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Braided Cobwebs: Cautionary Tales for Dynamic Pricing in Retail Electric Power Markets

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***Abstract*—This study investigates the effects of dynamic-price retail contracting on integrated retail and wholesale (IRW) power market operations. Performance is evaluated by means of carefully defined metrics for system stability and market participant welfare. The study is carried out for an IRW Test Case for which 500 households have price-responsive air-conditioning systems. It is shown that dynamic-price retail contracting can give rise to braided cobweb dynamics consisting of two interwoven cycles for power and price levels exhibiting either stability or instability depending on system conditions. Moreover, even in stable cases, dynamic-price retail contracts generally result in worse welfare outcomes for households than flat-rate retail contracts.**

***Index Terms*—Electric power system, integrated transmission and distribution operations, retail dynamic pricing, cobweb dynamics, system stability, market participant welfare**

NOMENCLATURE

***Parameters*:**

a	GenCo cost coefficient (\$/MWh)
α	Household parameter (Utils/\$) reflecting comfort-cost trade-off preferences
b	GenCo cost coefficient (\$/(MW)^{2}h)
c	Household demand coefficient (\$/MWh)
$c(m)$	$c(m) = c/[1+m]$
d	Household demand coefficient (\$/(MW)^{2}h)
$d(m)$	$d(m) = d/[1+m]$
m	Mark-up factor (scalar) for retail price

***Variables and Functions*:**

$\mathrm{E}^{w*}(m)$	Market equilibrium point for wholesale power and price outcomes as a function of m
$\mathrm{E}^{r*}(m)$	Market equilibrium point for retail power and price outcomes as a function of m
FD	LSE fixed demand bid (MW)
p	Generic symbol for a power level (MW)
p^{DA}	Power dispatch (MW) scheduled in the day-ahead market
p^{RT}	Power dispatch (MW) determined in the real-time market
p^{RET}	Retail power usage (MW)
π	Generic symbol for a power price (\$/MWh)
π^{DA}	Locational marginal price (\$/MWh) determined in the day-ahead market
π^{RT}	Locational marginal price (\$/MWh) determined in the real-time market
π^{RET}	Retail power price (\$/MWh)
R	Administratively-set flat rate (\$/MWh)
$\mathbf{r}(t)$	Retail power and price vector at time t

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I. INTRODUCTION

THIS study investigates the effects of dynamic-price retail contracting on integrated retail and wholesale (IRW) power market operations. Contract performance is evaluated by means of carefully defined metrics for system stability and market participant welfare. The latter metrics are separately calculated for wholesale Generation Companies (GenCos), Load-Serving Entities (LSEs), and retail households.

The study is implemented by means of the IRW Test Bed [2], a computational platform permitting the integrated study of retail and wholesale power markets operating during successive days over linked distribution and transmission grids. Particular attention is focused on an IRW Test Case for which the distribution system consists of 500 households with price-responsive air-conditioning (A/C) systems. Household power needs are serviced by an LSE that procures power at wholesale prices and resells this power to households at retail prices.

A simplified version of the IRW Test Case with postulated linear aggregate demand curves for hourly household A/C power usage is first used to derive, analytically, a set of necessary and sufficient conditions for power system stability under dynamic-price retail contracting. A key finding is that the use of dynamic-price retail contracts induces braided cobweb dynamics consisting of two interwoven cobweb cycles for power and price outcomes. These braided cobweb cycles can exhibit either stability or instability depending on a small set of structural parameters characterizing power supply and demand conditions.

A dynamic welfare sensitivity study is then conducted for the full IRW Test Case with 500 households, differentiated by randomly assigned structural attributes, who have A/C systems locally managed by intelligent price-responsive controllers. Since the resulting aggregate demand curves for hourly household A/C power usage exhibit approximately linear forms, braided cobweb dynamics again arise.

The three systematically varied treatment factors for this sensitivity study are: (i) the form of retail contracts, either flat-rate or dynamic-price; (ii) a mark-up factor m that determines the percentage by which retail prices are marked up over wholesale prices in the case of dynamic-price retail contracts; and (iii) a preference parameter α^n for each household n that determines n's willingness to trade off comfort against energy cost savings. For treatments involving flat-rate contracts, the

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flat rate is set to ensure that the LSE managing the flat-rate contracts breaks even over time.

Welfare outcomes and energy costs are reported for a range of treatments for which cobweb dynamics are stable. A key finding is that, all else equal, a dynamic-price retail contract with a positive mark-up m results in *worse* welfare outcomes for households than a flat-rate retail contract.

This difficulty with dynamic-price retail contracts arises because these contracts are based on a one-way communication of retail prices from LSEs to households. The households locally optimize, taking retail prices as given. However, no attention is paid to the potentially adverse feedback effects of the resulting household power usage levels on future wholesale and retail prices. These longer-run adverse effects are only observed by considering IRW operations over successive days.

The organization of this paper is as follows. Section II briefly reviews the growing interest in demand-response initiatives. Section III discusses the key features and capabilities of the IRW Test Bed. The IRW Test Case, implemented via the IRW Test Bed, is presented and explained in Section IV.

Section V undertakes a comprehensive analytical study of the braided cobweb dynamics arising in a simplified version of the IRW Test Case with postulated linear downward-sloping aggregate demand curves for hourly household A/C power usage. A welfare sensitivity study for the full IRW Test Case with simulated household A/C power demands is outlined in Section VI, and key findings for this study are reported in Section VII. Concluding discussion is provided in Section VIII. Technical details regarding welfare metrics are provided in an appendix.

II. Growing Interest in Demand Response

In traditional U.S. power systems based on vertically integrated utilities, the power usage of residential, commercial, and industrial customers was assumed to be highly unresponsive to price changes. Utilities typically charged their customers a flat hourly rate for power usage, plus additional fixed charges, on an extended (e.g., monthly) basis.

A critical utility task, referred to as *load following*, was then to ensure the continual balancing of real-time power usage and line losses with real-time power generation, whatever form this power usage took. Customers thus became accustomed to extracting power from the grid without any consideration of its actual production cost or environmental impact.

This traditional conception of customer power usage as externally determined load in need of balancing has been carried forward into restructured U.S. wholesale electric power systems. Although, in principle, *load-serving entities (LSEs)* participating in day-ahead markets are permitted to submit hourly demand bids for the next-day power needs of their customers in two parts – a price-responsive demand schedule and a fixed power amount – most LSE hourly demand takes a fixed form.[1]

As far back as 2002, power economists have forcefully argued the need for participants on both sides of a power market, buyers and sellers, to be able to express their reservation values[2] for power in order to achieve an efficient pricing of power; see, e.g., [4], [5, Chapter 1-1], [6], and [7]. However, given the relatively primitive state of metering technology, it was not feasible for power customers to adjust their power usage in real time in response either to system operator commands or to automated signals.

Consequently, despite some experimentation with broadly stepped time-varying rates (e.g., peak-load pricing and time-of-use plans), power customers continued to play a largely passive role in power system operations. Even today, most U.S. households and small businesses still have traditional meters and pay a flat rate (cents/kWh) for their power usage plus a lump-sum dollar charge for fixed costs [8].

This traditional relationship between power buyers and sellers is now beginning to change. Recent breakthroughs in metering technology, referred to as *Advanced Metering Infrastructure (AMI)*, have radically improved the potential for more active customer participation [9]. AMI broadly refers to an integrated system of meters, communication links (wired or wireless), and data management processes that permits rapid two-way communication between power customers and the agencies (e.g., utilities) that manage their power supplies.

In particular, AMI enables the implementation of *demand-response (DR)* initiatives designed to encourage fuller demand-side participation in power system operations. Looking ahead to increased AMI penetration, power system researchers are exploring three basic types of DR initiatives:[3]

> *(i) Incentive-Based Load Control:* Down/up adjustments in the power usage of household and business devices are undertaken, either in response to direct requests from designated parties,[4] or via device switches under the remote control of designated parties, with compensation at administratively set rates.
> *(ii) Dynamic Pricing:* Down/up power usage adjustments are undertaken by households and/or businesses in response to changes in power prices communicated to them by designated parties.
> *(iii) Transactive Energy System (TES):* Demands and supplies for power and ancillary services by households and businesses are determined by decentralized bid/offer-based transactions.

The implementation of these DR initiatives can result in curtailments (or increases) in total power withdrawal from the grid, or in shifts in the timing of power withdrawals from the grid with no significant change in total power withdrawal. In some cases, DR resources might be willing and able to offset curtailments (or increases) in their power withdrawals from

[1] For example, the percentage of cleared price-sensitive demand in ISO-New England's day-ahead market has remained nearly constant at around 27% from 2012-2016; see [3, Fig. 3-20].

[2] As detailed in [4], a *buyer's reservation value* for a good or service at a particular point in time is defined to be the buyer's maximum willingness to pay for the purchase of an additional unit of this good or service at that time. A *seller's reservation value* for a good or service at a particular point in time is defined to be the minimum payment that the seller is willing to receive for the sale of an additional unit of this good or service at that time.

