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Money balances in the production function: Nonlinear tests of model stability and measurement issues – two sides of the same coin? ☆

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ABSTRACT

Past empirical attempts to test the role of money in the production function following the Sinai and Stokes (1972) preliminary Cobb–Douglas model specification estimated in 1929–1967, using yearly data, have focused on estimating alternative production function models, such as CES and Translog, and experimenting with alternative specifications of the monetary variable. Most research in the United States on this topic has involved four basic datasets: annual data in the period 1929–1967, nonfinancial quarterly data in the period 1953:1 to 1977:3, annual data in the period 1930–1978 and annual data in the period 1959–1985. The current research uses MARS modeling, general additive modeling, flexible least squares and VAR methods to assess whether there is evidence of nonlinearity and/or model structural change that is impacted by whether a monetary variable has been added to the model specification or a different period is under study. VAR modeling is used with the nonfinancial quarterly dataset to assess whether shocks in the financial sector, as measured by log real M2, can impact the real sector. Since a significant impact is found on log capital, log labor and log real output, the implication is that the real sector is not isolated from the financial sector. One way to think of this is that shocks to the financial sector can have dynamic effects on the real sector.

1. Introduction

Past empirical attempts to further test the role of the financial sector in the production function, following the Sinai and Stokes (1972) preliminary Cobb–Douglas model specification in the period 1929–1967, using the Christensen and Jorgenson (1969, 1970) annual data, have focused on estimating alternative production function models, such as the Boyes–Kavanaugh (1981) CES model and Short (1979) and Simos (1981) experiments with a translog production function model. In addition, there have been a number of experiments with alternative specifications of the monetary variable, including Sinai and Stokes (1989), using interest rates as a possible shift parameter. Most research in the United States on this topic has involved four basic datasets: annual data in the period 1929–1967 discussed in Sinai and Stokes (1972), nonfinancial quarterly data in the period 1953:1 to 1977:3 obtained from DRI and discussed in Sinai and Stokes (1981b), annual data in the period 1930–1978, discussed in Nguyen (1986) and Sinai and Stokes (1989), and annual data in the period 1959–1985, discussed by Benzing (1989). As noted by Sinai and Stokes (1972) and mentioned by Fisher (1974), measured increasing returns to scale were a concern whether or not a financial variable was in the specification of the production function. In an alternative and related

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line of research, Neuburger and Stokes (1974, 1975) investigated the effect of a measure of financial market efficiency unique to Germany and Japan on real output in a test of the Gerschenkron (1962) hypothesis. Measured increasing returns to scale were less of a problem in these papers. This research will not be treated further in this paper, which is focused on the role of real balances in an aggregate production function but should be thought of as an alternative way to model the financial sector's effect on real output. Sinai and Stokes (1989) reported results using the 1929–1967 data but modeled the financial sector using an interest rate shifter with and without real balances in the equation. This formulation did not correct the estimated increasing returns to scale found in Sinai and Stokes (1972) and is not investigated further here.

The theoretical arguments for a role for real monetary balances in the production function have been developed by Bailey (1962), Nadiri (1969, 1970) and others. The seminal survey paper by Fisher (1974) summarized some of this material and urged caution. In Fisher's view the measured increasing returns to scale obtained by Sinai and Stokes (1972) for their Cobb–Douglas production function were not credible. Sinai and Stokes (1975, Table 1) noted that the finding of increasing returns to scale was not just confined to models containing real balances. Sinai and Stokes (1972, p. 294) noted that "the Cobb–Douglas functions we estimate exhibit increasing returns to scale, a result that is consistent with Bodkin and Klein's (1967) estimates of the Cobb–Douglas for the period 1909–1949." The persistence of findings of increasing returns to scale in production function research of this kind suggests this finding appears to be inherent to the data and/or model specification and sets the stage for further analysis, which is one of the motivations for the current paper. As an example, the sum of the coefficients on labor and capital in a Cobb–Douglas model estimated with an OLS model not containing TIME is 1.835. If time is added, then the sum falls to 1.634. If LnM1 is added to a model without TIME, the sum is 1.767. As TIME is added the sum falls to 1.674. While the measured increasing returns to scale appear to be inherent to the Christensen and Jorgenson (1969, 1970) data itself, it is still an open question whether this is the case for Cobb–Douglas models using other datasets and for other periods?

Whether the estimated returns to scale found is caused by measurement error in the inputs to the production function or whether it is due to an incorrect functional form of the production function is at issue. The functional form of the estimated model could be inappropriate because it has shifted over time or is inherently wrong for any period. The Cobb–Douglas function is

$$Y = Ae^{\lambda t}L^{\alpha}K^{\beta}M^{\gamma}e \tag{1.1}$$

which can be estimated in log form as

$$\ln(y) = \ln(A) + \lambda t + \alpha\ln(L) + \beta\ln(K) + \gamma\ln(M) + \ln(e) \tag{1.2}$$

where Y, L, K and M are output, labor, capital and a real monetary variable such as M1 or M2. In Sinai and Stokes (1972, footnote 5) the reported Kmenta (1967, pp. 180–181) test suggested that the Cobb–Douglas function was more appropriate than the CES function for the Christensen and Jorgenson (1969, 1970) data, although others argued for CES and translog functions. To control for the effect of the functional form, in an initial test, only the Cobb–Douglas form of the model, with and without TIME and with and without a real balances variable, is used to calculate the returns to scale for different datasets in different periods in results presented in Table 1. Only OLS results are shown to isolate the effects on the point estimates of the coefficients. Dataset A was first studied by Sinai and Stokes (1972) in models estimated using second-order GLS over the period 1929–1967. Consult this paper for data sources. Dataset B is the original dataset estimated from 1930–1967. It should be compared with dataset D, which is the Nguyen (1986) dataset estimated for the same period where the measured returns to scale appear lower than in dataset B but still show increasing returns to scale. Dataset C is the Nguyen data for the period 1930–1978, where for model 16 containing LnM1 and TIME, the returns to scale were 1.12395. Model 14, the Nguyen dataset estimated with only TIME, where the returns to scale were 1.0721, makes little sense since the capital coefficient was negative. Dataset E, studied by Benzing (1989), covers a later period, 1959–1985, and still finds increasing returns to scale in all specifications. The Sinai and Stokes (1981a, 1981b) disaggregate quarterly dataset that is for the non-financial sector was estimated in the period 1953:1–1977:3 also finds increasing returns to scale. The implication from the results reported in Table 1 is that the finding of increasing returns to scale that troubles Fisher (1974) is not unique to the Sinai and Stokes (1972) dataset nor to the use of annual data nor the period of estimation nor to models containing or not containing real balances, nor whether the whole economy is modeled or just the nonfinancial sector. What is not tested in this table is whether the increasing returns found is due to the restrictive assumptions of the Cobb–Douglas functional form of the model.

Fisher (1974, pp. 530–531) raises a number of further issues, paraphrased below, that warrant further investigation:

> First, a production function containing real balances is based on given exchange arrangements that may change with the variables that enter the production function. The aggregation that is required to have a production function for the whole economy may ignore these changes.
>
> Second, one has to ask whether the measure of real balances used is an adequate and stable index of resources freed from transactions.
>
> Third, over time, unless the measure of technical progress is adequately modeled, real money balances will not continue to reflect the resources freed from transactions.

Table 1
Measure of returns to scale for OLS Cobb–Douglas models.

