Financial Bilateral Contract Negotiation in Wholesale Electricity Markets Using Nash Bargaining Theory

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***Abstract*—Bilateral contracts are important risk-hedging instruments constituting a major component in the portfolios held by many electric power market participants. However, bilateral contract negotiation is a complicated process as it involves risk management, strategic bargaining, and multi-market participation. This study analyzes a financial bilateral contract negotiation process between a generation company and a load-serving entity in a wholesale electric power market with congestion managed by locational marginal pricing. Nash bargaining theory is used to model a Pareto-efficient settlement point. The model predicts negotiation outcomes under various conditions and identifies circumstances in which the two parties might fail to reach an agreement. Both analysis and simulation are used to gain insight regarding how these negotiation outcomes systematically vary in response to changes in the participants' risk preferences and price biases.**

***Index Terms*—Wholesale electricity market, financial bilateral contract, negotiation, locational marginal price, Nash bargaining theory, risk aversion, conditional value-at-risk.**

NOMENCLATURE

Bus i	Location of a GenCo and LSE in a financial bilateral contract negotiation.
P_G	GenCo's fixed production rate (MW).
A_G	GenCo's risk-aversion factor.
A_L	LSE's risk-aversion factor.
T	Contract period (hours).
λ_Σ	Sum of LMPs realized at bus i during T.
E^P	Expectation calculated using true probability measure $P(\lambda_\Sigma)$.
K_G	Bias affecting probability measure $Q_G(\lambda_\Sigma)$.
K_L	Bias affecting probability measure $Q_L(\lambda_\Sigma)$.
E^G	Expectation calculated by GenCo using biased probability measure Q_G.
E^L	Expectation calculated by LSE using biased probability measure Q_L.
α	Confidence level for GenCo and LSE.
$CVaR_\alpha^P$	Conditional Value-at-Risk calculated using true probability measure P.
$CVaR_\alpha^G$	Conditional Value-at-Risk calculated by GenCo using biased probability measure Q_G.
$CVaR_\alpha^L$	Conditional Value-at-Risk calculated by LSE using biased probability measure Q_L.
M	Hourly contract amount (MW).
M^R	Lower bound for negotiated contract amount.
M^U	Upper bound for negotiated contract amount.
S	Hourly contract strike price ($/MWh).
S^R	Lower bound for negotiated strike price.
S^U	Upper bound for negotiated strike price.
u_G	GenCo's return-risk utility function.
u_L	LSE's return-risk utility function.
π_G	GenCo net earnings.
π_L	LSE net earnings.
π_G^0	GenCo net earnings if no contract is signed.
π_L^0	LSE net earnings if no contract is signed.

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I. INTRODUCTION

COSTLY lessons learned from the California energy crisis in 2000-01 were that overreliance on spot markets can lead to extremely volatile prices as well as a market design vulnerable to gaming. The bilateral contracts for longer-term trades that were disallowed by the California regulators could have reduced spot price volatility, discouraged gaming behaviors by power traders, and provided a much-needed risk-hedging instrument for the three largest investor-owned utilities.

Bilateral contracting is the most common form of trade arrangement in many electricity markets. Examples include the continental European electricity market, the Texas (ERCOT) wholesale power market, the Nordic electricity market, and the Japanese electric power exchange [1]. Traders in these markets routinely hedge their price risks by signing bilateral contracts. An example of such a contract is a *Contract-For-Difference (CFD)* that specifies a strike price ($/MWh) at which a particular MW amount is to be exchanged at a particular reference location during a particular contract period. If the actual price at the reference location differs from the strike price, the advantaged party is required to "make whole" the disadvantaged party by paying the difference [2, Section V.A].

Given the prominent role played by negotiated bilateral contracts in power markets, a crucial question is how the parties to such contracts successfully negotiate the terms of their contracts. The negotiation process can be extremely complicated, involving considerations of both risk management and strategic gaming.

In particular, a participant in a bilateral contract negotiation will typically be concerned not only with expected net earnings but also with *risk*, i.e., the possibility of adverse deviations from expected net earnings. Consequently, the participant will presumably try to negotiate a contract that achieves a satisfactory trade-off between expected net earnings and risk in accordance with its risk preferences.

From a game theoretic perspective, each party to a negotiation must always keep in mind that a strategy of trying to unilaterally improve its own return at the expense of the other party will typically be self-defeating [3]. Although a party could insist on pushing the point of agreement in its favor, this effort will be in vain if the other party then decides to walk away. A typical bilateral contract negotiation process involves elements of both cooperation and competition [4]. Moreover, these considerations of risk and strategic gaming can arise across several distinct markets at the same time.

Within the field of power economics, only a few researchers to date have studied the bilateral contract negotiation process. Khatib and Galiana [5] propose a practical process in which the bargainers take both benefits and risks into account. They claim that their proposed process will lead to agreement on a mutually beneficial and risk-tolerable forward bilateral contract. Song et al. [6] and Son et al. [7] analyze bidding strategies in a bilateral market in which GenCos submit bids to loads. Necessary and sufficient conditions for the existence of a Nash equilibrium in bidding strategies are then derived. In a series of studies, Kockar et al. [8]–[10] examine important issues arising for mixed pool/bilateral trading. Although the number of studies focusing on bilateral contract negotiation in electric power markets is small, a large number of electric power researchers have examined the related topics of risk management and portfolio optimization [2], [11]–[26].