[3] For surveys covering all three types of DR research, see [10], [11], [12], [13]. For DR deployment in the U.S., see [14], [15], [16].

[4] These designated parties can be system operators or utilities. They can also be intermediaries who manage collections of customer-owned energy resources in accordance with system operational requirements.

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the grid by resorting to local "behind the meter" generation and storage facilities, such as an on-site wind turbine or a small-scale battery system with no grid connection.

A key goal of type-(i) DR initiatives is to permit ancillary services to be extracted from demand-side resources in support of system *reliability*. A key goal of type-(ii) DR initiatives is to enhance system *efficiency* by permitting household and business customers to express their reservation values for power at different times and locations. A key goal of type-(iii) DR initiatives is to enhance the reliability *and* efficiency of system operations by enabling a balancing of demands and supplies for power and ancillary services across an entire electrical infrastructure on the basis of household and business reservation values.

Researchers focusing on type-(i) and type-(ii) DR initiatives have primarily stressed metering, control, and planning aspects for system operators and power customers. For example, refs. [17, Chapters 2-3] and [18] investigate the ability of type-(i) DR programs to provide reserve services for system operators. Ref. [19] studies various forms of control strategies for type-(i) initiatives designed to maximize the net benefits of building residents subject to constraints.

Ref. [20] develops a type-(ii) smart controller for a household air-conditioning system. The controller uses stochastic dynamic programming to determine optimal next-day power usage in accordance with household comfort/cost trade-off preferences, conditional on price signals for next day power usage and a forecast for next-day weather. Ref. [21] develops a two-stage co-optimisation framework for a customer who installs a battery system in stage 1 and decides among several offered type-(i) and type-(ii) DR programs in stage 2.

However, some work has explored the effects of type-(ii) DR initiatives on power system operations over time. For example, ref. [22] develops an agent-based computational platform to study the effects of price-responsive power demand by commercial buildings, modeled as autonomous agents with reinforcement learning capabilities. These building agents compete to offer DR services into a wholesale power market operating over a transmission grid. Also, ref. [23] reports results from a preliminary investigation of the effects of price-responsive household demands for power on integrated transmission and distribution system operations over time.

A *Transactive Energy System (TES)* is a set of economic and control mechanisms that permits the balancing of demands and supplies for power across an entire electric power system while still maintaining system reliability [24, p. 11] . Researchers focusing on type-(iii) DR initiatives are interested in understanding the potential effects of proposed TES designs on the operations of an entire electric power system. As detailed in ref. [25], TES designs are typically based on two-way communications between collections of *distributed energy resource (DERs)* and various forms of entities that manage the power usage of these DER collections.

As clarified below, the results of this study raise cautionary concerns about the potentially destabilizing effects of type-(ii) dynamic-price retail contracts based on one-way communication. These results suggest that an increased research emphasis on type-(iii) TES designs based on two-way communication would be desirable.

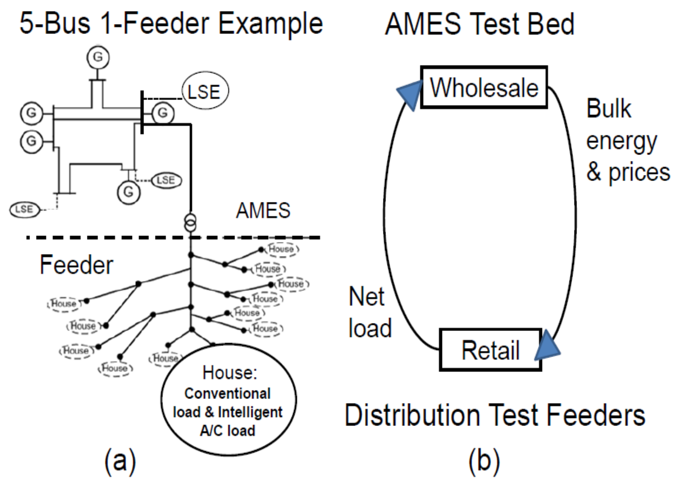

Fig. 1. (a) Simple illustration of the IRW Test Bed; and (b) the basic IRW feedback loop.

III. The IRW Test Bed

As illustrated in Fig. 1, the *Integrated Retail and Wholesale (IRW) Test Bed* [2] is an agent-based computational platform permitting researchers to study the integrated grid-constrained operations of retail and wholesale power sectors over successive days. The IRW Test Bed has five key components:

- *C1:* An ETP modeling for the *thermal state of a household over time*;
- *C2:* A C++ implementation of an *intelligent price-responsive controller* for each household's A/C system;
- *C3:* A GridLAB-D (C++) implementation of *conventional (non-price-responsive) household loads*;
- *C4:* An AMES (Java) implementation of a *wholesale power sector*;
- *C5:* A *MySQL database server* to handle data storage and transfer among components C1-C4.

With regard to C1, the IRW Test Bed uses the physics-based *Equivalent Thermal Parameter (ETP)* model [26] to represent each household's A/C system. The basic ETP model supposes that the thermal state of a household at any given time is described by inside air and mass temperatures whose movement over time can be expressed by a parameterized system of linear differential equations.

With regard to C2, each household's A/C system is managed by an intelligent price-responsive A/C controller, as developed in [20]. At the beginning of each day this controller uses stochastic dynamic programming to determine the household's optimal next-day A/C power usage. This optimal power usage is based on the household's comfort/cost trade-off preferences, forecasted (or announced) next-day retail power prices, and forecasted next-day environmental conditions (outside air temperature, humidity, and solar radiation).

With regard to C3, the IRW Test Bed uses GridLAB-D [27] to model conventional household loads. GridLAB-D is an open-source agent-based computational platform for the modeling of distribution systems that provides detailed physics-based models for a wide variety of appliances and equipment owned by residential, commercial, and industrial customers.

With regard to C4, the IRW Test Bed uses a suitably modified version of AMES (V3.0) to model a wholesale power sector whose loads include the LSE-managed power usage of retail households. AMES (*A*gent-based *M*odeling of *E*lectricity *S*ystems) [28] is an open-source agent-based computational platform that captures key features of actual U.S. centrally-managed wholesale power markets.

More precisely, as depicted in Fig. 2, AMES (V3.0) models the operations of a *Day-Ahead Market (DAM)* and a *Real-Time Market (RTM)* managed by an *Independent System Operator (ISO)*. The DAM and RTM operate in tandem over a high-voltage AC transmission grid during successive days. Congestion on the grid is handled by *locational marginal prices (LMPs)*, i.e., by the pricing of power in accordance with the location and timing of its injection into, or withdrawal from, the transmission grid.

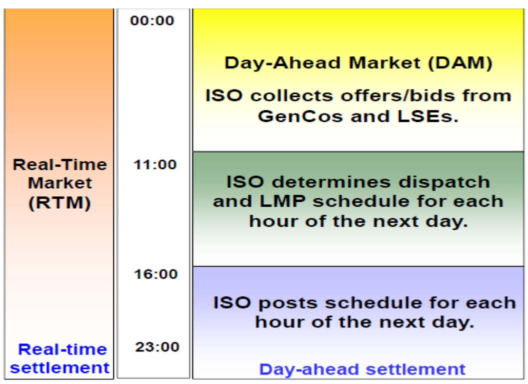

Fig. 2. Daily operation of the day-ahead and real-time markets in the IRW Test Bed, implemented via AMES (V3.0).

The daily operations of the DAM and RTM in the IRW Test Bed proceed roughly as follows. During the morning of each day D a collection of user-specified *Generation Companies (GenCos)* and *Load-Serving Entities (LSEs)* submit into the day-D DAM a collection of supply offers and demand bids, respectively, for all 24 hours H of day D+1. For each hour H, these offers and bids take the following general form:

$$\text{GenCo Price-Responsive Supply: } \pi = a + 2bp\,; \qquad (1)$$

$$\text{LSE Price-Responsive Demand: } \pi = c - 2dp\,; \qquad (2)$$

$$\text{LSE Fixed Demand: } p = \text{FD}\,, \qquad (3)$$

where: π (\$/MWh) denotes price, p (MW) denotes power, FD (MW) denotes a fixed (non-price-responsive) demand for power, and a (\$/MWh), b (\$/(MW)^{2}h), c (\$/MWh), and d (\$/(MW)^{2}h) are positive coefficients. The power levels in eqns. (1) through (3) represent constant power levels to be maintained during the entire hour H, either as injections into the grid (power supplies) or as withdrawals from the grid (power demands).

Given these offers and bids for the day-D DAM, the ISO solves a *Security-Constrained Economic Dispatch (SCED)* optimization problem subject to standard transmission line, generation capacity, and power balance constraints in order to determine scheduled GenCo dispatch levels and an LMP π^{DA}(B,H,D+1) (\$/MWh) at each transmission bus B for each hour H of day D+1.[5] A GenCo is paid π^{DA}(B,H,D+1) for each MW it is scheduled to inject at transmission bus B during hour H of day D+1, and an LSE must pay π^{DA}(B,H,D+1) for each MW its customers are scheduled to withdraw at transmission bus B during hour H of day D+1.