Model	Dates DSN	Constant	Labor	Capital	M1	M2	Time	RTS	RSS
1	1929–1967 A	−3.938	1.4508	0.3838				1.8346	4.34E−02
2	1929–1967 A	−3.449	1.1547	0.5012	0.111			1.7669	3.72E−02
3	1929–1967 A	−3.852	1.2947	0.4149		0.1158		1.8254	4.00E−02
4	1929–1967 A	−13.064	1.3414	0.2924			5.21E−03	1.6338	3.75E−02
5	1929–1967 A	−8.95	1.1967	0.4058	0.0713		3.04E−03	1.6738	3.60E−02
6	1929–1967 A	−11.536	1.2986	0.3195		0.04505	4.36E−03	1.6631	3.72E−02
7	1930–1967 B	−3.93	1.4496	0.3836				1.8332	4.32E−02
8	1930–1967 B	−3.2996	1.051	0.5447	0.1517			1.7474	3.56E−02
9	1930–1967 B	−3.85	1.2742	0.4196		0.1323		1.8261	3.98E−02
10	1930–1967 B	−16.9	1.3007	0.2549			7.39E−03	1.5556	3.58E−02
11	1930–1967 B	−12.467	1.0648	0.4083	0.1072		5.12E−03	1.5803	3.26E−02
12	1930–1967 B	−15.122	1.2317	0.2904		0.06717	6.40E−03	1.5893	3.50E−02
13	1930–1978 C	−5.73	1.7513	0.2320				1.9833	1.72E−01
14	1930–1978 C	−40.31	1.1184	−0.0463			2.05E−02	1.0721	6.71E−02
15	1930–1978 C	−18.435	0.8186	0.3121	0.24507		8.56E−03	1.3758	4.80E−02
16	1930–1978 C	−34.161	0.9992	0.0114		0.1134	1.72E−02	1.1240	6.40E−02
17	1930–1967 D	−6.1553	1.746	0.3203				2.0663	1.01E−01
18	1930–1967 D	−34.703	1.2207	0.0604			1.70E−02	1.2811	3.69E−02
19	1930–1967 D	−23.3897	1.02261	0.2471	0.14035		1.09E−02	1.4101	3.26E−02
20	1930–1967 D	−28.759	1.102	0.1208		0.1095	1.38E−02	1.3323	3.44E−02
21	1959–1985 E	−1.9963	0.90587	0.4617				1.3676	2.44E−02
22	1959–1985 E	−3.4038	0.69194	0.5053	0.4333			1.6306	1.46E−02
23	1959–1985 E	−1.2786	0.5711	0.1567		0.5178		1.2456	4.73E−03
24	1959–1985 E	17.171	1.071	0.6866			−0.0111	1.7576	2.38E−02
25	1959–1985 E	−4.705	0.6799	0.4903	0.4352		7.50E−04	1.6054	1.46E−02
26	1959–1985 E	18.077	0.7378	0.3836		0.5179	−1.12E−02	1.6393	4.12E−03
27	1953:1–1977:7 F	−2.326	1.4984	0.4037				1.9021	6.91E−02
28	1953:1–1977:7 F	−1.792	1.3525	0.4968	−0.15417			1.6951	6.03E−02
29	1953:1–1977:7 F	−2.3857	1.5154	0.366	0.0615			1.9429	6.81E−02
30	1953:1–1977:7 F	0.2917	0.9040	0.3283			3.75E−03	1.2323	1.69E−02
31	1953:1–1977:7 F	0.4015	0.8996	0.2349	0.1284		4.55E−03	1.2629	1.32E−02
32	1953:1–1977:7 F	0.3044	0.9097	0.2345		0.1458	3.97E−03	1.2900	1.15E−02

Notes: Labor, Capital, M1 and M2 are in natural log form. RTS = Returns to Scale. REE = e′e. Datasets A and B are from Sinai and Stokes (1972). Datasets C and D are from Nguyen (1986). Dataset E is from Benzing (1989). Dataset F is from Sinai and Stokes (1981a, 1981b).

The current research uses MARS modeling, general additive modeling, flexible least squares and VAR methods to assess whether there is evidence of nonlinearity and/or model structural change whether a monetary variable has been added to the model specification or a different period is under study. VAR modeling is used with the quarterly nonfinancial dataset to assess if shocks in the financial sector can impact the real sector, and if so how? This research tests whether there are linkages between the financial sector as measured by log real M1 or M2 on the real sector variables as measured by log capital, log labor and log output. Prior to getting to these topics is a brief discussion of measurement issues.

2. Measurement issues

The Griliches (1986, p. 1469) survey of data issues contrasted two alternative approaches. The first approach involved measurement problems in the data themselves, such as those that arise from aggregation. Problems of this nature were raised by Fisher (1974) as noted earlier. The second approach is "that there are no data problems only model problems in econometrics. For any set of data there is the 'right model'." Fisher (1974, p. 518, footnote 5) alluded to some of these problems as they pertain to real balances. In the present paper only Fisher's second and third issues are studied, leaving the detailed discussion of aggregation problems in the monetary variable to further work. The effect of other variables in the model on the estimated monetary and other input variables is investigated.

The research design employed in this paper is to relax the assumption of a linear / log-linear models and identify possible nonlinear models, using automatic detection methods such as GAM (Hastie and Tibshirani, 1990) and MARS (Friedman, 1991) for the four datasets. The objective is to contrast the findings for the different periods rather than the usual approach of using instrumental variable techniques to correct for possible measurement issues in the variables. Bound, Brown, and Mathiowetz (2001, 3708) in their comprehensive survey of measurement issues caution

> "Standard methods for correcting for measurement error bias, such as instrumental variables estimation, are valid when errors are classical and the underlying model is linear, but not, in general, otherwise. While statisticians and econometricians have been quite clear about the assumptions built into procedures they have developed to correct for measurement error, empirical economists have often relied on such procedures without giving much attention to the plausibility of the assumptions they are explicitly or implicitly making about the nature of the measurement error. Not only can standard fixes not solve the underlying problem, they can make things worse!"

As noted by Lucas (2013) in the last 60 years there have been substantial changes in the monetary sector. For example, the ratio of demand deposits to GDP fell from 30% to 5% in 1950 to 2000 alone. Against such changes, it is unreasonable to expect to have a stable linear or log-linear model. Using four datasets involving research in the real money balances in the production function literature studied by Sinai and Stokes (1972, 1975, 1977, 1981a, 1981b) and others as a base case, a number of tests are made to help answer these questions.

3. Brief review of the literature and a discussion of the theory behind the estimated models

Sinai and Stokes (1972) added real money balances to a Cobb–Douglas production function using annual data developed by Christensen and Jorgenson (1969, 1970) in a preliminary attempt to test whether there was any empirical evidence to support adding real balances to the aggregate production function. Footnote 1 of Sinai and Stokes (1972) outlines the key theoretical literature that argues that real balances are in fact a producer's good. At issue is the correct functional form of the production function and whether this is invariant over time. In the Sinai and Stokes (1972) paper, which they indicated was preliminary, it was argued that the functional form of the model was not settled, nor was their general agreement on the correct monetary variable. However, the Kmenta (1967) test suggested the Cobb–Douglas function was a reasonable choice for preliminary work. Sinai and Stokes (1989) experimented with interest rate variables being used as a proxie for the role of the monetary sector on the production sector by use of a shift factor in the production function. In response to the original article, a number of authors argued for other forms of the production function, such as CES models or trans log models. A disadvantage of this research design is that the functional form of the model to be tested is determined before the empirical test is performed. In the present paper, in contrast, general nonlinear modeling techniques, from which the discussion below has been adapted from Stokes (1997), are employed to attempt to determine if there is evidence for nonlinearity in the Cobb–Douglas function that would suggest that further experimentation is warranted.