This study analyzes a negotiation process between a *generation company (GenCo)* and a *load-serving entity (LSE)* for a financial bilateral contract,[1] taking into account considerations of risk management, strategic gaming, and multi-market interactions. Given the small amount of previous research on this technically challenging problem, a relatively simple form of power purchase agreement is used to permit the derivation of analytical and computational findings with clear interpretable results. Our goal is to provide a basic foundation upon which future research on financial bilateral negotiation in power markets can build.

Specifically, to model the financial bilateral negotiation process between the GenCo and LSE, we introduce a key tool from cooperative game theory: namely, Nash bargaining theory. In contrast to non-cooperative game theory (e.g., Nash equilibrium), cooperative game theory assumes that participants in strategic situations are able to bargain directly with each other to reach binding (enforceable) decisions. As will be clarified below, Nash bargaining theory is a particular cooperative-game modeling of a negotiation process that constrains negotiated outcomes to satisfy basic fairness and efficiency criteria thought to be important in real-world bargaining situations. It also identifies circumstances in which the parties to the negotiation might fail to reach an agreement.

We use Nash bargaining theory to study how negotiated outcomes between the GenCo and LSE depend on their relative aversion to risk and on the degree to which their price estimates are biased. In reaching these negotiated outcomes, the GenCo and LSE each take into account their perceived trade-off between risk and expected return. Risk is measured using *Conditional Value at Risk (CVaR)*, a risk measure now widely adopted in financial practice. Making use of both analytical modeling and computational experiments, we carefully investigate how the negotiated outcomes for the GenCo and LSE vary systematically in response to changes in their risk preferences and price biases.

The remainder of this paper is organized as follows. Section II describes the model of a contract-for-difference negotiation process between a GenCo and an LSE. Technical results regarding Nash bargaining outcomes for the GenCo and LSE under different structural conditions are derived in Section III. A five-bus wholesale power market test case is used in Section IV to determine the sensitivity of Nash bargaining outcomes. Concluding remarks and a discussion of future extensions are provided in Section V.

II. Analytical Formulation of a Financial Bilateral Contract Negotiation Problem

A. Overview

This section develops an analytical model of a GenCo G and an LSE L attempting to negotiate the terms of a financial bilateral contract in order to hedge price risk in a day-ahead energy market with congestion managed by locational marginal prices (LMPs). Both G and L are located at the same bus, so the price risk they face arises from their uncertainty regarding future LMP outcomes at their common bus.

Each participant G and L is assumed to express its preferences over possible terms for its negotiated contract by means of a return-risk utility function. Each participant is assumed to know the utility function of the other participant. Thus, expressed in standard game theory terminology, the negotiation process between G and L is a two-player cooperative game with a commonly known payoff matrix.

The day-ahead energy market in which G and L participate entails core features of actual restructured day-ahead energy markets in the U.S. Specifically, during each operating day D a market operator runs *DC optimal power flow (DC-OPF)* software to determine hourly dispatch schedules and LMPs for the day-ahead energy market on day D+1. It is assumed that each GenCo reports its true cost and capacity conditions to the ISO. The DC-OPF is implemented as in Yu et al. [27] except that, for simplicity, ancillary services aspects are omitted.

B. The GenCo's Perspective

To be concrete, GenCo G is assumed to own a single power plant located at bus i. The production of the power plant is set

[1] In U.S. ISO-managed electric power markets such as the Midwest ISO, a bilateral transaction that involves the physical transfer of energy through a transmission provider's region is referred to as a *physical bilateral transaction*. A bilateral transaction that only transfers financial responsibility within and across a transmission provider's region is referred to as a *financial bilateral transaction*.

at a fixed rate P_G (MW) for which its outage risk is assumed to be zero. Since the plant's production rate is fixed, G is not permitted to bid strategically in the day-ahead energy market. For simplicity, it is assumed that G has a long-term supply contract for fuel, implying its fuel costs per MW of production are essentially fixed. The total variable production cost ($/h) for G's power plant in any hour h is given by

$$TVC(P_G) \quad = \quad aP_G + bP_G^2 \tag{1}$$

Under the above assumptions, the only risk facing G is price risk induced by the variability of LMP outcomes at its own bus i. In an attempt to reduce its price risk, suppose G enters into a financial bilateral contract negotiation with an LSE L, also located at bus i.

More precisely, suppose G and L attempt to negotiate the hourly contract amount M (MW) and strike price S ($/MWh) for a *contract-for-difference* over a specified period from hour 1 to hour T. Let LMP_i^h denote the LMP realized at bus i for any hour h during the period. Under the terms of this CFD, if LMP_i^h differs from the strike price S, then the advantaged party must compensate the disadvantaged party. For example, if S exceeds LMP_i^h, the advantaged buyer L must pay the disadvantaged seller G an amount [S - LMP_i^h]·M; and conversely.