The RTM runs each hour of each day.[6] At the start of the RTM for hour H on day D+1, the ISO is assumed to know the actual power usage at each transmission bus B for hour H of day D+1. The ISO then solves a SCED optimization problem to resolve any discrepancies between the generation *scheduled* in the day-D DAM for dispatch at each transmission bus B during hour H of day D+1, conditional on day-D LSE demand bids, and the generation needed at each transmission bus B during hour H of day D+1 to balance *actual* power withdrawals. Any needed adjustments in the DAM-scheduled power supplies and demands at bus B for hour H of day D+1 are settled at π^{RT}(B,H,D+1), the RTM LMP (\$/MWh) at bus B for hour H of day D+1.

IV. IRW Test Case

A. Overview

This section describes a simplified version of the IRW Test Bed, referred to as the *IRW Test Case*. This test case will be used to explore how a switch from flat-rate to dynamic-price retail contracts for households affects IRW system operations over time under systematically varied parameter specifications. Particular attention will be paid to the effects of this switch on system stability and welfare outcomes.

B. IRW Test Case Assumptions

The IRW Test Case includes one GenCo and one LSE located at the same transmission bus B. Hence, transmission grid congestion is not an issue. The LSE serves as a wholesale power purchasing agent for 500 households populating a distribution system connected to the transmission system at transmission bus B. The GenCo has sufficient capacity to meet the power needs of the 500 households.

The power usage of each household arises solely from its A/C system. As detailed in [20], the A/C load of each household is managed by an intelligent controller that uses stochastic dynamic programming to determine optimal next-day power usage in accordance with household comfort/cost trade-off preferences, conditional on retail price signals for next day power usage and a forecast for next-day outside air temperature. The dynamics governing the thermal state of each household n are represented by a simplified ETP model with inside air temperature T_a^n as the only state variable, as follows:

$$\frac{dT_a^n(t)}{dt} = z_1^n[T_o(t) - T_a^n(t)] - z_2^n p^n(t)\,, \qquad (4)$$

where $T_o(t)$ denotes outside air temperature at time t, $p^n(t)$ denotes household n's A/C power level (ON or OFF) at time t, and the coefficients z_1^n and z_2^n are positively valued.

[5] In the LMP expression π^{DA}(B,H,D+1), the term D+1 refers to the delivery day for the DAM conducted on the market settlement (clearing) day D.

[6] In actual U.S. centrally-managed wholesale power markets the RTM is conducted at least once every five minutes.

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Given these assumptions, the feedback loop determining retail prices and real-time loads over time for the IRW Test Case is as depicted in Fig. 3. A more detailed description of this feedback loop will now be provided, starting from the lower right. For ease of notation, all references to transmission bus B are suppressed in this description.

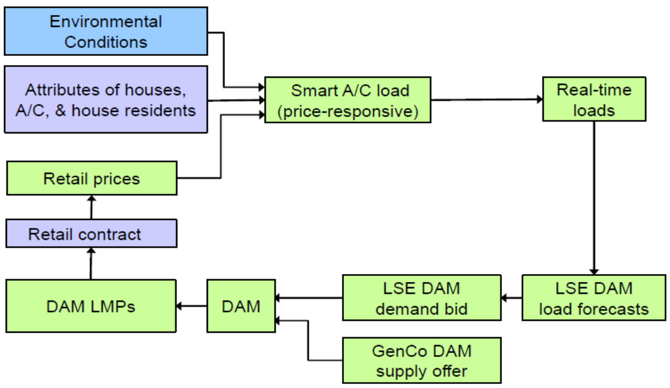

Fig. 3. Feedback loop for the IRW Test Case. Since retail prices and real-time loads are not affected by RTM imbalance adjustments, these adjustments are not depicted.

The supply offer submitted by the GenCo to the day-D DAM for any specific hour H of day D+1 takes form (1). This supply offer gives the GenCo's reservation value (marginal production cost) for each successive MW of power it might be required to generate during hour H of day D+1; that is, it gives the GenCo's *marginal cost function*.

The demand bid submitted by the LSE to the day-D DAM for any specific hour H of day D+1 takes the fixed form (3). This demand bid is the LSE's forecast for the aggregate load of its household customers during hour H of day D+1.

In this study it is assumed the LSE sets this forecast equal to the actual aggregate load observed for its household customers during hour H on day D-1. The simple form of this LSE load forecast can be supported on four grounds.

First, the LSE must make this forecast before the realization of household loads for day D. Second, although the LSE understands that household power usage can be price responsive, the exact form of this price responsiveness depends on two factors that the LSE cannot directly observe: (i) the comfort/cost preferences of each household resident; and (ii) the structural attributes of each house, which determine how outside weather conditions affect inside air temperature. Third, the only external forcing terms considered in this study are routine weather variations exhibiting high degrees of serial correlation during any particular hour across successive days. Fourth, household A/C power usage is strongly correlated with weather conditions.

The retail contracts offered by the LSE to the 500 households take one of two possible forms: (i) a *flat-rate* retail contract with a flate rate R ($/MWh) set to ensure the LSE breaks even over time; or (ii) a *dynamic-price* retail contract with one-way communication (LSE to households) in which DAM LMPs (marked up by a percentage $m \geq 0$) are passed through to households as next-day retail prices. More precisely, under dynamic-price retail contracts, the *retail price* charged by the LSE to each household withdrawing power for usage during hour H of day D+1 is given by

$$\pi^{\mathsf{RET}}(\text{H,D+1}) = [1+m]\pi^{\mathsf{DA}}(\text{H,D+1})\,, \tag{5}$$

where π^{DA}(H,D+1) ($/MWh) is the LMP determined in the day-D DAM for hour H of day D+1.[7]

As detailed in [20], the *utility (benefit)* U^n(H,D+1) attained by household n from the purchase and use of electric power during any hour H of any day D+1 takes the form

$$\text{Comfort}^n(\text{H,D+1}) - \alpha^n \text{EnergyCost}^n(\text{H,D+1})\,. \tag{6}$$

In eqn. (6), the comfort (Utils) attained by household n depends on the inside air temperature experienced by n during hour H of day D+1. Also, the energy cost ($) charged to n depends on two factors: (i) n's power usage during hour H of day D+1; and (ii) the form of n's retail contract, either flat-rate or dynamic-price.

The key parameter α^n (Utils/$) in eqn. (6) is a preference parameter measuring the degree to which household n is willing to trade off comfort against energy cost. The higher the value of α^n, the greater the weight that n places on energy cost savings relative to thermal comfort.[8]

V. Analysis of Braided Cobweb Dynamics for the IRW Test Case under Dynamic Pricing

A. Overview

Suppose retail contracts for the IRW Test Case take a dynamic-price form. In this case the LSE uses two simple adaptive methods, one to determine its wholesale demand bids on behalf of its household customers, and one to set retail prices for its household customers.

The manner in which these two adaptive methods enter into daily IRW power system operations will now be explained in some detail. The dependence of price and quantity terms on the transmission bus B is suppressed for ease of notation.

First, at the opening of the DAM on each day D the LSE engages in *adaptive load forecasting*. Specifically, the LSE participates in the day-D DAM by submitting 24 demand bids, one bid p^{DA}(H,D+1) for each hour H of day D+1. The demand bid p^{DA}(H,D+1) is the LSE's forecast for the aggregate load of its household customers during hour H of day D+1. As discussed in Section IV-B, the LSE sets this forecast equal to the aggregate load observed for its household customers during hour H on day D-1.

Second, at the close of the DAM on each day D the LSE engages in *adaptive advance retail pricing*. Specifically, the

[7]Retail prices are typically reported in cents/kWh, not in $/MWh, which would require a conversion factor be given on the right-hand side of eqn. (5). This price conversion is ignored in the current study for ease of exposition.

[8]Roughly defined, α^n measures the benefit to n of an additional dollar of income. It permits costs measured in dollars to be expressed in benefit units (Utils), so that benefit/cost trade-offs can be calculated. The precise sense in which α^n quantifies the trade-off between comfort and energy cost for n is explained in detail in [20, Appendix]. The value of α^n can be derived as the shadow price for n's budget constraint in a more fully articulated constrained benefit maximization problem: namely, the maximization of n's benefit from consumption of multiple goods/services (including thermal comfort) subject to a budget constraint. Thus, α^n measures n's *marginal benefit of income* at the optimization point, given by the rate of change in the maximized value of n's benefit with respect to a change in n's income.

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LSE announces 24 retail prices to its household customers, one retail price π^{RET}(H,D+1) for each hour H of day D+1. As in eqn. (5), the retail price π^{RET}(H,D+1) is an m-percent mark-up of the LMP π^{DA}(H,D+1) determined in the day-D DAM for hour H of day D+1. Note that π^{DA}(H,D+1) is the price the LSE must itself pay for each MW of power it purchases in the day-D DAM for its household customers during hour H of day D+1.

