I' = I^*$. Note that any allocation V with unequal components I_j is Pareto dominated by a feasible allocation with equal components – namely, the allocation constructed from V by setting all of its components equal to its minimum component. This is so, because any production of intermediate good in excess of this minimum component uses up additional units of labor (at the cost of leisure) with no subsequent gain in output for any agent. Consequently, to establish that $V^* = (I^*, \ldots, I^*)$ is Pareto efficient, it suffices to show that V^* is not Pareto dominated by any feasible allocation with equal components. But this follows immediately from the determination of I^* as the feasible intermediate good level that uniquely maximizes utility for the "representative" agent assuming all agents select the same feasible intermediate good level.

Suppose, instead, that $0 \leq I' < I^*$. Then A^* lies *strictly* to the left of A' along the line $le + c = L$; see Fig. 5. Consequently, the feasible intermediate good vector V^* with $I_j = I^*$ for $j = 1, \ldots, N$ gives each agent j a strictly higher utility than the feasible intermediate good vector V' with $I_j = I'$ for $j = 1, \ldots, N$. It follows that V^* Pareto dominates V', i.e., V' is *not* Pareto efficient. Q.E.D.

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Online Lecture Materials for Walrasian General Equilibrium

L. Tesfatsion, "Introduction to Walrasian General Equilibrium Modeling"
`https://www2.econ.iastate.edu/tesfatsi/WalrasIntro.pdf`

L. Tesfatsion, "Walrasian General Equilibrium: Benchmark of Coordination Success?"
`https://www2.econ.iastate.edu/tesfatsi/Walbench.pdf`

L. Tesfatsion, "Robustness of Walrasian General Equilibrium Modeling" (Slide-Set)
`https://www2.econ.iastate.edu/tesfatsi/WalrasianGERobustness.LT.pdf`

L. Tesfatsion, "Game Theory: Basic Concepts and Terminology (Notes & Slide-Set)"
`https://www2.econ.iastate.edu/tesfatsi/gamedef.pdf`

L. Tesfatsion, "Non-Walrasian Equilibrium: Illustrative Examples"
`https://www2.econ.iastate.edu/tesfatsi/nonwalra.pdf`