After signing a CFD with hourly contract amount M and strike price S, the combined net earnings of G from its day-ahead energy market sales and its CFD, conditional on any given realization of LMP_i^h values over the contract period from hour 1 to hour T, are given by

$$\begin{aligned}\pi_G(M,S) \quad &= \quad \sum_{h=1}^{T}[LMP_i^h \cdot P_G - TVC(P_G)] \\ &\quad + \sum_{h=1}^{T}[(S - LMP_i^h)\cdot M] \end{aligned}\tag{2}$$

Let the net earnings attained by G from its day-ahead energy market sales be denoted by

$$\begin{aligned}\pi_G^0(\lambda_\Sigma) \quad &\equiv \quad \sum_{h=1}^{T}[LMP_i^h \cdot P_G - TVC(P_G)] \\ &= \lambda_\Sigma \cdot P_G - \sum_{h=1}^{T} TVC(P_G) \quad , \end{aligned}\tag{3}$$

where

$$\lambda_\Sigma \quad \equiv \quad \sum_{h=1}^{T} LMP_i^h \quad . \tag{4}$$

Then G's net earnings function (2) can be expressed as

$$\pi_G(M,S) \quad = \quad \pi_G^0(\lambda_\Sigma) + [T\cdot S - \lambda_\Sigma]\cdot M \tag{5}$$

Note that the time-value of money is not considered in G's net earnings function (2). The introduction of a discount rate could easily be incorporated to obtain a standard present-value representation for intertemporal net earnings without changing the analysis below. However, for expositional simplicity it is assumed that the contract period T for the CFD under study here is of such short duration that the discount rate across all hours of T can be set to zero.

GenCo G is a profit-seeking company that negotiates contract terms in an attempt to attain a favorable tradeoff between expected net earnings and financial risk exposure. To accomplish this, it makes use of a *return-risk utility function* to measure its relative preferences over return-risk combinations.

The best-known example of a return-risk utility function is the mean-variance utility function traditionally used in finance to evaluate portfolios of financial assets (e.g., stock holdings). Often mean-variance utility functions are specified in a simple parameterized linear form: u(mean,variance) = mean - A·variance.

Modern finance has moved away from the use of variance as a measure of financial risk for two key reasons. First, the return rates for many financial instruments appear to have thick-tailed probability density functions (PDFs) for which second moments (hence variances) do not exist.[2] Second, in financial contexts, upside deviations from expected returns are desirable; only downside deviations satisfy the intuitive idea that "riskiness" should refer to the possibility of "adverse consequences."

Consequently, in place of variance, modern financial researchers now frequently measure the financial risk of an asset portfolio in terms of single-tail measures such as *value-at-risk (VaR)* and *conditional-value-at-risk (CVaR)*. Basically, for any given confidence level α, the VaR of a portfolio is given by the smallest number l such that the probability that the loss in portfolio value exceeds l is no greater than (1-α). In contrast, the CVaR of a portfolio is defined as the *expected* loss in portfolio value during a specified period, conditional on the event that the loss is greater than or equal to VaR. Thus, CVaR informs a portfolio holder about expected loss conditional on the occurrence of an unfavorable event rather than simply indicating the probability of an unfavorable event.[3]

In this study the return-risk utility function of GenCo G is assumed to have the following parameterized linear form:

$$\begin{aligned}&u_G(E^G(\pi_G), CVaR_\alpha^G(-\pi_G)) \\ &\qquad = \quad E^G(\pi_G) - A_G \cdot CVaR_\alpha^G(-\pi_G)\end{aligned}\tag{6}$$

In (6), $E^G(\pi_G)$ denotes G's expected net earnings, and $CVaR_\alpha^G(-\pi_G)$ denotes the CVaR associated with G's "loss function," i.e., the negative of G's net earnings function (2), conditional on any given confidence level α. The parameter A_G in (6) is G's *risk-aversion factor* that determines G's

[2] A PDF $f(x)$ for a random variable X is said to be *thick tailed* if $f(x)$ and/or $f(-x)$ approaches zero relatively slowly (e.g., relative to a normal PDF) as $|x|$ approaches infinity. If the convergence is sufficiently slow, the usual integral characterization for a second moment will not be well defined for this PDF. In practical terms, this means that the sample variance formed for such a PDF on the basis of N samples will diverge to infinity almost surely as N becomes arbitrarily large.

[3] See [2], [27], [28], and [29] for a more detailed discussion of the meaning of VaR and CVaR and of the conceptual and technical advantages of CVaR relative to VaR. This study adopts the original Rockafeller and Uryasev [28] convention of defining CVaR as the right tail of a loss distribution so that CVaR is a direct measure of risk, i.e., CVaR increases as risk increases. Some risk-management researchers prefer to define CVaR as the left tail of a net earnings distribution.

preferred tradeoff between expected net earnings and risk exposure as measured by CVaR.

C. The LSE's Perspective

On each day D the LSE L submits a demand bid to purchase power at bus i from the day-ahead energy market for day D+1 in order to service retail customer load at bus i on day D+1. This demand bid consists of a 24-h load profile. Retail customers at bus i pay L a regulated rate f ($/MWh) for electricity.