The most basic nonlinear method used is the general additive model (GAM) developed by Hastie and Tibshirani (1990) that forms the expectation of y given $x_1, \ldots, x_p$ as

$$E(y|x_1, \ldots, x_p) = \alpha + f_1(x_1) + f_2(x_2) + \cdots + f_p(x_p) \tag{3.1}$$

where $f_i(x_i)$ can be approximated by a modeler determined polynomial of a given degree. After the model is estimated containing these nonlinear terms then, holding all other terms fixed, the algorithm forces each variable in turn to be linear. The increase in $e'e$ that is obtained by imposing linearity on that term can be tested to see if there is significant nonlinearity in that variable, conditional on the other variables being allowed to be nonlinear.

The multivariate adaptive regression splines (MARS) method developed by Friedman (1991) can be thought of as a generalization of a threshold model with the added advantage that the knots are data-determined rather than imposed by the modeler. Following the simplified treatment in Stokes (1997), the MARS model can be written as

$$y = f(x_1, \ldots, x_p) + e \tag{3.2}$$

involving N observations on p right-hand side variables, $x_1, \ldots, x_p$. The MARS procedure attempts to approximate the nonlinear function $f()$ by

$$\hat{f}(X) = \sum_{j=1}^{s} c_j K_j(X), \tag{3.3}$$

where $\hat{f}(X)$ is an additive function of the product basis functions $\{K_j(X)\}_{j=1}^{s}$ associated with the s sub regions $\{R_j\}_{j=1}^{s}$ and c_j is the coefficient for the jth product basis function. If all sub regions include the complete range of each of the right-hand side variables, then the coefficients $\{c_j\}_{j=1}^{s}$ can be interpreted as just OLS coefficients of variables or interactions among variables. The usual OLS assumption that all variables are "switched on" is met. However, the MARS procedure can identify the sub regions under which the coefficients are stable, and other regions when they are zero. In addition, it is possible to search for and detect any possible interactions up to a maximum number of possible interactions controllable by the user. For example, assume the model has only one right-hand side variable x and in the population

$$y = \begin{cases} \alpha + \beta_1 x + e & \text{for } x > 100, \\ \alpha + \beta_2 x + e & \text{for } x < 100. \end{cases} \tag{3.4}$$

In terms of the MARS notation, this is written

$$y = \alpha' + c_1 (x - \tau^*)_+ - c_2 (\tau^* - x)_+ + e, \tag{3.5}$$

where $\tau^* = 100$ and $()_+$ is the right $(+)$ truncated spline function, which takes on the value 0 if the expression inside $()_+$ is negative and its actual value if the expression inside $()_+$ is > 0. Here $c_1 \equiv \beta_1$ and $c_2 \equiv \beta_2$. In terms of Eq. (3.4), $K_1(X) \equiv (x - \tau^*)$ and $K_2(X) \equiv (\tau^* - x)$. Note that the derivative of the spline function is not defined for values of x at the knot value of 100. Friedman (1991) suggests using either a linear or cubic approximation to determine the exact y

value. The later Hastie and Tibshirani MARS Fortran code used in this analysis does not use this approximation and the resulting models are thus easier to interpret and implement in programs such as Excel. It should be stressed that once the transformed vectors $(x-\tau^*)_+, (\tau^*-x)_+$ in Eq. (3.5) are determined, OLS is used to solve for the coefficients (α', c_1, c_2). Modifications were made to the Hastie–Tibshirani code to produce SE's and t scores. Unlike OLS and GAM models, since the MARS procedure allows data shrinkage, the transformed vectors are usually highly significant when the OLS step is performed. Many potential variables may not, in fact, be in the final model. An important diagnostic of a MARS model is the number of times a vector is non-zero and the total number of non-zero vectors by observation. It has been found that with many financial models, for a substantial portion of the data there are few, if any, vectors in the model save for the constant. For these observations the market appears efficient.

Before estimation of a MARS model, the user selects the maximum number of knots (NK) to consider and the highest order interaction to investigate (MI). A number of switches are available to control the degree of complexity of the estimated model. An example of an interaction model for $y=f(x,z)$ follows.

$$y=\alpha+c_1\left(x-\tau_1^*\right)_+ + c_2\left(\tau_1^*-x\right)_+ + c_3\left(x-\tau_1^*\right)_+\left(z-\tau_2^*\right)_+ + e \tag{3.6}$$

implies that

$$y=\begin{cases}\alpha+c_1x-c_1\tau_1^*+e & \text{for } x>\tau_1^* \text{ and } z<\tau_2^*,\\ \alpha-c_2x+c_2\tau_1^*+e & \text{for } x<\tau_1^*,\\ \alpha+c_1x-c_1\tau_1^*+c_3(xz-\tau_1^*z-\tau_2^*x+\tau_1^*\tau_2^*)+e & \text{for } x>\tau_1^* \text{ and } z>\tau_2^*.\end{cases} \tag{3.7}$$

As an aid in determining the degree of model complexity, Friedman (1991) suggested using a modified form of the generalized cross validation criterion (MGCV),

$$MGCV=\left[(1/N)\sum_{i=1}^{N}\left(y_i-\hat{f}(X)\right)^2\right]\Big/\left[1-\left[C(M)^*/N\right]^2\right], \tag{3.8}$$

where there are N observations, $\hat{f}(X_i)\equiv\hat{y}_i$ and $C(M)^*$ is a complexity penalty. The default is to set $C(M)^*$ equal to a function of the effective number of parameters. The formula used is

$$C(M)^*=C(M)+\delta M. \tag{3.9}$$

GAM and MARS models can be simulated over a range of values of a specific variable, conditional on set values of the other variables in the model, to graphically produce a leverage plot that will indicate over which values the estimate partial derivative with respect to that variable changes. An argument can be made that a variable such as lnK, lnL, LnM1 or lnM2 might have measurement issues that arise due to shocks to the system. This argument builds on the fact that while the shock will impact the left-hand side, it will not be captured in the right-hand side variable. To investigate this possibility, VAR modeling using Monte Carlo integration as suggested by Sims (1980), Sims, Stock, and Watson (1990) and Sims and Zha (1999) was performed on quarterly disaggregate data first used by Sinai and Stokes (1981b). In the two-variable case, which is shown to reduce notational clutter, a model of this type between x and y can be written

$$\Phi(B)\begin{bmatrix}x_t\\ y_t\end{bmatrix}=\begin{bmatrix}e_{1t}\\ e_{2t}\end{bmatrix} \tag{3.10}$$

which can be expanded to

$$\begin{bmatrix}\phi_{11}(B) & \phi_{12}(B)\\ \phi_{21}(B) & \phi_{22}(B)\end{bmatrix}\begin{bmatrix}x_t\\ y_t\end{bmatrix}=\begin{bmatrix}e_{1t}\\ e_{2t}\end{bmatrix} \tag{3.11}$$

where Granger (1969) causality from x to y implies that $\phi_{21}(B)\neq 0$ where $\phi_{ij}(B)$ is a polynomial in the lag operator B $(B^k x_t\equiv x_{t-k})$, with m terms. Often of interest is the effect of a shock in the x and y equations. To measure this effect requires transforming the VAR model in Eq. (3.11) to a VMA model, given $\Phi(B)$ is invertible, or

$$\begin{bmatrix}x_t\\ y_t\end{bmatrix}=\Theta(B)\begin{bmatrix}e_{1t}\\ e_{2t}\end{bmatrix} \tag{3.12}$$

where $\Theta(B)\equiv[\Phi(B)]^{-1}$. The terms in $\Theta(B)$measure the dynamic response of each of the endogenous variables to a shock to the system. Eq. (3.12) can be expanded to

$$\begin{bmatrix}x_t\\ y_t\end{bmatrix}=\begin{bmatrix}\theta_{11}(B) & \theta_{12}(B)\\ \theta_{21}(B) & \theta_{22}(B)\end{bmatrix}\begin{bmatrix}e_{1t}\\ e_{2t}\end{bmatrix}. \tag{3.13}$$

The term $\theta_{21}(B)$ measures the effect of shocks in the x equation on the y variable conditional of the shocks hitting the y variable while. If $\theta_{ij}(B)=0$ for $i\neq j$, then each endogenous variable is not impacted from shocks coming from the other endogenous variables. To investigate this possibility later in the results section of this paper, Rats software version 8.2 routine @mcgraphirf, Doan (2010, p. 495), is used to calculate using Monte Carlo integration $\theta_{ij}(B)$ bounds on the moving average estimates. In general, the number of lags in the VAR model m is not the number of lags in $\theta_{ij}(B)$, which we will call q. In the results reported later, $m = 2$ and $q = 20$. B34S version 8.11F has been used to calculate the other results reported in the paper.