These two adaptive LSE methods, together with other features of the IRW Test Case, give rise to dynamic market cycling behavior known in the economics literature as *cobweb dynamics* [31]. However, a unique aspect of this cobweb dynamics for the IRW Test Case is that the cycling is *braided.*

More precisely, as depicted in Fig. 4, this braided cobweb dynamics consists of two intertwined cycles determining retail and wholesale outcomes on alternative days. For example, starting on any day D-1, one cycle determines retail outcomes for days D-1, D+1, D+3,... and the other cycle determines retail outcomes for days D, D+2, D+4,... .

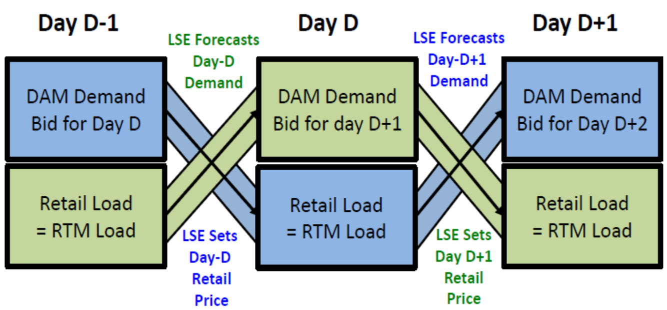

Fig. 4. Braided cobweb dynamics resulting for the IRW Test Case under dynamic-price retail contracts.

This section provides a comprehensive analysis of the braided cobweb dynamics arising for the IRW Test Case when retail contracts take a dynamic-price form. Section V-B presents a summary explanation of these findings through a series of graphical depictions. Section V-C provides a detailed analytical derivation of these findings.

Seven key assumptions characterizing the IRW Test Case with dynamic-price retail contracts, explained with care in Section IV, are listed below in summary form for easy later reference. These assumptions reflect the basic simplifying IRW Test Case assumption that grid congestion is not an issue, hence power and price outcomes do not have to be differentiated by transmission bus location.

Seven Key Assumptions Characterizing the IRW Test Case under Dynamic Pricing:

- **(A1)** RTM load p^{RT}(H,D) is equal to actual retail load p^{RET}(H,D) for each hour H of each day D.
- **(A2)** The GenCo's RTM supply offer for each hour H of each day D takes form (1).
- **(A3)** The ISO ensures RTM market clearing for each hour H of each day D.
- **(A4)** The LSE's DAM demand bids take the fixed-demand form (3). Specifically, on each day D the LSE sets its day-D DAM demand bid p^{DA}(H,D+1) for each hour H of day D+1 equal to the actual retail load p^{RET}(H,D-1) observed during hour H of day D-1.
- **(A5)** The GenCo's DAM supply offer for each hour H of each day D takes form (1).
- **(A6)** The ISO ensures DAM market clearing for each hour H of each day D.
- **(A7)** The LSE makes use of a dynamic-price retail contract with mark-up $m \geq 0$. That is, on each day D-1 the LSE sets the retail price π^{RET}(H,D) for hour H of day D equal to $[1+m]\pi^{\mathsf{DA}}$(H,D), the DAM LMP determined on day D-1 for hour H of day D with an m-percent mark-up.

In addition, for the purposes of this analytical section only, the following auxiliary assumption is imposed:

Auxiliary IRW Test Case Assumption Temporarily Imposed for Analytical Purposes:

- **(A8)** For each hour H of each day D, the aggregate demand curve for A/C power usage determined by the 500 household price-responsive A/C controllers takes the linear form

$$\pi^{\mathsf{RET}} = c - 2dp^{\mathsf{RET}}\,, \tag{7}$$

where the coefficients c and d are positively valued.

Given (A7) and (A8), the actual retail power usage $p^{\mathsf{RET}}(H,D)$ of households during any hour H of any day D is determined from

$$[1+m]\pi^{\mathsf{DA}}(\text{H,D}) = c - 2dp^{\mathsf{RET}}(\text{H,D})\,, \tag{8}$$

where π^{DA}(H,D) denotes the price of power previously determined in the day-(D-1) DAM for hour H of day D. For later purposes, note that the determination of p^{RET}(H,D) via (8) is equivalent to determining p^{RET}(H,D) by means of the modified demand relationship

$$\pi^{\mathsf{DA}}(\text{H,D}) = c(m) - 2d(m)p^{\mathsf{RET}}(\text{H,D})\,, \tag{9}$$

where $c(m) = c/[1+m]$ and $d(m) = d/[1+m]$.

B. Graphical Preview of Analytical Findings

The LSE is the connecting link between the wholesale and retail power sectors for the IRW Test Case. The LSE purchases bulk power at wholesale prices in the DAM and the RTM and then resells this power to households at retail prices.

Fig. 5 depicts supply and demand relationships for the IRW Test Case under the dynamic-pricing assumptions (A1)-(A8), assuming $m > 0$ and $c(m) > a$. The supply curve, a plot of (1), represents the GenCo's marginal production costs in the wholesale power market. The aggregate demand curve, a representation for the aggregate power demand arising in the retail sector from household price-responsive A/C systems, is expressed in two forms: (i) with original coefficients c and d, using the retail price, as in (7); and (ii) with m-modified coefficients $c(m)$ and $d(m)$, using the DAM LMP, as in (9).

Specifically, Fig. 5 depicts supply and demand relationships for the IRW Test Case for any fixed hour H over successive days. Market equilibrium, which might or might not be attained, is a two-point cycle consisting of successive back-and-forth movements between the vector $\mathrm{E}^{w*}(m)$ of wholesale

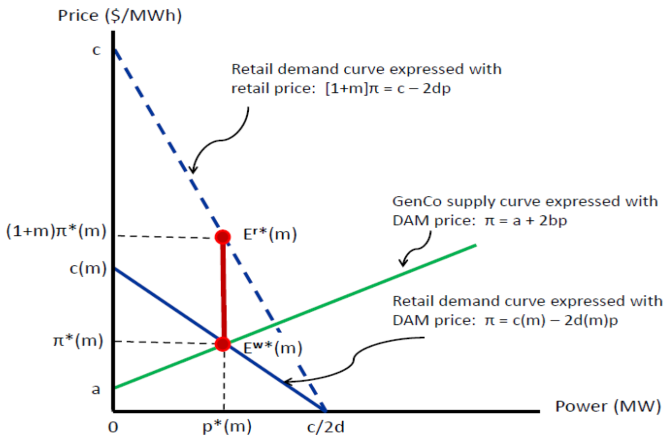

Fig. 5. Market supply and demand relationships for the IRW Test Case with dynamic-price retail contracts for any fixed hour over successive days, given a price mark-up $m > 0$ such that $c(m) > a$. The market equilibrium, which might or might not be attained, is a two-point cycle consisting of persistent back-and-forth movement between $\mathrm{E}^{w*}(m)$ and $\mathrm{E}^{r*}(m)$.

power and price outcomes and the vector $\mathrm{E}^{r*}(m)$ of retail power and price outcomes.

To understand better the nature of this cycle, suppose the LSE's fixed demand bid submitted into the day-(D-1) DAM for hour H of day D induces a vertical demand curve at the power level $p^*(m)$. To balance this fixed demand, the GenCo must be scheduled to produce $p^*(m)$ during hour H of day D. Consequently, the day-(D-1) DAM LMP for hour H of day D must be set at $\pi^*(m)$, as determined from the GenCo's supply curve. The resulting DAM market clearing point $\mathrm{E}^{w*}(m)$, depicted in Fig. 5, takes the following specific numerical form:

$$\mathrm{E}^{w*}(m) = \begin{bmatrix} p^*(m) \\ \pi^*(m) \end{bmatrix} = \begin{bmatrix} \frac{[c(m)-a]}{2[b+d(m)]} \\ \frac{[ad(m)+bc(m)]}{[b+d(m)]} \end{bmatrix} . \quad (10)$$

The LSE then sets the retail price for hour H of day D equal to $[1+m]\pi^*(m)$. This retail price induces a retail power usage (load), determined from the retail demand function. As depicted in Fig. 5, it follows by straightforward calculation that this retail power usage is equal to $p^*(m)$ in (10). Let

$$\mathrm{E}^{r*}(m) = \begin{bmatrix} p^*(m) \\ [1+m]\pi^*(m) \end{bmatrix} \quad (11)$$

denote these retail power and price outcomes. The LSE then submits $p^*(m)$ as its fixed-demand bid for the day-D DAM for hour H of day D+1, and the entire process repeats.