At the end of day D the LSE is charged the price LMP_i^h ($/MWh) for its cleared demand for hour h of day D+1, where LMP_i^h is the LMP determined by the market operator for bus i in hour h via DC-OPF. Any deviation between L's cleared demands and its actual demands for day D+1 are resolved in the real-time market for day D+1 using real-time market LMPs. However, for simplicity, it is assumed that this deviation is zero.

The risk faced by L on each day D arises from its uncertainty regarding the prices it will be charged for its cleared demand. As detailed in Section II-B, it is assumed that L attempts to partially hedge its price risk at bus i by entering into a negotiation with GenCo G at bus i for a CFD contract over a given contract period from hour 1 to hour T. The negotiable terms of this CFD consist of an hourly contract amount M (MW) and an hourly strike price S ($/MWh).

Suppose L and G have signed a CFD for a contract amount M at a strike price S. Let P_{Li}^h denote L's cleared day-ahead market demand at bus i for any hour h during the contract period. Then the combined net earnings of L from its day-ahead energy market purchases and its CFD, conditional on any given realization of LMP_i^h values over the CFD contract period from hour 1 to hour T, are given by

$$\pi_L(M,S) = \sum_{h=1}^{T}[P_{Li}^h \cdot (f - LMP_i^h)] + \sum_{h=1}^{T}[(LMP_i^h - S)\cdot M] \quad (7)$$

As was done for G, let the net earnings of L from its day-ahead energy market purchases be denoted by

$$\pi_L^0 \equiv \sum_{h=1}^{T}[P_{Li}^h \cdot (f - LMP_i^h)] \quad (8)$$

Then, using (4), the net earnings function (7) for L can be expressed as

$$\pi_L(M,S) = \pi_L^0 + [\lambda_\Sigma - T\cdot S]\cdot M \quad (9)$$

Finally, similar to G, it is assumed that L uses a return-risk utility function to represent its preferences over combinations of expected net earnings and risk. In particular, it is assumed L's utility function takes the following parameterized linear form:

$$u_L(E^L(\pi_L), CVaR_\alpha^L(-\pi_L)) = E^L(\pi_L) - A_L \cdot CVaR_\alpha^L(-\pi_L) \quad (10)$$

In (10), $E^L(\pi_L)$ denotes L's expected net earnings, and $CVaR_\alpha^L(-\pi_L)$ denotes the CVaR associated with L's "loss function," i.e., the negative of its net earnings function (7), conditional on any given confidence level α. The parameter A_L in (10) is L's *risk-aversion factor* that determines L's preferred tradeoff between expected net earnings and risk exposure as measured by CVaR.

D. Effects of GenCo and LSE Price Estimation Biases on Expected Price and Perceived Risk

This section examines how biases in the PDFs used by GenCo G and LSE L to represent their uncertainty about the LMP outcomes at their bus i affect their price expectations and perceived risk exposure. These results will be used in Section IV to determine how these biases affect the outcomes of financial bilateral contract negotiation between G and L.

As seen in (5) and (9), the derivatives of the net earnings functions π_G and π_L with respect to the contract amount M depend on prices only through the LMP summation term λ_Σ defined in (4). Consequently, price biases distort the contract amount M negotiated by G and L only to the extent that these price biases affect the PDFs used by G and L for λ_Σ.

Suppose the true uncertainty in λ_Σ over the contract period can be represented by a probability measure P defined over a sigma-field $\mathcal{F}$ of measurable subsets of a sample space Ω of elementary events, i.e., by the probability space $(\Omega, \mathcal{F}, P)$. Suppose, instead, that G and L perceive this uncertainty to be described by probability spaces $(\Omega, \mathcal{F}, Q_G)$ and $(\Omega, \mathcal{F}, Q_L)$, respectively, where Q_G and Q_L differ from P by constant shift factors K_G and K_L as follows:

$$Q_G(\lambda_\Sigma + K_G) = P(\lambda_\Sigma) \quad (11)$$
$$Q_L(\lambda_\Sigma + K_L) = P(\lambda_\Sigma) \quad (12)$$

The constant shift factors K_G and K_L will cause the first moments (means) of Q_G and Q_L to deviate from the first moment (mean) of P, assuming these first moments exist. However, any higher moments of P will be unchanged by these constant shift factors.

Let the corresponding PDFs for λ_Σ under the three different probability measures P, Q_G, and Q_L be denoted by $f_P(\lambda_\Sigma)$, $f_{Q_G}(\lambda_\Sigma)$, and $f_{Q_L}(\lambda_\Sigma)$. These probability measures and corresponding PDFs satisfy the following relationships:

$$dP(\lambda_\Sigma) = f_P(\lambda_\Sigma)d\lambda_\Sigma \quad (13)$$
$$dQ_G(\lambda_\Sigma) = f_{Q_G}(\lambda_\Sigma)d\lambda_\Sigma \quad (14)$$
$$dQ_L(\lambda_\Sigma) = f_{Q_L}(\lambda_\Sigma)d\lambda_\Sigma \quad (15)$$

It follows from these relationships that

$$f_{Q_G}(\lambda_\Sigma + K_G) = f_P(\lambda_\Sigma) \quad (16)$$
$$f_{Q_L}(\lambda_\Sigma + K_L) = f_P(\lambda_\Sigma) \quad (17)$$

Fig. 1 illustrates relationships (16) and (17) for a particular configuration of biases.