All models considered up to now have assumed the coefficients are fixed over the estimation period or have a fixed nonlinear function. Kalaba and Tesfatsion (1989, p. 1215) drop this assumption and define flexible least squares (FLS) as

> "... the collection of all coefficient sequence estimates b which yield vector-minimal sums of squared residual measurement and dynamic errors for the given observations, i.e., which attain the 'residual efficiency frontier.' The frontier characterizes the efficient attainable trade-offs between residual measurement error and residual dynamic error for a time series model. In particular, the frontier reveals the cost in terms of residual measurement error that must be paid in order to attain the zero residual dynamic error (time consistent coefficients) required by ordinary least squares estimation."

One position on the residual efficiency frontier that constrains the dynamic error to be 0.0 at the cost of the highest measurement error corresponds to the OLS solution. Assuming N observations, define measurement error as

$$r_M^2(b; N) = \sum_{n=1}^{N} \left[y_n - x_n^T b_n\right]^2 \tag{3.14}$$

and dynamic error as

$$r_D^2(b; N) = \sum_{n=1}^{N-1} [b_{n+1} - b_n]^T [b_{n+1} - b_n]. \tag{3.15}$$

Define μ as the weight. For every $\mu \geqslant 0$ an incompatibility cost

$$\begin{aligned} C(b; \mu, N) &= \mu r_D^2(b; N) + r_M^2(b; N) \\ &= \mu \sum_{n=1}^{N-1} [b_{n+1} - b_n]^T [b_{n+1} - b_n] + \sum_{n=1}^{N} \left[y_n - x_n^t b_n\right]^2 \end{aligned} \tag{3.16}$$

can be defined. As $\mu \to \infty$ $C(b; \mu, N) \to$ OLS sum of squares since in this case the coefficients do not change or $b_{n+1} = b_n$ and $r_D^2(b; N) \to 0$. As $\mu \to 0$ $r_M^2(b; N) \to 0$ and the coefficients move so as to minimize $C(b; \mu, N)$. Let the unique coefficient sequence that minimizes the incompatibility cost be defined as the flexible least squares (FLS) solution at time N, conditional on μ,

$$b^{FLS}(\mu, N) = \left(b_1^{FLS}(\mu, N), \ldots, b_N^{FLS}(\mu, N)\right) \tag{3.17}$$

from which

$$\begin{aligned} r_M^2(\mu, N) &= r_M^2\left(b^{FLS}(\mu, N); N\right), \\ r_D^2(\mu, N) &= r_D^2\left(b^{FLS}(\mu, N); N\right). \end{aligned} \tag{3.18}$$

Kalaba and Tesfatsion (1989) describe in detail how best to solve (3.18). The solution requires that $X(n)$ for $n = 1, N$ be of rank K. This rules out, for example, dummy variables being in the model. OLS is a limiting case of FLS when $\mu = \infty$.

$$\lim_{\mu \to \infty} b_n^{FLS}(\mu, N) = b^{OLS}(N), \quad 1 \leqslant n \leqslant N. \tag{3.19}$$

For all μ values

$$b^{OLS}(N) = \left[\sum_{n=1}^{N} x_n x_n^T\right]^{-1} \sum_{n=1}^{N} x_n x_n^T b_n^{FLS}(\mu, N) \tag{3.20}$$

which can be used as a useful check on the calculation. Eq. (3.20) illustrates that OLS estimates are nested inside FLS estimates. If $X(N)$ has full rank K, there exists a constant $K \times 1$ coefficient vector b such that

$$b_n^{FLS}(\mu, N) = b, \quad 1 \leqslant n \leqslant N \tag{3.21}$$

if and only if $b = b^{OLS}(N)$ and

$$\left[x_n^T b^{OLS}(N) - y_n\right] x_n = 0, \quad 1 \leqslant n \leqslant N. \tag{3.22}$$

Table 2
OLS and GAM estimates of the production function for LnY.

Variable	OLS	GAM	NL_Pval	Lin_Res
TIME	0.397E−02	0.376E−2	1.000	0.1085E−01
	(21.48)	(23.56)		
LnK	0.2345	0.2304	1.000	0.1082E−01
	(13.97)	(15.90)		
LnL	0.9097	0.9513	1.000	0.1002E−01
	(27.09)	(32.81)		
LnM2	0.1458	0.1569	0.609	0.8136E−02
	(6.65)	(8.285)		
CONSTANT	0.3044	0.1365		
	(2.33)	(1.211)		
e′e	1.153E−2	7.86E−3		

LnY is the dependent variable. t scores and Z scores are listed under OLS and GAM coefficients respectively. NL_Pval is the significance of a test for nonlinearity for that variable. A value of 0.95 indicates that nonlinearity is significant at the 5% level. Lin_Res is e′e assuming that the variable is restricted to be linear.

The first order conditions for the incompatibility cost, (3.23), to be a minimum are

$$\begin{aligned} 0 &= \left[x_1^T b_1 - y_1\right]x_1 - \mu[b_2 - b_1], \quad 1 < n < N, \\ 0 &= \left[x_n^T b_n - y_n\right]x_n - \mu[b_{n+1} - b_n] + \mu[b_n - b_{n-1}], \\ 0 &= \left[x_N^T b_N - y_N\right]x_N - \mu[b_N - b_{N-1}], \quad n = N. \end{aligned} \tag{3.23}$$

4. Testing the disaggregate data for nonlinearity

Sinai and Stokes (1981b) used nonfinancial disaggregate data in the period 1953:1 to 1977:3 which was obtained from DRI. In Table 1 the estimated returns to scale were measured in models 27–32. For model 32 containing both TIME and LnM2, the measured returns to scale was 1.29 and $e'e = 0.01153$. The OLS results are shown in Table 2 column 1 and represent a base case from which to measure the cost of assuming linearity. The GAM results of the same model, shown in columns 2–4, show $e'e = 0.00786$ and TIME, LnK and LnL highly nonlinear as seen in column 3. If, in turn, each of these variables were restricted to be linear, e′e would increase to 0.01085, 0.01082 or 0.01002, respectively, as shown in column 4. The log of M2 was found not to be nonlinear. Note that this analysis was in the 1953–1977 period and did not include the depression and the World War II which were in the original Sinai and Stokes (1972) 1929–1967 annual data.

Using the MARS procedure, a more complex model was developed that allowed for interactions and resulted in $e'e = 0.003365$ or roughly 50% of the corresponding value found with the GAM model and substantially less that what was found for the OLS model. The MARS findings are shown in Table 3. A term such as $0.7249 * \mathrm{MAX}(\mathrm{LnK} - 6.678)$ indicates that for values of $\mathrm{LnK} > 6.678$ the coefficient is 0.7249. The term $-0.5664 * \mathrm{MAX}(6.678 - \mathrm{LnK})$ indicates that for values of $\mathrm{LnK} < 6.678$ the coefficient is 0.5664. This is visually shown in Fig. 2 that in addition takes into account the effect of other LnK interactions. A MARS model is best evaluated in terms of such a leverage plot at focus values of the independent variables. In this case the medians were used. The column NON Zero in Table 3 indicates the number of times and the percentage of times a certain knot is in the data at a non-zero value. For the above knots the times were 50 and 48 respectively.