Thus, market equilibrium in Fig. 5 is the 2-point cycle that alternates between the vector $\mathrm{E}^{w*}(m)$ of wholesale power and price outcomes and the vector $\mathrm{E}^{r*}(m)$ of retail power and price outcomes. Note that RTM operations are not depicted in Fig. 5. The reason for this is that the RTM is an imbalance adjustment mechanism designed to handle discrepancies between DAM scheduled generation and real-time loads. However, no such discrepancies arise in the market equilibrium.

A key issue is whether the market participants in the IRW Test Case are able to attain this market equilibrium. Comparing the form of $\mathrm{E}^{w*}(m)$ in (10) to the form of $\mathrm{E}^{r*}(m)$ in (11), it is clear that the hour-H *wholesale* power and LMP outcomes (p^{DA}(H,D),π^{DA}(H,D)) converge to $\mathrm{E}^{w*}(m)$ over successive days D if and only if the hour-H *retail* power and price outcomes (p^{RET}(H,D-1),π^{RET}(H,D-1)) converge to E^{r*}(m) over successive days D-1.

These two intertwined wholesale and retail processes constitute the braided cobweb dynamics for the IRW Test Case. Given any start-day D-1, the study of the wholesale process is most naturally undertaken by choosing a *wholesale-start point*, i.e., a start-point W0 on the plot of the GenCo's supply function (1); and the study of the retail process is most naturally undertaken by choosing a *retail-start point*, i.e., a start-point R0 on the plot of the retail demand function (7).

Fig. 6 depicts these two braided cobweb cycles. The wholesale-start cycle is depicted as W0 $\rightarrow$ A1 $\rightarrow$ A2 ..., and the retail-start cycle is depicted as R0 $\rightarrow$ B1 $\rightarrow$ B2 ... .

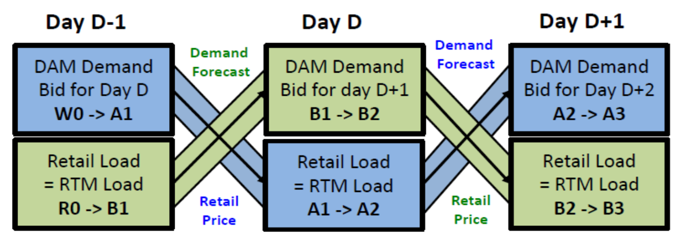

Fig. 6. Depiction of the two braided cobweb cycles for the IRW Test Case under dynamic-price retail contracting, differentiated by start point. The wholesale-start cycle begins at W0 and the retail-start cycle begins at R0.

As will be established analytically in Section V-C, below, the type of cobweb dynamics exhibited by each of these braided cobweb cycles is entirely determined by the sign of $[b - d(m)]$, where b is the slope coefficient for the GenCo's supply offer (1) and $d(m)$ is the slope coefficient for the m-modified retail demand function (9). In particular, assuming the start point W0 (R0) does not coincide with $\mathrm{E}^{w*}(m)$ ($\mathrm{E}^{r*}(m)$), a necessary and sufficient condition for wholesale (retail) power and price outcomes to converge to $\mathrm{E}^{w*}(m)$ ($\mathrm{E}^{r*}(m)$) is $b < d(m)$.

This analysis is previewed here using graphical depictions for illustrative simulation runs. For concreteness, attention is focused on the cobweb dynamics exhibited by the retail-start cycle R0 $\rightarrow$ B1 $\rightarrow$ B2 ... for a particular hour H on successive days D, commencing on day D-1 from a start-point R0 on the plot of the retail demand function (7). For increased graphical clarity, the mark-up m for the dynamic-price retail contracts is set to zero, which implies $\mathrm{E}^{w*}(0) = \mathrm{E}^{r*}(0) \equiv \mathrm{E}^*$.

Figs. 7–9 illustrate the three possible types of cobweb dynamics that can be exhibited by the retail-start cycle, conditional on the magnitude of $[b - d(0)]$, assuming a retail-start point R0 that is not coincident with E^*. In each case the power system repeatedly cycles through four market phases. The first occurrences of these four market phases correspond to the movements R0 $\rightarrow$ B1 $\rightarrow$ B2 $\rightarrow$ B3 $\rightarrow$ B4. The second occurrence of the first market phase then corresponds to the movement B4 $\rightarrow$ B5.

Specifically, at the start point R0, values are determined for retail power usage (load) and the retail price for day D-1, which then determine RTM load for day D-1 by assumption (A1). The RTM load on day D-1 determines the RTM LMP

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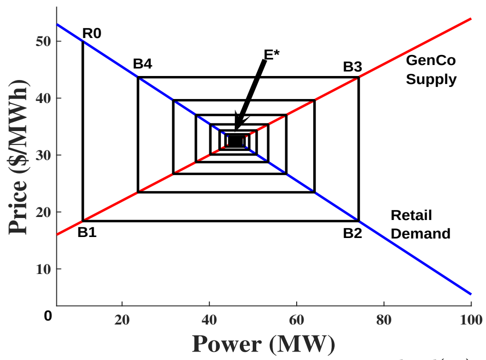

Fig. 7. Illustration of convergent cobweb dynamics ($b<d(m)$) for the IRW Test Case with dynamic-price retail contracts. Parameter settings: m=0, a=14, b=0.2, c=55.5, d=0.25, and start point R0=(11,50).

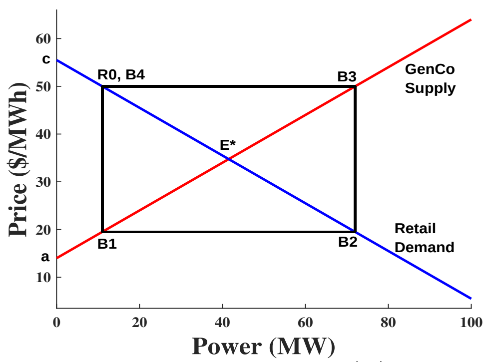

Fig. 8. Illustration of a fixed cobweb cycle (b=$d(m)$) for the IRW Test Case with dynamic-price retail contracts. Parameter settings: m=0, a=14, b=0.25, c=55.5, d=0.25, and start point R0=(11,50).

for day D-1, by assumptions (A2) and (A3), which moves the system from R0 to B1. The LSE then uses RTM load on day D-1 as its day-D DAM demand bid (in fixed demand form) by assumption (A4), which determines the day-D DAM LMP through DAM market clearing by assumptions (A5) and (A6). The LSE then sets the day D+1 retail price equal to $[1+m]$ times the day-D DAM LMP, by assumption (A7), which determines retail power usage for day D+1 by assumption (A8); this moves the system from B1 to B2.

At B2, the retail load for day D+1 determines the RTM load for day D+1, by (A1), which then determines the RTM

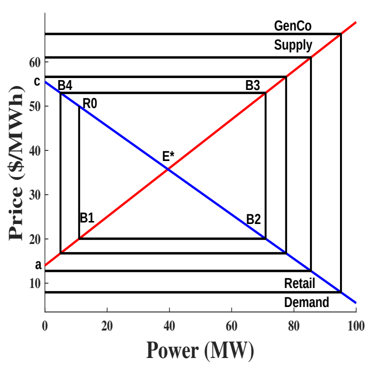

Fig. 9. Illustration of divergent cobweb dynamics ($b>d(m)$) for the IRW Test Case with dynamic-price retail contracts. Parameter settings: m=0, a=14, b=0.3, c=55.5, d=0.25, and start point R0=(11,50).

LMP for day D+1 by assumptions (A2) and (A3); this moves the system from B2 to B3. The LSE uses the RTM load for day D+1 to determine its day-(D+2) DAM demand bid, by assumption (A4), which then determines the day-(D+2) DAM LMP by assumptions (A5) and (A6). The LSE then uses this day-(D+2) DAM LMP, multiplied by $[1+m]$, to set the retail price for day-(D+3), by assumption (A7), which determines retail power usage by assumption (A8). This moves the system from B3 to B4, and the entire process then repeats.

C. Analytical Derivation of Findings

As discussed in Section V-B, and illustrated in Fig. 6, assumptions (A1)–(A8) for the IRW Test Case with dynamic-price retail contracts induce market dynamics having a braided cobweb form. That is, two cycles are intertwined with each other, a wholesale-start cycle and a retail-start cycle. Starting on any day D-1, the wholesale-start cycle determines retail outcomes on days D, D+2, D+4, . . ., and the retail-start cycle determines retail outcomes on days D-1, D+1, D+3, ...

For concreteness, this section focuses on the retail-start cycle. Note that a wholesale-start cycle starting on any arbitrarily selected day D-1 can be decomposed into a wholesale start on day D-1 followed by a retail-start cycle starting on day D. As will be established below in Proposition 2, the long-run dynamical behavior of the retail-start cycle is entirely determined by the relative size of two exogenously given parameters, regardless of the start day. Consequently, the wholesale-start cycle always exhibits the same long-run dynamical behavior as the retail-start cycle.