Using the above relationships, the effects of the constant shift factors K_G and K_L on the expectation and CVaR for λ_Σ can be derived. These derivations are summarized in the following key theorem, proved in Appendix A.

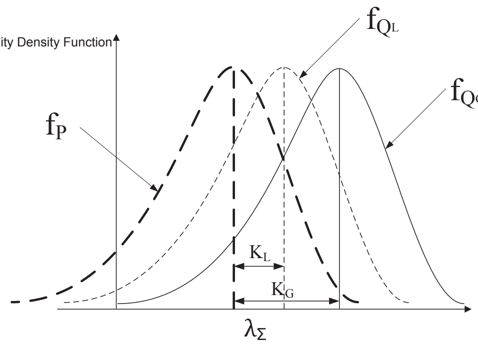

Fig. 1. Relationships among true and biased probability density functions for λ_Σ given $K_G > K_L > 0$

Theorem 1: Given any confidence level $\alpha \in (0,1)$, the expectation and CVaR$_\alpha$ measure for λ_Σ under the true probability measure P and the biased probability measures Q_G and Q_L satisfy the following relationships:

$$E^G(\lambda_\Sigma) = E^P(\lambda_\Sigma) + K_G \quad (18)$$
$$CVaR^G_\alpha(\lambda_\Sigma) = CVaR^P_\alpha(\lambda_\Sigma) + K_G \quad (19)$$
$$E^L(\lambda_\Sigma) = E^P(\lambda_\Sigma) + K_L \quad (20)$$
$$CVaR^L_\alpha(\lambda_\Sigma) = CVaR^P_\alpha(\lambda_\Sigma) + K_L \quad (21)$$

III. A Nash Bargaining Theory Approach

Section III-A reviews Nash bargaining theory in general terms. The theory is then applied in Section III-B to the financial bilateral contract negotiation set out in Section II.

A. Nash Bargaining Theory: General Formulation

Consider two utility-seeking players attempting to agree on a settlement point $\boldsymbol{u} = (u_1, u_2)$ in a compact convex *utility possibility set* $U \subseteq \Re^2$. If the two players fail to reach an agreement, the default outcome is a *threat point* $\boldsymbol{\zeta} = (\zeta_1, \zeta_2)$ satisfying $\boldsymbol{\zeta} \in U$ and

$$U \cap \{\boldsymbol{x} \in \Re^2 : \boldsymbol{x}_j > \zeta_j \text{ for } j = 1 \text{ or } j = 2\} \neq \emptyset \quad (22)$$

Let the set of all bargaining problems $(U, \boldsymbol{\zeta})$ satisfying the above assumptions be denoted by D. For each $(U, \boldsymbol{\zeta}) \in D$, define the *barter set* as follows:

$$B(U, \boldsymbol{\zeta}) \quad \equiv \quad U \cap \{\boldsymbol{x} \in \Re^2 : \boldsymbol{x} \geq \boldsymbol{\zeta}\} \quad (23)$$

Nash [30] defined a *bargaining solution* to be any function $f{:}D \rightarrow \Re^2$ that assigns a unique outcome $f(U, \boldsymbol{\zeta}) \in B(U, \boldsymbol{\zeta})$ for every bargaining problem $(U, \boldsymbol{\zeta}) \in D$. Nash characterized four axioms considered to be essential for any fair and efficient bargaining process: invariance under positive linear affine transformations; symmetry; independence of irrelevant alternatives; and Pareto efficiency; see [31] for details. He then proved that there is a unique bargaining solution that satisfies these four axioms. Specifically, for any given bargaining problem $(U, \boldsymbol{\zeta}) \in D$ satisfying these four axioms, Nash's bargaining solution $f^*(U, \boldsymbol{\zeta}) \equiv (u_1^*, u_2^*) \in B(U, \boldsymbol{\zeta})$ is the unique solution to the following problem: maximize $(u_1 - \zeta_1)(u_2 - \zeta_2)$ with respect to the choice of $(u_1, u_2) \in B(U, \boldsymbol{\zeta})$. Hereafter the function f^* will be referred to as the *Nash Bargaining Solution*.

B. Application of Nash Bargaining Theory to the Contract Negotiation Problem for GenCo G and LSE L

Consider once again the financial bilateral contract problem set out in Section II. GenCo G and LSE L are engaged in a negotiation for a contract-for-difference at their common location, bus i.

Suppose G and L use Nash bargaining theory in an attempt to negotiate the contract amount M and strike price S for this CFD. Assume the threat point $\boldsymbol{\zeta}$ is given by the utility levels expected to be attained by G and L if no contract is signed:

$$\zeta_1 \quad \equiv \quad u_G(E^G(\pi^0_G), CVaR^G_\alpha(-\pi^0_G)) \quad (24)$$
$$\zeta_2 \quad \equiv \quad u_L(E^L(\pi^0_L), CVaR^L_\alpha(-\pi^0_L)) \quad (25)$$

Suppose, also, that the feasible negotiation ranges for M and S are nonempty closed intervals: $M^R \leq M \leq M^U$, and $S^R \leq S \leq S^U$.[4]

The utility possibility set U for G and L's CFD bargaining problem is then given by the set of all possible utility outcomes (6) and (10) for G and L as M and S vary over their feasible negotiation ranges. The barter set for this bargaining problem $(U, \boldsymbol{\zeta})$ takes the form $B \equiv \{(u_G, u_L) \in U : u_G \geq \zeta_1, u_L \geq \zeta_2\}$. Finally, the Nash bargaining solution for this CFD bargaining problem is calculated as follows:

$$\max_{(\boldsymbol{u_G}, \boldsymbol{u_L}) \in \boldsymbol{B}} \quad (u_G - \zeta_1)(u_L - \zeta_2) \quad (26)$$

The following key theorem, proved in Appendix B, establishes that the barter set B for this CFD bargaining problem is always convex even though the utility possibility set U can fail to be convex.