Leverage plots are shown in Fig. 1 and Figs. 3–4 for the other variables in the model. In Fig. 1 the break in the TIME coefficient is around observation 50, i.e., the end of the first quarter of 1965. In Fig. 2 the contribution of LnK is shown. Around 6.675 the change in the elasticity of output with respect to LnK is reduced. In Fig. 3 the leverage of LnL is reduced after 3.85. Fig. 4 shows that at the median values of the other variables in the system, the leverage of LnM2 is fixed at 6.66 until the $\mathrm{LnM2} = 4$, when the leverage starts to increase. Note that all these results are conditional on the other variables NOT in the graph being set at their medians. For these settings the GAM and OLS results are quite similar. The MARS procedure, however, detects the nonlinearity. The importance of these findings is that they are consistent with a model where there are changes in model parameters as a function of values not, just time. Such changes will not usually be picked up by techniques such as recursive residual analysis, which was used in Sinai and Stokes (1989).

5. Time series results

A VAR model with 2 lags was inverted and using Monte Carlo Integration bounds were obtained for 20 impulse response values. Two models were estimated, one with time and one without time (Figs. 5, 6). The constant and time, when in the model, were assumed to be deterministic and are thus not a part of the graphs. The idea was that shocks impact the left-hand side variable lnY. One way to think of this is that the right-hand side variables are miss-measured because the lagged shocks are not reflected in their value. Note that while the contemporary shocks of each equation are constrained by the solution of the normal equations not to be related to the right-hand side variables, lagged shocks do not have this

Table 3
MARS model of production function for LnY.

MARS model coefficients							SE	*t*	NON Zero	%
LnY	= 6.1591874						0.46495119E−02	1325	99	100.000
	−0.34753341	∗ max(	LnL{ 0}	−	3.8476338	, 0.0)	0.13022978	−2.66	50	50.505
	−0.60202199	∗ max(	3.8476338	−	LnL{ 0}	, 0.0)	0.72684012E−01	−8.28	48	48.485
+	0.72486722	∗ max(	LnK{ 0}	−	6.6779608	, 0.0)	0.67546816E−01	10.7	50	50.505
	−0.56644389	∗ max(	6.6779608	−	LNK{ 0}	, 0.0)	0.24740604E−01	−22.9	48	48.485
+	0.89362826E−02	∗ max(	TIME{ 0}	−	49.000000	, 0.0)	0.63953241E−03	13.9	50	50.505
	−0.16518614E−02	∗ max(	49.000000	−	TIME{ 0}	, 0.0)	0.21927065E−03	−7.53	48	48.485
	−0.17675943E−01	∗ max(	TIME{ 0}	−	66.000000	, 0.0)	0.24140141E−02	−7.32	31	31.313
		∗ max(	LnK{ 0}	−	6.6779608	, 0.0)				
	−0.30324118E−01	∗ max(	66.000000	−	TIME{ 0}	, 0.0)	0.53221862E−02	−5.69	18	18.182
		∗ max(	LnK{ 0}	−	6.6779608	, 0.0)				
+	0.99731075E−02	∗ max(	TIME{ 0}	−	60.000000	, 0.0)	0.21734428E−02	4.58	39	39.394
		∗ max(	LnL{ 0}	−	3.8476338	, 0.0)				
+	0.77192080E−01	∗ max(	60.000000	−	TIME{ 0}	, 0.0)	0.19731354E−01	3.91	10	10.101
		∗ max(	LnL{ 0}	−	3.8476338	, 0.0)				
+	0.45538074	∗ max(	LNK{ 0}	−	6.6779608	, 0.0)	0.13829854	3.29	40	40.404
		∗ max(	LnM2{ 0}	−	3.9900632	, 0.0)				
	−0.84529648	∗ max(	LNK{ 0}	−	6.2116803	, 0.0)	0.23552356	−3.58	30	30.303
		∗ max(	3.8476338	−	LnL{ 0}	, 0.0)				
Residual sum of squares					3.364792933899840E−03					

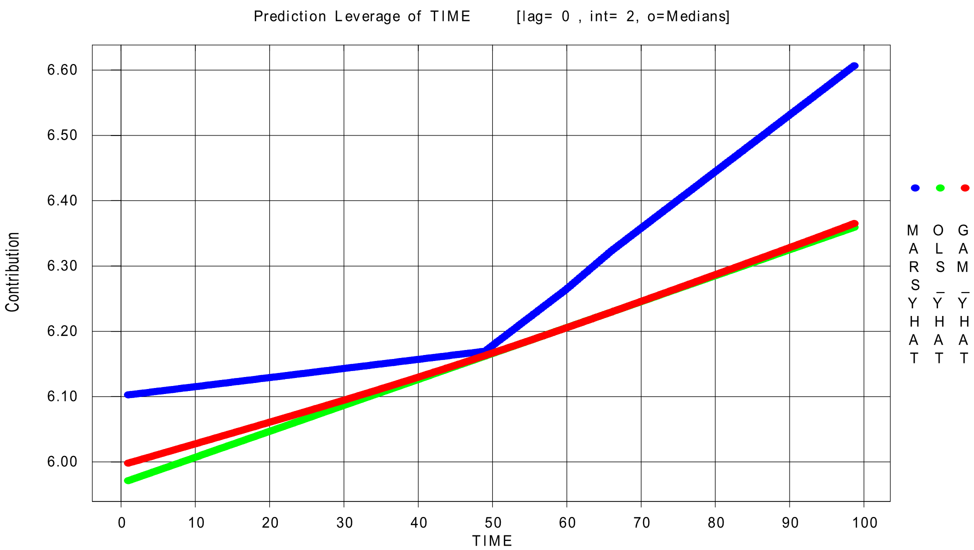

Fig. 1. Leverage of TIME.

property. Variance decomposition of the variance of LnY show the relative importance by lag of the shocks in the proxie for the financial sector LnM2 (Tables 4, 5). The finding that shocks in LnM2 significantly impact LnL, LnK and LnY with a lag is consistent with the view that the real sector is not immune to the financial sector. What is unique to the dataset used is the fact that it does not contain the financial sector labor, capital or output.

6. Flexible least squares

Sinai and Stokes (1989), using the 1929–1967 data, report recursive residual analysis as suggested by Brown, Durbin, and Evans (1975) for Cobb–Douglas models with and without time and with either M1 or M2. The estimated capital coefficient was found to be negative from 1934 to 1949 or 1950. This finding suggests that the data for capital seriously overstates what is available in those years. Recursive residual analysis is Bayesian in spirit, since the coefficients are estimated conditional on the data up to that point. Flexible least squares, while allowing coefficients to move, uses the complete dataset and is not Bayesian. As noted in the theory section, OLS is a special case of FLS, which is implicitly testing the OLS assumption of fixed coefficient estimates over the sample period.