Consider an IRW Test Case satisfying assumptions (A1) through (A8). Let H denote any particular hour of a 24-hour day, hereafter suppressed in all notation for easier readability. Let D-1 denote any given day, and let

$$\text{R0} = (p^{\mathsf{RET}}(\text{D-1}), \pi^{\mathsf{RET}}(\text{D-1}))^T \tag{12}$$

denote an arbitrarily selected point on the plot of the retail demand function (7) for hour H of day D-1. For each $t = 0, 1, 2, \ldots$, define

$$s(t) = ([\text{D-1}] + 4t)\,; \tag{13}$$

$$\mathbf{r}(t) = (p^{\mathsf{RET}}(s(t)), \pi^{\mathsf{RET}}(s(t))^T\,. \tag{14}$$

By construction, the $\mathbf{r}(t)$ process represents retail power and price outcomes for the IRW Test Case during hour H on every fourth day, starting from R0 on day D-1.

PROPOSITION 1: Let R0 and $\mathbf{r}(t)$ be defined as in (12) and (14). Then the dynamical behavior of the $\mathbf{r}(t)$ process is determined by the following system of linear difference equations:

$$\mathbf{r}(t) \;=\; \mathbf{q}(m) + \mathbf{J}(m)\mathbf{r}(t-1)\,,\ \forall t = 1, 2, \ldots; \tag{15}$$

$$\mathbf{r}(0) \;=\; \text{R0}\,, \tag{16}$$

where:

$$\mathbf{q}(m) = [\mathbf{I} + \mathbf{M}(m)]\mathbf{v}(m)\,; \tag{17}$$

$$\mathbf{J}(m) = [\mathbf{M}(m)]^2\,; \tag{18}$$

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$$\mathbf{v}(m) = \begin{bmatrix} \frac{[c(m)-a]}{2d(m)} \\ \frac{(1+m)[ad(m)+bc(m)]}{d(m)} \end{bmatrix} ; \quad (19)$$

$$\mathbf{M}(m) = (-b/d(m)) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ; \quad (20)$$

$$\begin{bmatrix} c(m) \\ d(m) \end{bmatrix} = \begin{bmatrix} \frac{c}{1+m]} \\ \frac{d}{[1+m]} \end{bmatrix} . \quad (21)$$

Proof of Proposition 1: By assumption, R0 in (12) satisfies the retail demand function (7) for hour H of day D-1, implying

$$p^{\mathsf{RET}}(\text{D-1}) = \frac{[c - \pi^{\mathsf{RET}}(\text{D-1})]}{2d} . \quad (22)$$

The following additional relationships can be verified for IRW Test Case outcomes over the next four days by successive application of assumptions (A1) through (A8).

By (A1): $p^{\mathsf{RT}}(\text{D-1}) = p^{\mathsf{RET}}(\text{D-1})$ (23)

By (A2),(A3): $\pi^{\mathsf{RT}}(\text{D-1}) = a + 2bp^{\mathsf{RT}}(\text{D-1})$ (24)

By (A4): $p^{\mathsf{DA}}(\text{D+1}) = p^{\mathsf{RT}}(\text{D-1})$ (25)

By (A5),(A6): $\pi^{\mathsf{DA}}(\text{D+1}) = a + 2bp^{\mathsf{DA}}(\text{D+1})$ (26)

By (A7): $\pi^{\mathsf{RET}}(\text{D+1}) = [1+m]\pi^{\mathsf{DA}}(\text{D+1})$ (27)

By (A8): $p^{\mathsf{RET}}(\text{D+1}) = [c - \pi^{\mathsf{RET}}(\text{D+1})]/2d$ (28)

For each $j = 0, 1, 2, \ldots$, define

$$k(j) = [\text{D-1}] + 2j ; \quad (29)$$

$$\mathbf{y}(j) = (p^{\mathsf{RET}}(k(j)), \pi^{\mathsf{RET}}(k(j))^T . \quad (30)$$

Note that relationship (28) for day D+1 has the same form as relationship (22) for day D-1. By a simple induction argument, it follows that relationships (23) through (28) can be recursed forward indefinitely for days $k(j-1)$ to $k(j)$ for each $j = 1, 2, \ldots$. Consequently, it follows by straightforward calculation that

$$\mathbf{y}(j) = \mathbf{v}(m) + \mathbf{M}(m)\mathbf{y}(j-1) , \; j = 1, 2, \ldots . \quad (31)$$

Finally, for each $t = 1, 2, \ldots$, note that $s(t) = k(2t)$ and $\mathbf{r}(t)$=$\mathbf{y}(2t)$. Thus, for each $t = 1, 2, \ldots$, one has

$$\begin{aligned} \mathbf{r}(t) &= \mathbf{y}(2t) \quad (32) \\ &= \mathbf{v}(m) + \mathbf{M}(m)\mathbf{y}(2t-1) \\ &= \mathbf{v}(m) + \mathbf{M}(m)[\mathbf{v}(m) + \mathbf{M}(m)\mathbf{y}(2t-2)] \\ &= [\mathbf{I} + \mathbf{M}(m)]\mathbf{v}(m) + [\mathbf{M}(m)]^2\mathbf{y}(2t-2) \\ &= [\mathbf{I} + \mathbf{M}(m)]\mathbf{v}(m) + [\mathbf{M}(m)]^2\mathbf{r}(t-1) \\ &= \mathbf{q}(m) + \mathbf{J}(m)\mathbf{r}(t-1) . \end{aligned}$$

QED

COROLLARY 1: Let R0, $\mathbf{r}(t)$, and $\mathbf{J}(m)$ be defined as in Proposition 1. For each $t = 0, 1, 2, \ldots$, define

$$\mathbf{z}(t) = \mathbf{r}(t) - \mathrm{E}^{r*}(m) , \quad (33)$$

where $\mathrm{E}^{r*}(m)$ is the retail market equilibrium point given by (11). Then

$$\mathbf{z}(t) \; = \mathbf{J}(m)\mathbf{z}(t-1) , \forall t = 1, 2, \ldots ; \quad (34)$$

$$\mathbf{z}(0) \; = \text{R0} - \mathrm{E}^{r*}(m) . \quad (35)$$

Proof of Corollary 1: By simple recursion and manipulation of terms,

$$\begin{aligned} \mathbf{z}(t) &= \mathbf{q}(m) + \mathbf{J}(m)\mathbf{x}(t-1) - \mathrm{E}^{r*}(m) \quad (36) \\ &= \mathbf{q}(m) + \mathbf{J}(m)\big[\mathbf{x}(t-1) - \mathrm{E}^{r*}(m)\big] \\ &\quad + \mathbf{J}(m)\mathrm{E}^{r*}(m) - \mathrm{E}^{r*}(m) \\ &= \mathbf{q}(m) + \mathbf{J}(m)\mathbf{z}(t-1) + [\mathbf{J}(m) - \mathbf{I}]\mathrm{E}^{r*}(m) \\ &= \mathbf{J}(m)\mathbf{z}(t-1) + \big(\mathbf{q}(m) + [(b/d(m))^2 - 1]\mathrm{E}^{r*}(m)\big) \\ &= \mathbf{J}(m)\mathbf{z}(t-1) + \mathbf{u}(m) . \end{aligned}$$

It then follows by straightforward calculation and definition (11) for $\mathrm{E}^{r*}(m)$ that $\mathbf{u}(m) = \mathbf{0}$. QED

If the start-point R0 for the $\mathbf{r}(t)$ process in Proposition 1 coincides with the retail market equilibrium point $\mathrm{E}^{r*}(m)$, it follows immediately from Corollary 1 that $\mathbf{r}(t)$ = $\mathrm{E}^{r*}(m)$ for all $t = 0, 1, \ldots$; hence, no dynamics are generated. Consequently, the following proposition focuses on cases for which R0 does not coincide with $\mathrm{E}^{r*}(m)$.

PROPOSITION 2: Let R0 and $\mathbf{r}(t)$ be defined as in Proposition 1. Suppose R0 $\neq \mathrm{E}^{r*}(m)$. Then, ignoring non-negativity restrictions on power and price levels, the long-run dynamical behavior of the $\mathbf{r}(t)$ process is entirely determined by the sign of $[b - d(m)]$, as follows:

Case 1 (Cobweb Convergence): If $b < d(m)$, $\mathbf{r}(t)$ converges to $\mathrm{E}^{r*}(m)$ as $t \to \infty$.

Case 2 (Cobweb Fixed Cycle): If $b = d(m)$, $\mathbf{r}(t)$ = R0 for all $t = 0, 1, \ldots$.

Case 3 (Cobweb Divergence): If $b > d(m)$, the components of $\mathbf{r}(t)$ diverge to plus or minus ∞.