Theorem 2: Suppose the previously given restrictions on the CFD bargaining problem for G and L all hold. Suppose, also, that the lowest possible strike price S^R is less than S^{R^*} as defined in (27), the highest possible strike price S^U is greater than S^{U^*} as defined in (28), and $0 \leq M^R < P_G$. Then the Nash barter set B for the CFD bargaining problem for G and L is a non-empty, compact, convex subset of $\Re^2$. Specifically, the barter set B is a compact right triangle when conditions (29) and (30) both hold (cf. Fig. 2); the barter set B reduces to the no-contract threat point when inequality (30) does not hold (cf. Fig. 3); and the barter set B is a compact right triangle when (29) does not hold but (30) holds (cf. Fig. 4).

[4]The reason why we include consideration of contract amounts M greater than the GenCo's fixed generation level P_G is that the LSE might have a much larger amount of load to serve than P_G. In this case, if the LSE is extremely risk averse, it might be willing to pay a significant "risk premium" to the GenCo to sign a CFD contract in an amount greater than P_G in order to hedge its risk. The GenCo might be willing to sign the CFD because its expected net earnings outweigh the price risk associated with its short sale.

$$S^{R^*} = \min\{\frac{E^P(\lambda_\Sigma) + A_G CVaR_\alpha^P(\lambda_\Sigma) + (1+A_G)K_G}{T[1+A_G]}, \frac{E^P(\lambda_\Sigma) - A_G CVaR_\alpha^P(-\lambda_\Sigma) + (1+A_G)K_G}{T[1+A_G]}, \frac{E^P(\lambda_\Sigma) + (1+A_L)K_L - A_L CVaR_\alpha^P(-\lambda_\Sigma)}{T[1+A_L]}\} \tag{27}$$

$$S^{U^*} = \max\{\frac{E^P(\lambda_\Sigma) + A_G CVaR_\alpha^P(\lambda_\Sigma) + (1+A_G)K_G}{T[1+A_G]}, \frac{E^P(\lambda_\Sigma) - A_G CVaR_\alpha^P(-\lambda_\Sigma) + (1+A_G)K_G}{T[1+A_G]}, \frac{E^P(\lambda_\Sigma) + (1+A_L)K_L + A_L CVaR_\alpha^P(\lambda_\Sigma)}{T[1+A_L]}\} \tag{28}$$

$$\frac{dCVaR_\alpha^L(-\pi_L(M^U,S^R))}{dM} > \frac{A_G - A_L}{A_L[1+A_G]}E^P(\lambda_\Sigma) - \frac{A_G[1+A_L]}{A_L[1+A_G]}CVaR_\alpha^P(\lambda_\Sigma) + \frac{1}{A_L}K_L - \frac{1+A_L}{A_L}K_G + TS \tag{29}$$

$$\frac{dCVaR_\alpha^L(-\pi_L(M^R,S^R))}{dM} < \frac{A_G - A_L}{A_L[1+A_G]}E^P(\lambda_\Sigma) + \frac{A_G[1+A_L]}{A_L[1+A_G]}CVaR_\alpha^P(-\lambda_\Sigma) + \frac{1}{A_L}K_L - \frac{1+A_L}{A_L}K_G + TS \tag{30}$$

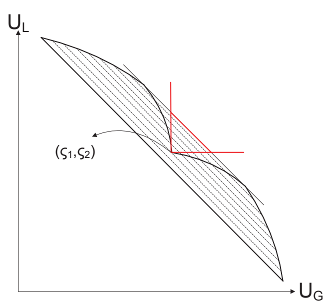

Fig. 2. Illustration of the utility possibility set U and barter set B for GenCo G and LSE L when (29) and (30) both hold. The barter set is a right triangle.

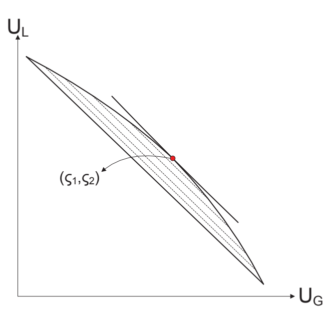

Fig. 3. Illustration of the utility possibility set U and barter set B for GenCo G and LSE L when (30) fails to hold. The barter set reduces to the non-contract threat point.