Flexible least squares results, using the methods of Kalaba and Tesfatsion (1989) and a modification of their Fortran code, were calculated for the original Sinai and Stokes (1972) dataset to test for the stability of the TIME variable and are shown in Fig. 7. The findings show that for a number of periods, such as the first nine years and WW II war years, the TIME

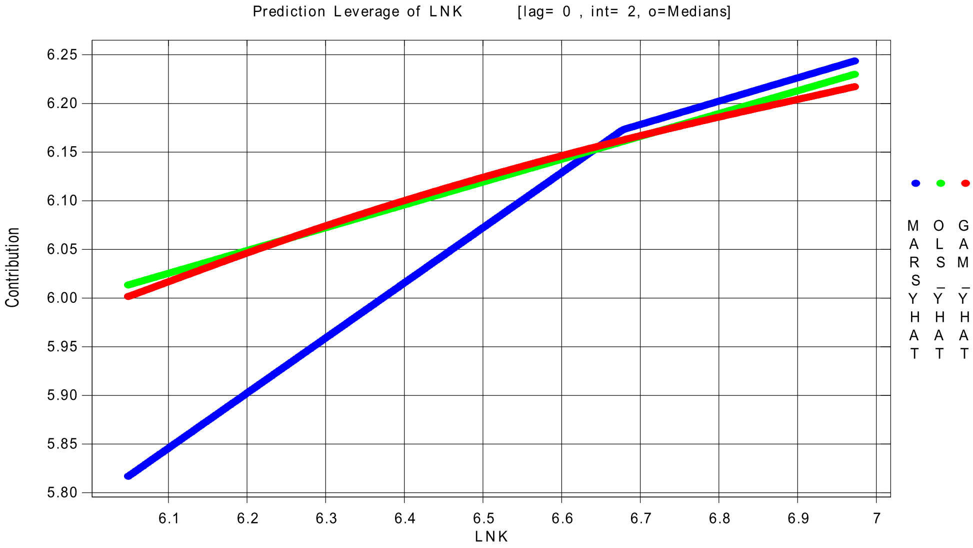

Fig. 2. Leverage of LnK.

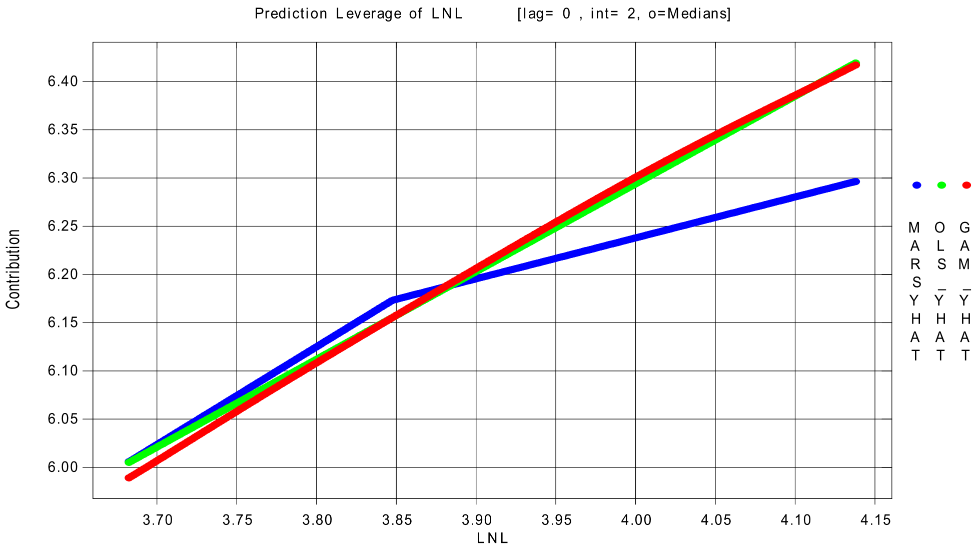

Fig. 3. Leverage of LnL.

coefficient was negative. After 1949 the coefficient was again positive. Since TIME enters the Cobb–Douglas as $e^{\lambda t}$, given the time period was 20, or 1949, and the coefficient was −0.006, the net effect was 0.8869.

7. GAM analysis of Benzing 1959–1985 data

Yearly data in the period 1959–1985 was studied by Benzing (1989). As noted by Lucas (2013, p. 45) in the period 1950–2000, due to the effects of regulation Q that limited payment of interest on demand deposits, the ratio of demand deposits (M1) to GDP fell from 30% to 5%. This major structural change should impact the stability of money in the production function relationship, especially for M1 in the latter period. To present evidence for this hypothesis OLS and GAM models are shown in Table 6 to see if nonlinearity shows up in the monetary variables in the function. Since in the limit $\beta_{i,OLS} \approx \beta_{i,GLS}$, only OLS and GAM results will be shown.

The first thing to note is that for the M1 and M2 models, only the monetary variable is significantly nonlinear in both cases as shown in columns 3 and 7. For the M1 model, time is also found to be nonlinear. Looking first at the M1 model, the residual sum of squares fell from 1.458E−02 to 2.902E−03 when linearity in logs was not imposed and the GAM approach was used. If linearity is imposed on the LnM1 variable, with all other variables potentially allowed to be nonlinear, the residual sum of squares increases to 0.664E−2. We find at a significance level of 99.96% that the linear restriction is binding. For the LnM2 model, the reduction in the residual sum of squares was from 4.120E−03 to 2.163E−03. Imposing linearity

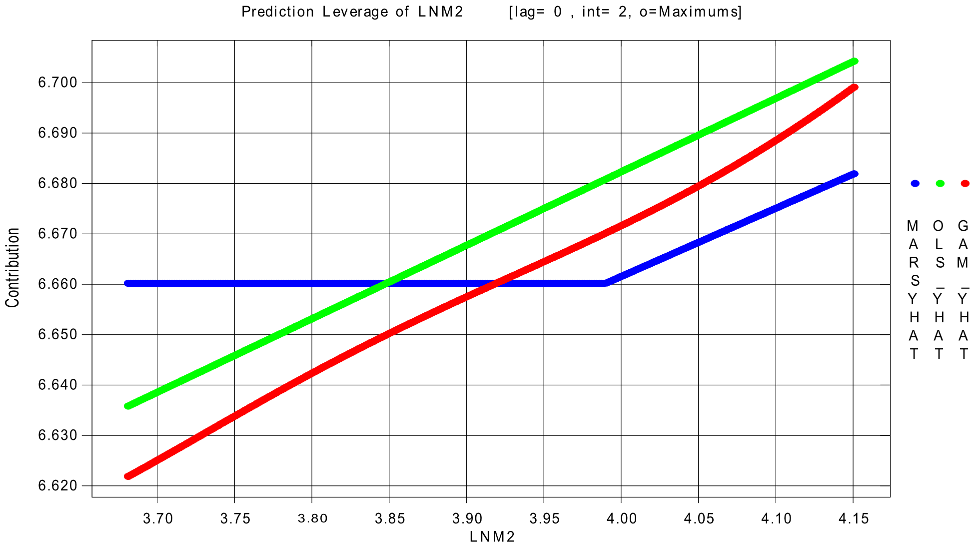

Fig. 4. Leverage of LnM2.

on only the LnM2 variable results in an increase in the residual sum of squares to 0.3711E−2, which is significant at the 98.17% level. At the very least, these results are suggestive that the log-linear Cobb–Douglas specification is not appropriate. Another finding is that the GAM model containing LnM2 has all variables significant while in the GAM LnM1 model only LnL, TIME and LnM1 are significant.

8. Conclusion

The finding of increasing returns to scale in the original Sinai and Stokes (1972) production function model, while consistent with a similar finding of Bodkin and Klein (1967) using earlier data in a Cobb–Douglas production function without real balances, is found in a number of datasets in different periods, whether or not they have money balances and/or time in the production function. The fact that the addition of these real balances and time variables alters both the measured returns to scale and the labor and capital coefficients suggests that a production function without these variables may not be correctly specified.

A GAM model with log labor, log capital, log real M2 and time in the period 1953:1 to 1977:3 using quarterly data showed nonlinearity for all variables except log real M2. OLS, MARS and GAM leverage plots were used to show how the coefficients moved, depending on the values of the variables. This finding is consistent with Griliches (1986, p. 1469) argument that “there are no data problems only model problems in econometrics.”