Proof of Proposition 2: From Corollary 1,

$$\begin{aligned} \mathbf{z}(t) &= \mathbf{J}(m)\mathbf{z}(t-1) \quad (37) \\ &= [\mathbf{J}(m)]^2\mathbf{z}(t-2) \\ &= \ldots = [\mathbf{J}(m)]^t\mathbf{z}(0) , \end{aligned}$$

where $\mathbf{z}(t)$ = $[\mathbf{x}(t)$ - $\mathrm{E}^{r*}(m)]$ and $[\mathbf{J}(m)]^t$ = $(b/d(m))^{2t}\mathbf{I}$. If $b < d(m)$, it follows from (37) that $\mathbf{z}(t) \to 0$ as $t \to \infty$. Hence, $\mathbf{r}(t) \to \mathrm{E}^{r*}(m)$ as $t \to \infty$. This establishes Case 1.

Suppose, instead, that $b = d(m)$. It then follows from (37) that $\mathbf{z}(t)$ = $\mathbf{z}(0)$ for all $t = 0, 1, \ldots$, which implies $\mathbf{r}(t)$ = $\mathbf{r}(0)$ = R0 for all $t = 0, 1, \ldots$. This establishes Case 2.

Finally, suppose $b > d(m)$. Consider, first, what happens if R0 lies to the *left* of $\mathrm{E}^{r*}(m)$ along the plot of the retail demand function (7); see Fig. 5. This implies the components of $\mathbf{z}(0)$ = $(z_1(0), z_2(0))^T$ satisfy $z_1(0) < 0$ and $z_2(0) > 0$. It then follows from (37) that $z_1(t)$ diverges to $-\infty$ as $t \to \infty$ and $z_2(t)$ diverges to $+\infty$ as $t \to \infty$. The reverse divergence directions for $z_1(t)$ and $z_2(t)$ can easily be shown to hold if R0 instead lies to the *right* or $\mathrm{E}^{r*}(m)$ along the plot of the retail demand function (7). This establishes Case 3. QED

Consider once again Figs. 7–9. These three figures illustrate the three possible types of cobweb dynamics that can be exhibited by power and price outcomes for the IRW Test Case, given dynamic-price retail contracts with a zero markup ($m = 0$) and a start point R0 on the plot of the retail

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demand function (7) that is not coincident with the retail market equilibrium point $\mathrm{E}^* \equiv \mathrm{E}^{r*}(0)$.

Note that the long-run dynamical behavior of power and price outcomes depicted in these figures can in fact be inferred from the relative positions of R0, $\mathbf{r}(1) \equiv$ B4, and E* along the depicted demand function. As established by Corollary 2 in ref. [1, Sec. V.C], this finding holds for any IRW Test Case satisfying the dynamic-price assumptions (A1)-(A8), regardless of the magnitude of the mark-up $m \geq 0$.

VI. A Dynamic Welfare Sensitivity Design

This section explains the design of a dynamic welfare sensitivity study undertaken for the full IRW Test Case described in Section IV.[9] The hourly A/C power demands for each of the 500 households in the full IRW Test Case are simulated demands determined by an intelligent price-responsive A/C controller, as in [20].

In particular, for dynamic-price treatments, the simplifying auxiliary assumption (A8) that directly postulates *linear* aggregate demand curves for hourly household A/C power usage is *not* imposed. However, as detailed in [1, Appendix], simulation studies show that these demand curves are well approximated by linear downward-sloping functions over a broad range of power levels. Thus, braided cobweb dynamics still arise.

For example, Fig. 10 depicts household power usage as a function of retail price for hour 18:00 over 20 successive days for a dynamic-price treatment. As can be seen, the trend line drawn through this scatter-plot is linear with a negative slope over a broad power range.[10]

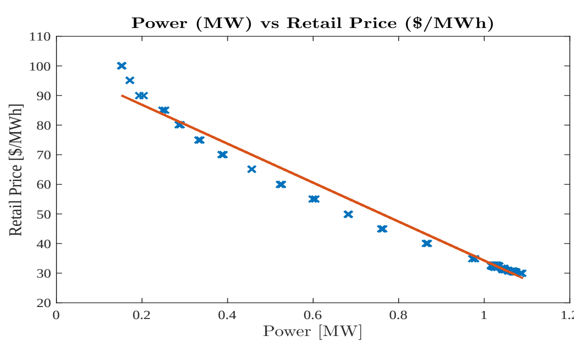

Fig. 10. Simulation-generated scatter plot for retail prices and corresponding aggregate household A/C power demands for hour 18:00 over twenty successive simulated days, with corresponding trend line, for a dynamic-price treatment with $b = 0.0001$ and $m = 0$.

The three treatment factors for this sensitivity study are: (i) the form of retail contracts, either flat-rate or dynamic-price; (ii) the mark-up m in (5) that determines the percentage by which retail prices are marked up over wholesale prices in the case of dynamic-price retail contracts; and (iii) the household comfort-cost trade-off parameter α in (6). Four values are tested for the mark-up m: 0.0, 0.2, 0.4, and 0.6. Also, four values are tested for the trade-off parameter α: 0, 1000 ("Small"), 2000 ("Medium"), and 3000 ("Large").

As noted in Section IV-B, the thermal dynamics governing each of the 500 households in the full IRW Test Case are represented by a simplified ETP model for which inside air temperature, T_a, is the sole state variable. Specifically, this simplified ETP model is as depicted in eqn. (4) for any given household n. Positive values were randomly assigned to the parameters z_1^n and z_2^n in eqn. (4) for each household n at the beginning of the sensitivity study and then maintained throughout all subsequent simulation runs.

Additional attributes maintained at fixed settings for all simulation runs are as follows:

- A/C rating for each household n;
- Initial inside air temperature for each household n;
- Bliss point temperature (70^o F) for each household n;
- Daily outside air temperature profile (commonly experienced across all households);
- Initial retail prices communicated by the LSE to all households during the first two simulated days.

For the flat-rate case, the retail price charged to households in each simulation run was set at a fixed *break-even* rate R ($/MWh) for the entire simulation, i.e., at a fixed rate R for which hourly LSE net earnings are at least zero and are as close to zero as possible. The break-even value for R was found by trial and error for each flat-rate simulation run.[11]

Below are the flat rates R determined for different values of α for the flat-rate retail contracting outcomes reported in Section VII.

- R = 27.9 ($/MWh) for α = 0;
- R = 27.7 ($/MWh) for α = 1000 (Small);
- R = 27.5 ($/MWh) for α = 2000 (Medium);
- R = 27.2 ($/MWh) for α = 3000 (Large).

Note these R values are a *decreasing* function of α.

Finally, each simulation run in the sensitivity study consisted of 20 simulated days. Hourly LSE, GenCo, and household welfare outcomes and hourly energy cost outcomes, calculated during simulated days 11 to 20, were used to obtain average daily outcomes. The calculation of the hourly welfare outcomes is summarized in an appendix to this study and explained in detail in ref. [1, Sec. VI].

VII. Welfare Sensitivity Findings for the Full IRW Test Case

As in Section VI, consider the full IRW Test Case described in Section IV with simulated household A/C power demands. Table I reports average daily welfare and energy cost outcomes for a range of treatments exhibiting cobweb convergence. Only treatments with α and m values commonly set across all 500 households are shown.

[9] Complete source code and data files for this sensitivity study, including external forcing terms (e.g., outside air temperature), maintained parameter values, and functional forms, can be accessed at [29].

[10] As indicated in Fig. 10, a linear approximation is not a good fit for the aggregate hourly household A/C power demand curves in the boundary region where power approaches zero; rather, the curves become exponential. This is not surprising since household power demands are for a critical purpose: namely, maintaining inside air temperature at a comfortable level.

[11] As carefully explained in [1, Sec. VI-D], LSE net earnings are zero for the IRW Test Case under flat-rate contracts if the IRW Test Case is initialized at the market equilibrium point (p^*, π^*) where retail demand equals GenCo supply and the flat rate R is set equal to π^*. However, none of the simulation runs for the welfare sensitivity study were initialized at this equilibrium point.

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TABLE I
AVERAGE DAILY WELFARE AND ENERGY COST OUTCOMES FOR THE FULL IRW TEST CASE WITH SIMULATED HOUSEHOLD A/C POWER DEMANDS UNDER A RANGE OF TREATMENTS EXHIBITING COBWEB CONVERGENCE.

	m	α = 0		α = Small		α = Medium		α = Large	
		Flat	Dynamic	Flat	Dynamic	Flat	Dynamic	Flat	Dynamic
Avg. LSE Net Earnings ($/day)	0.0	0.00	0.00	0.04	0.00	0.08	0.00	0.04	0.00
	0.2		9.11		7.67		6.17		4.79
	0.4		18.21		14.84		11.4		8.35
	0.6		27.32		21.51		15.69		10.95
Avg. Gen Net Earnings ($/day)	0.0	12.59	12.59	10.96	10.99	9.24	9.30	7.59	7.64
	0.2		12.59		10.66		8.62		6.73
	0.4		12.59		10.32		7.98		5.89
	0.6		12.59		9.97		7.33		5.16
Avg. Utility (Utils/day)	0.0	10446.29	10446.29	8581.52	8587.73	6707.30	6733.29	4879.86	4913.59
	0.2		10446.29		8216.86		5996.66		3870.66
	0.4		10446.29		7845.11		5275.68		2882.13
	0.6		10446.29		7473.19		4563.41		1965.76
Avg. Energy Cost ($/day)	0.0	1.82	1.82	1.58	1.58	1.33	1.33	1.09	1.09
	0.2		2.19		1.84		1.48		1.15
	0.4		2.55		2.08		1.60		1.17
	0.6		2.92		2.29		1.67		1.17

The following regularities are seen in the outcomes reported in Table I. Each of these regularities is subject to an "all else equal" qualifier.