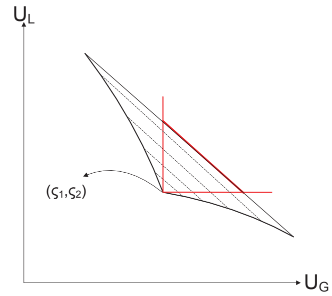

Fig. 4. Illustration of the utility possibility set U and barter set B for GenCo G and LSE L when (29) does not hold but (30) holds. The barter set is a right triangle.

IV. Simulation Results

A. Five-Bus Test Case and Experimental Design

This section reports on computational CFD bargaining experiments conducted using a modified version of the benchmark five-bus test case presented in [32]. As depicted in Fig 5, the key changes are the addition of GenCo G6 at Bus 3 that owns and operates a power plant at Bus 3, and a more detailed modeling of LSE 2 at Bus 3.

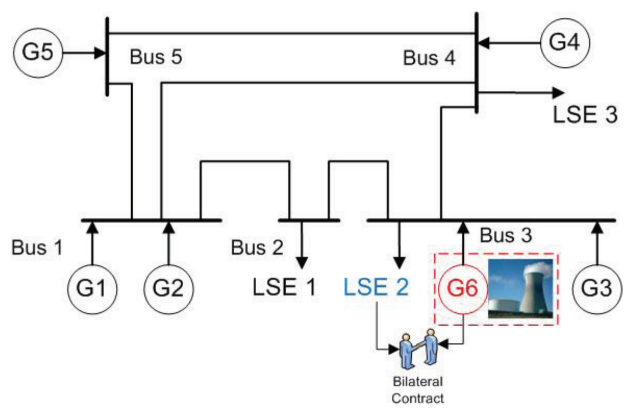

Fig. 5. Five-bus test case used for computational experiments.

More precisely, G6 is assumed to have the characteristics of the profit-seeking risk-averse GenCo G described in Section II-B, and LSE 2 is assumed to have the characteristics of the profit-seeking risk-averse LSE L described in Section II-C. To hedge their price risk at Bus 3, G6 and LSE 2 enter into a negotiation process for a CFD. As in Section III-B, this CFD negotiation process is modeled as a Nash bargaining problem, and outcomes are obtained via a Nash bargaining solution as in (26).

The two types of experiments reported below examine how the outcomes of this CFD bargaining problem are affected by systematic variations in structural conditions. The first set of experiments investigates the effects of absolute and relative changes in the risk-aversion factors A_G and A_L for G6 and LSE 2, assuming zero price bias. The second set of experiments investigates the effects of absolute and relative changes in the price bias factors K_G and K_L affecting the estimates formed by G6 and LSE 2 for λ_Σ, the sum of

LMPs at Bus 3 during the CFD contract period, conditional on particular risk aversion settings. For simplicity, these price bias factors are assumed to be proportional to E[λ_Σ].

As in Section II-B, G6's power plant is assumed to have a quadratic *total variable cost (TVC) function* given by (1). The parameters characterizing this TVC function are set as follows: $b = 0.005$ and $a = 10.0$. G6's fixed production rate P_G is set at 300 MW. The regulated retail resale rate f for LSE 2 is set at \$25/MWh. Also, the confidence level α for all CVaR evaluations for both GenCo G6 and LSE 2 is set at 0.95. All line capacities, reactances, and cost and capacity data for GenCos G1 through G5 are set as in the benchmark five-bus test case from [32].

The CFD contract period for G6 and LSE 2 is assumed to be one month, "June." The "true" daily average load during this month was generated via a truncated multivariate normal distribution. To make the case study more realistic, the parameters for the mean vector and covariance matrix for this distribution were estimated from MISO load data for June 2006 [33]. The daily average load and load autocorrelation function used for sample generation are provided in Tables I and II. The variance of the daily average load was set at 834.5748 MW^2. The hourly load was approximated by multiplying the daily total load by an hourly load weight factor equal to the load weight factor for the historical data.

TABLE I
DAILY AVERAGE LOAD FOR THE FIVE-BUS TEST CASE DURING THE CONTRACT MONTH ("JUNE").

June 1	June 2	June 3	June 4	June 5
337.01 MW	319.10 MW	285.94 MW	268.12 MW	318.61 MW
June 6	June 7	June 8	June 9	June 10
329.53 MW	335.84 MW	336.94 MW	316.81 MW	270.06 MW
June 11	June 12	June 13	June 14	June 15
250.76 MW	297.36 MW	310.81 MW	322.45 MW	338.52 MW
June 16	June 17	June 18	June 19	June 20
360.43 MW	341.99 MW	312.55 MW	351.49 MW	349.64 MW
June 21	June 22	June 23	June 24	June 25
363.59 MW	367.08 MW	336.56 MW	300.43 MW	285.71 MW
June 26	June 27	June 28	June 29	June 30
329.89 MW	335.36 MW	336.34 MW	337.69 MW	336.93 MW

TABLE II
AUTOCORRELATION FUNCTION FOR DAILY AVERAGE LOAD FOR THE FIVE-BUS TEST CASE DURING THE CONTRACT MONTH ("JUNE").