A VAR model of a production function containing real M2, both with and without time with two lags using quarterly data in the period 1953:1 to 1977:3, was inverted into its VMA form. Shocks in the log real M2 variable were shown to have a significant impact on the labor variable, the capital variable and the output variable. The fact that these shocks impact labor and capital is consistent with the fact that there are measurement problems in the data themselves that are a function of unexpected shocks in the monetary variable. Recall that this is the first view of data management suggested by Griliches (1986). The economic implication of this result is that shocks in the financial sector, as measured by the LnM2 variable, have significant effects on the real sector variables. Real balances are thus not neutral.

Using the longer yearly data in the period 1929–1967 flexible least squares findings show that for a number of periods, such as the first nine years and the WW II war years, the TIME coefficient was found to be negative. After 1949 the coefficient was again positive. Since TIME enters the Cobb–Douglas as $e^{\lambda t}$, given the period was 20, and the coefficient was −0.006, the net effect was 0.8869.

Finally, to test the effect of the ratio of M1 to GDP falling from 30% to 5% in the period 1950–2000 as noted by Lucas (2013), nonlinearity tests were performed on the model, using the Benzing 1959–1985 data, with the result that in a model containing TIME and LnM1, both were found to be nonlinear while labor and capital were found not to be nonlinear. If LnM2 is substituted for LnM1, then only LnM2 is found to be nonlinear. These results are different from what was found for the earlier 1929–1967 period when it was labor and capital that were found to be nonlinear. A possible result was that in the latter period there were structural changes in the real sector (labor and capital) while in the 1959–1985 period the major change appears to be in the way real balances are entering the production process. Clearly, more work has to be done on improving the monetary variable to be used in the production function.

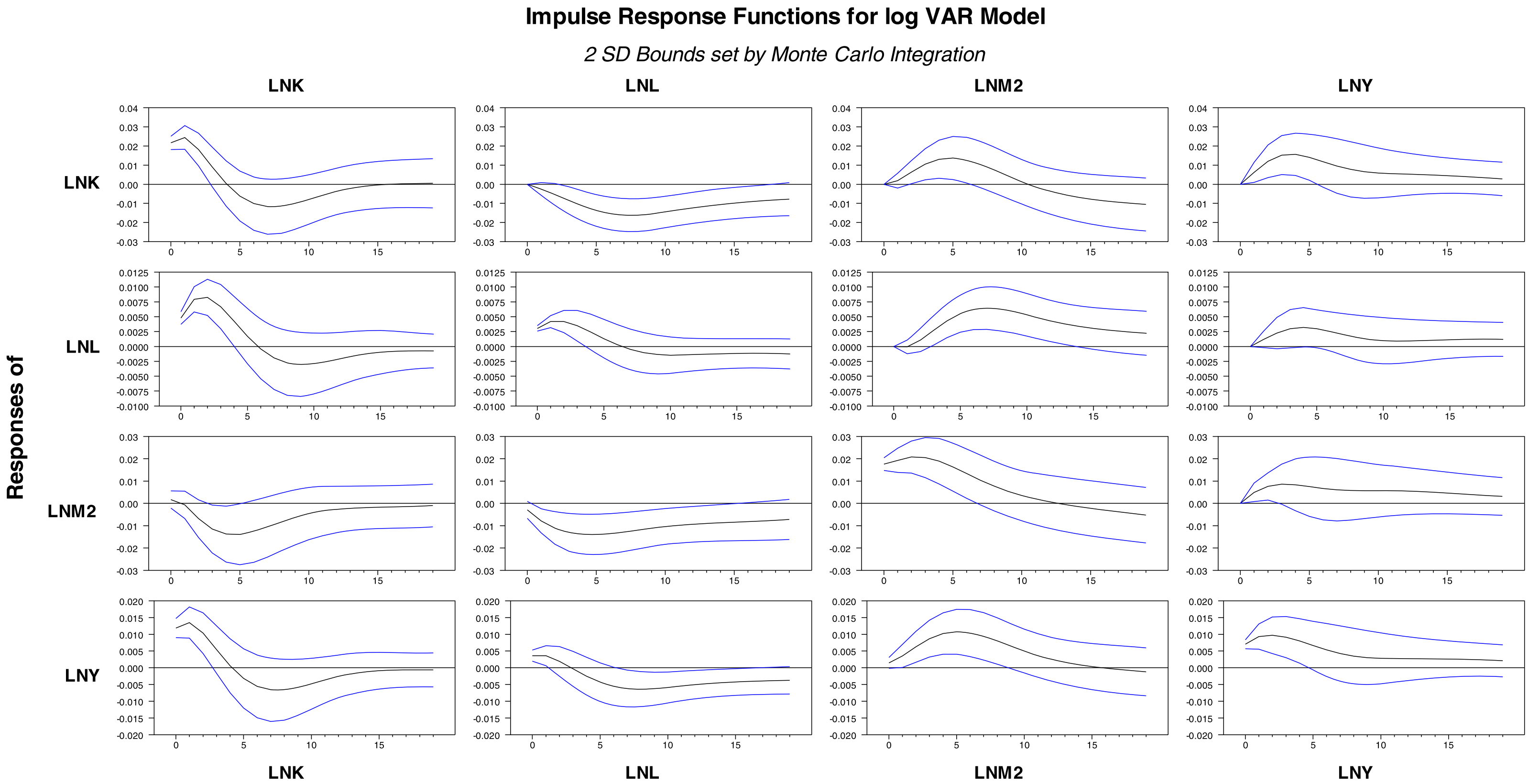

Fig. 5. Impulse response functions for dynamic Cobb–Douglas model with time.