- *Overall Welfare Findings:*
  - Suppose households care about energy cost ($\alpha > 0$). Then a dynamic-price retail contract with a *positive* mark-up m results in a *better* welfare outcome for the LSE and *worse* welfare outcomes for the GenCo and households than a flat-rate retail contract.
  - Suppose households do *not* care about energy cost (α=0). Then GenCo and household welfare are independent of the retail power price, hence independent of the form of the retail contract; but LSE welfare is higher under a dynamic-price retail contract than under a flat-rate retail contract for any $m > 0$.
- *LSE Welfare:*
  - Given dynamic-price contracting, LSE welfare *increases* as m *increases*.
  - Given a dynamic-price retail contract with $m > 0$, LSE welfare *decreases* as α *increases*.
- *GenCo and Household Welfare:*
  - Given $\alpha > 0$ and dynamic-price contracting, GenCo and household welfare *decrease* as m *increases*.
  - Given $\alpha > 0$, GenCo and household welfare are *higher* under a dynamic-price retail contract with a *zero* mark-up m than under a flat-rate retail contract.
  - GenCo and household welfare both *decrease* as α *increases*.
- *Energy Cost:*
  - Given dynamic-price retail contracting, average daily energy cost *increases* as m *increases*.
  - Average daily energy cost *decreases* as α *increases*.
  - The same average daily energy cost is realized under a dynamic-price retail contract with zero mark-up ($m = 0$) and a flat-rate retail contract.

Perhaps the most surprising regularity reported in Table I is that, given $\alpha > 0$, households are better off under a flat-rate retail contract than under a dynamic-price retail contract with a positive mark-up m. A key reason for this finding is that the dynamic-price retail contracts are based on one-way communication. The LSE communicates next-day retail prices to households, who then optimize their next-day welfare conditional on these prices. However, households pay no attention to the possibly adverse feedback effects of their resulting next-day power usage on future retail and wholesale power prices, hence on their longer-term welfare outcomes.

VIII. CONCLUSION

Economists have known for decades that possibly divergent "cobweb cycles" can arise for prices and quantities in market models for which a lag exists between the decision to produce a nonstorable good and its actual production. Economic research on this topic remains active; see, e.g., [30], [31], [32]. Power engineers have raised similar concerns for real-time electric power markets; see, e.g., [33], [34], [35].

These previous cobweb-cycle studies convey an important common message: namely, proposed new market initiatives must be designed with care in order to avoid adverse unintended consequences. Relatively simple single-market models are used to illustrate this point.

The present study differs from this previous work in three key ways. First, the study makes use of an agent-based computational platform, the IRW Test Bed, to undertake a comparative test-case study of two distinct demand-response (DR) designs for an end-to-end power system encompassing both transmission and distribution level operations. Second, the end-to-end power system operates over time as an open-ended dynamic process. Third, the end-to-end power system incorporates important operational features of actual U.S. centrally-managed wholesale power markets.

A key finding of our test-case study is that dynamic-price retail contracting can give rise to unstable braided cobweb dynamics. Moreover, even in convergent cases, we find that households are generally worse off under dynamic-price retail contracts than under a flat-rate contract for which the flat rate is set to assure the contract provider "breaks even" over time.

The basic explanation for these findings is the one-way nature of dynamic-price communication. Households optimize their current power usage in response to successively received retail price signals. However, no one is paying attention to the potentially adverse feedback effects of the resulting household power usage levels on future wholesale and retail prices. These longer-run effects can only be discerned by considering IRW power system operations over successive days.

This explanation suggests that researchers and policy makers should give increased attention to DR designs based on two-way communication. For example, as noted in Section II and discussed at length in [25], "transactive energy system" designs based on two-way price-bid communications permit system managers to give advance consideration to the price-response functions (bids) of distributed energy resource (DER) owners as a means for anticipating the longer-run impacts of any price signals communicated to these DER owners.

Many simplifying assumptions regarding physical constraints, institutional arrangements, and decision-maker behaviors are of course made in the current test-case study. Thus, our test-case findings are cautionary rather than definitive.

For example, the current study assumes a simple adaptive form for an LSE's DAM demand bids, i.e., an LSE's forecasts for the next-day loads of its retail customers. In future studies we will consider more carefully the manner in which an LSE might be able to enhance its net earnings through alternative forms of bidding. This issue is challenging due to informational constraints. The precise manner in which external system conditions affect the next-day price-responsive power usage of retail customers depends on the private thermal states, structural attributes, preferences, and goals of these customers, and these private aspects can differ across different customers.

In addition, the current study assumes an LSE uses a simple method to determine retail prices for its retail customers: namely, it sets retail power prices equal to a percentage mark-up over wholesale power prices. The use of percentage price mark-ups is ubiquitous in real-world markets and in government fiscal policies (e.g., sales taxes). However, as seen in this study, the exact manner in which retail prices are set within a DR program has important effects on the program's performance. Consequently, it might be useful (and even necessary) to specify the form of this price setting as part of a DR program's design rather than leave it to the discretion of the program's participants.

Fortunately, the IRW Test Bed used to implement our current study is modular and extensible. Consequently, it will permit these and other issues to be addressed with care within the context of larger systems with greater degrees of empirical verisimilitude. This should facilitate increasingly sophisticated studies of DR designs, enhancing chances for their successful real-world implementation.

Appendix: Welfare Metrics for the Full IRW Test Case with Simulated Household Demand

Formulas for the calculation of hourly welfare outcomes for the LSE, the GenCo, and the households for the fully simulated IRW Test Case under both dynamic-price and flat-rate retail contracting are carefully derived and illustrated in [1, Secs. VI-C,D]. These derivations are briefly summarized below.

Suppose, first, that contracts take a dynamic-price form with mark-up $m \geq 0$, implying that assumptions (A1)-(A7) in Section V-A hold. Let H denote a particular hour of a 24-hour day, hereafter suppressed in all notation for easier readability; and let D-1 denote any particular day. Then *LSE welfare* for hour H of day D+1, measured in terms of net earnings (i.e., revenues minus costs), is given by

$$\begin{aligned}\text{LWelfare}^{\mathsf{DP}}(\text{D+1}) &= m\pi^{\mathsf{DA}}(\text{D+1})p^{\mathsf{RET}}(\text{D+1}) \\ &+ [\pi^{\mathsf{RT}}(\text{D+1}) - \pi^{\mathsf{DA}}(\text{D+1})][p^{\mathsf{DA}}(\text{D+1}) - p^{\mathsf{RT}}(\text{D+1})]\,. \end{aligned} \tag{38}$$

Also, *GenCo welfare* for hour H of day D+1, measured in terms of net earnings, is given by

$$\begin{aligned}\text{GWelfare}^{\mathsf{DP}}(\text{D+1}) &= \pi^{\mathsf{DA}}(\text{D+1})p^{\mathsf{RET}}(\text{D-1}) \\ &- \pi^{\mathsf{RT}}(\text{D+1})[p^{\mathsf{RET}}(\text{D-1}) - p^{\mathsf{RET}}(\text{D+1})] \\ &- \text{GCost(D+1)}\,. \end{aligned} \tag{39}$$

The term GCost(D+1) in (39) denotes the GenCo's total operating cost for the production of p^{RET}(D+1) during hour H of day D+1, which is given by the area under its supply offer (marginal cost curve) from $p = 0$ to $p = p^{\mathsf{RET}}$(D+1).

Suppose, instead, that contracts take a flat-rate form with flat-rate R. Then *LSE welfare* for hour H of day D+1, measured in terms of net earnings, is given by

$$\begin{aligned}&\text{LWelfare}^{\mathsf{R}}(\text{D+1}) = \\ &\quad [\pi^{\mathsf{RT}}(\text{D+1}) - \pi^{\mathsf{DA}}(\text{D+1})][p^{\mathsf{RET}}(\text{D-1}) - p^{\mathsf{RET}}(\text{D+1})] \\ &\quad + [R - \pi^{\mathsf{DA}}(\text{D+1})]p^{\mathsf{RET}}(\text{D+1})\,. \end{aligned} \tag{40}$$

GenCo welfare for hour H of day D+1, measured in terms of net earnings, again takes form (39).

Finally, for all treatment configurations, hourly household welfare (utility) is measured by (6), a weighted average of comfort and energy-cost savings (negative energy cost). The degree to which a household n is willing to sacrifice comfort for additional energy-cost savings is measured by the weight factor α^n in (6).

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