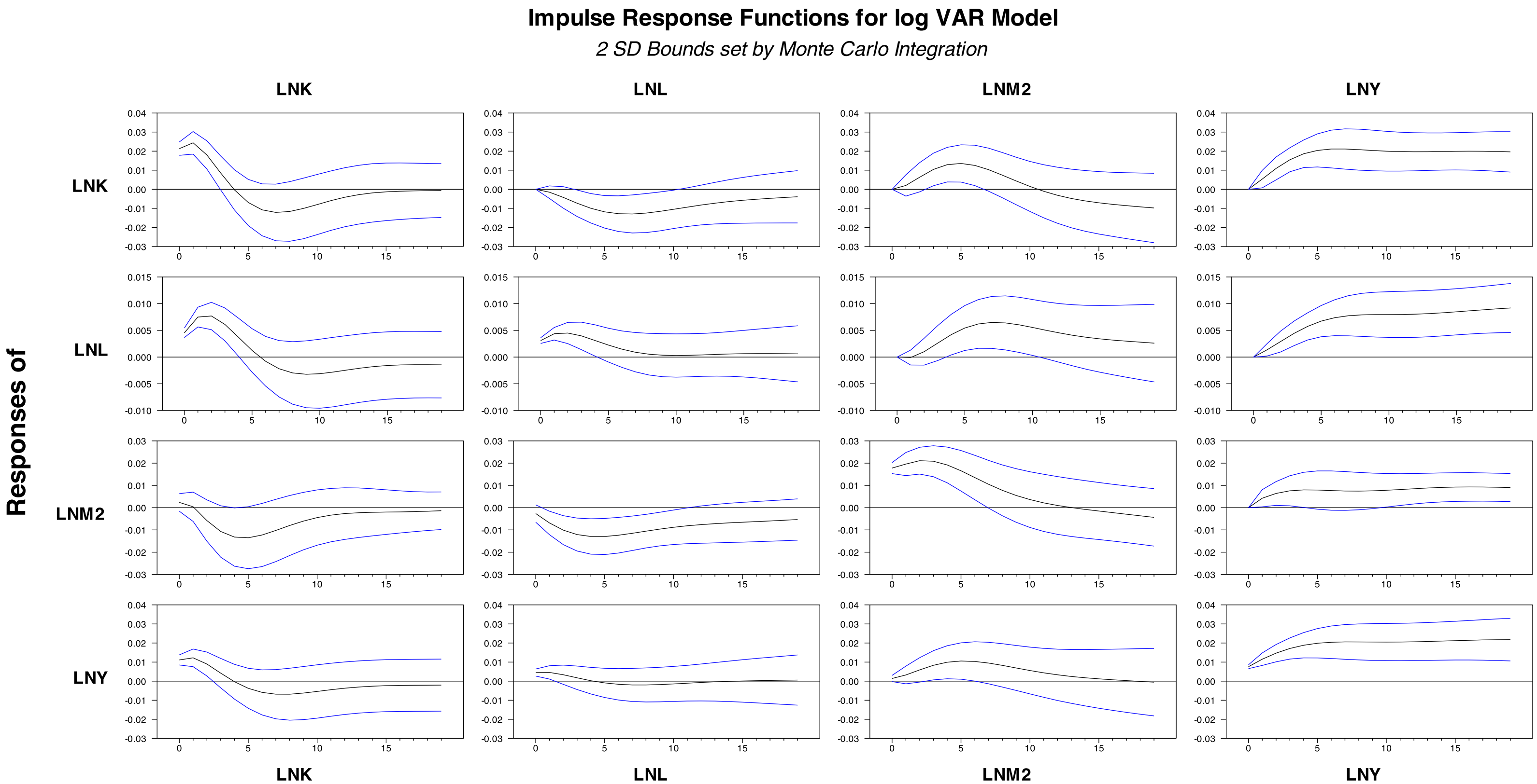

Fig. 6. Impulse response functions for dynamic Cobb–Douglas model without time.

Table 4
Decomposition of variance for series LnY for model with time.

Step	Std error	LnK	LnL	LnM2	LnY
1	0.01317462	67.482	6.702	1.179	24.638
2	0.02029810	59.305	7.328	4.347	29.019
3	0.02493349	49.612	6.641	11.977	31.770
4	0.02854555	39.176	5.281	23.715	31.829
5	0.03222686	30.944	4.235	35.692	29.129
6	0.03623711	26.647	4.053	44.102	25.198
7	0.04022200	25.472	4.578	48.359	21.590
8	0.04371243	25.820	5.437	49.819	18.925
9	0.04642593	26.578	6.365	49.872	17.185
10	0.04832020	27.231	7.230	49.381	16.158
11	0.04952266	27.624	7.980	48.771	15.624
12	0.05023191	27.779	8.601	48.216	15.404
13	0.05063888	27.776	9.101	47.757	15.367
14	0.05088547	27.692	9.500	47.385	15.423
15	0.05105767	27.579	9.825	47.078	15.518
16	0.05119907	27.459	10.104	46.819	15.618
17	0.05132907	27.341	10.358	46.596	15.705
18	0.05145643	27.226	10.602	46.403	15.769
19	0.05158605	27.113	10.842	46.240	15.804
20	0.05172143	27.001	11.077	46.112	15.809

Table 5
Decomposition of variance for series LnY for model without time.

Step	Std error	LnK	LnL	LnM2	LnY
1	0.01346959	60.011	9.879	0.776	29.334
2	0.02236818	45.583	10.178	1.009	43.230
3	0.02997737	31.829	10.227	2.773	55.172
4	0.03727621	21.977	9.600	4.957	63.466
5	0.04427464	15.771	8.837	6.525	68.867
6	0.05077949	11.997	8.290	7.208	72.506
7	0.05669220	9.625	8.073	7.151	75.151
8	0.06206972	8.031	8.195	6.633	77.141
9	0.06707510	6.898	8.626	5.920	78.556
10	0.07189286	6.076	9.312	5.202	79.410
11	0.07666490	5.481	10.182	4.575	79.763
12	0.08146750	5.042	11.159	4.064	79.735
13	0.08632055	4.703	12.172	3.656	79.469
14	0.09120971	4.420	13.167	3.327	79.086
15	0.09610673	4.169	14.110	3.056	78.666
16	0.10098248	3.938	14.988	2.828	78.246
17	0.10581330	3.725	15.801	2.636	77.838
18	0.11058310	3.531	16.556	2.473	77.440
19	0.11528320	3.356	17.263	2.337	77.044
20	0.11991087	3.201	17.932	2.224	76.644

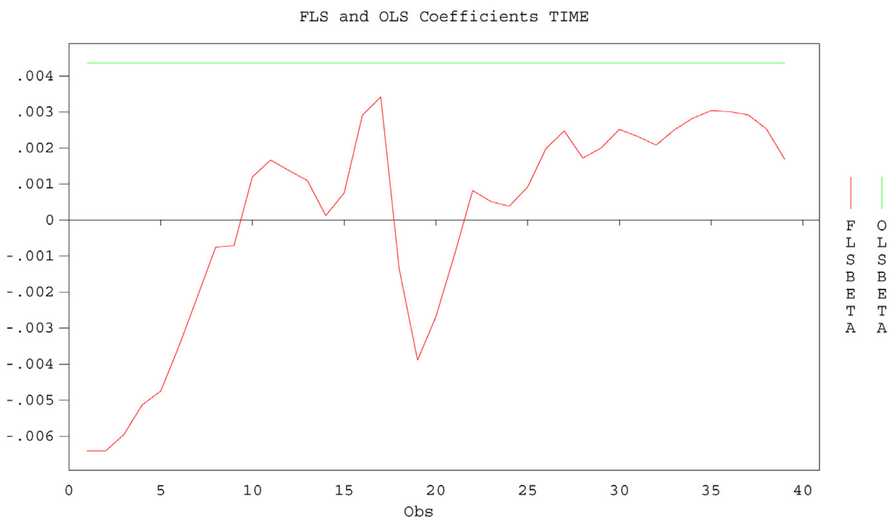

Fig. 7. FLS pattern of TIME using Sinai and Stokes (1972) 1929–1967 data.

Table 6
OLS and GAM models of the 1959–1985 benzing data using LnM1 and LnM2.

Col: Variable	1 OLS	2 GAM	3 NL_Pval	4 Lin_Res	5 OLS	6 GAM	7 NL_Pval	8 Lin_Res
Time	0.7500e−3 (0.062)	0.1343e−1 (1.98)	0.991	0.5306e−2	−0.1121e−1 (−1.80)	−3.523 (−2.61)	0.287	0.2374e−2
LnK	0.4903 (1.86)	0.444e−3 (0.302e−2)	0.796	0.3854e−2	0.3836 (2.74)	0.3310 (2.60)	0.425	0.2469e−2
LnL	0.6800 (2.45)	0.9488 (6.12)	0.736	0.3726e−2	0.7378 (5.26)	0.8189 (6.43)	0.349	0.2415e−2
LnM1	0.4352 (3.73)	0.2976 (4.56)	1.00	0.664e−2				
LnM2					0.5179 (10.25)	0.4323 (9.43)	0.982	0.3711e−2
Constant	−3.237 (−1.15)	−1.210 (−0.769)			−3.868 (−2.60)	−3.523 (−2.61)		
e′e	0.1484e−1	0.2902e−2			0.4120e−2	0.2163e−2		

LnY is the dependent variable. t scores and Z scores are listed under OLS and GAM coefficients respectively. NL_Pval is the significance of a test for nonlinearity for that variable. A value of 0.95 indicates that nonlinearity is significant at the 5% level. Lin_Res is e′e assuming that that variable is restricted to be linear.

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