Lag 0	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
1.00000	0.68366	0.22233	-0.09257	-0.16865	-0.04008
Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Lag 11
0.18943	0.36306	0.28063	0.12285	0.00094	-0.05240
Lag 12	Lag 13	Lag 14	Lag 15	Lag 16	Lag 17
-0.05279	-0.03001	-0.00707	0.00596	0.00903	0.00644
Lag 18	Lag 19	Lag 20	Lag 21	Lag 22	Lag 23
0.00251	-0.00028	-0.00137	-0.00125	-0.00065	-0.00011
Lag 24	Lag 25	Lag 26	Lag 27	Lag 28	Lag 29
0.00017	0.00022	0.00015	0.00005	-0.00001	-0.00004

Using the above modeling for hourly loads, 1000 sample paths were generated for hourly DC-OPF dispatch and LMP solutions for the day-ahead energy market over the contract month.To reduce the sample space and corresponding sample generation time and number of runs necessary for Monte Carlo simulation, recourse was made to *Latin Hypercube Sampling*, an efficient stratified sampling technique [34].

Given each experimental treatment, i.e., each setting for (A_G, A_L, K_G, K_L), these 1000 sample paths were used to formulate the return-risk utility functions (6) and (10) for G6 and LSE 2 as functions of the contract amount M and strike price S. The feasible negotiation ranges for M and S were set as follows:[5] $M \in [0, 600]$, and $S \in [15, 25]$. The unique Nash bargaining outcomes for M and S were then determined.

B. Findings

1) Risk-Aversion Treatment: This section examines the effects of changes in the risk-aversion factors A_G and A_L assuming zero price bias ($K_G = K_L = 0$).

Table III reports the Nash bargaining outcomes for the contract amount M and strike price S as A_G and A_L are systematically varied from 0.5 to 2.0. Moving from top to bottom in each column of Table III, the negotiated strike price S systematically decreases as G6's risk-aversion factor A_G is increased, holding fixed the risk-aversion factor A_L for LSE 2. Conversely, moving from left to right in each row, the negotiated strike price S systematically increases as LSE 2's risk-aversion factor A_L is increased, holding fixed the risk-aversion factor A_G for G6. In summary, all else equal, as each trader becomes more risk averse the negotiated strike price S moves in a direction that favors the other trader.

TABLE III
EFFECTS OF RISK-AVERSION FACTORS ON THE CONTRACT AMOUNT M AND STRIKE PRICE S DETERMINED THROUGH NASH BARGAINING.

A_G \ A_L	0.5	1	2
0.5	\$19.84/MWh 300.0 MW	\$19.93/MWh 284.4 MW	\$20.02/MWh 271.0 MW
1	\$19.74/MWh 300.0 MW	\$19.81/MWh 300.0 MW	\$19.88/MWh 291.8 MW
2	\$19.64/MWh 300.0 MW	\$19.71/MWh 300.0 MW	\$19.77/MWh 300.0 MW

For S sufficiently close to S^U = 25 (the setting for S^U used in all simulation runs), it follows from Lemma 4 that the net earnings of LSE 2 decrease with increases in the contract amount M whereas the net earnings of G6 increase with increases in M. This implies that LSE 2 will prefer a smaller M and G6 a larger M as S increases, all else equal. However, it is then unclear which way the actual negotiated contract amount M will move as the negotiated strike price S increases.

The findings in Table III reveal that, for each given risk aversion level A_G for G6, a *higher* risk aversion level A_L for LSE 2 results not only in a higher S but also in an M that

[5] As required by Theorem 2, it can be shown that the setting $S^R = 15$ is smaller than S^{R^*} in (27) and the setting $S^U = 25$ is greater than S^{U^*} in (28) for each tested configuration for (A_G, A_L, K_G, K_L).

is *either unchanged or lower*. Conversely, for each given risk aversion level A_L for LSE 2, a *lower* risk aversion level A_G for G6 results not only in a higher S but also in an M that is *either unchanged or lower*. In short, S and M tend to move inversely in Table III .

Another interesting regularity is observed in the diagonal elements of Table III. When LSE 2 and G6 have the same level of risk aversion, a lock-step change in risk aversion for both LSE 2 and G6 results in no change in the negotiated value for M. On the other hand, from the off-diagonal elements (0.5,1) and (1,2) of Table III it is seen that the negotiated outcomes for M and S can depend on the absolute levels of risk aversion for LSE 2 and G6; it is not only the relative levels that matter.

Finally, the reason why several combinations of A_G and A_L in Table III result in the same contract amount 300 MW is that GenCo G6 is fully hedged with a 300 MW contract because its (fixed) production rate is set at 300 MW. Therefore, when G6 is at least as risk averse as LSE 2, it is not surprising to see the contract amount M settle at 300 MW.

Figs. 6 and 7 display the effects of changes in the risk-aversion factor A_L for LSE 2 on the post-contract net earnings histograms for G6 and LSE 2, respectively, assuming the risk-aversion factor A_G for G6 is fixed at 1.0. As LSE 2 becomes more risk averse, its net earnings histogram shifts to the left, an unfavorable shift for LSE 2. On the other hand, the net earnings histogram for G6 shifts to the right, a favorable shift for G6. These net earnings findings provide additional support for the conclusion previously drawn from the more aggregated findings reported in Table III: namely, an increase in risk aversion for one party to the CFD bargaining process, all else equal, results in a worse outcome for this party and a more favorable outcome for the other party.

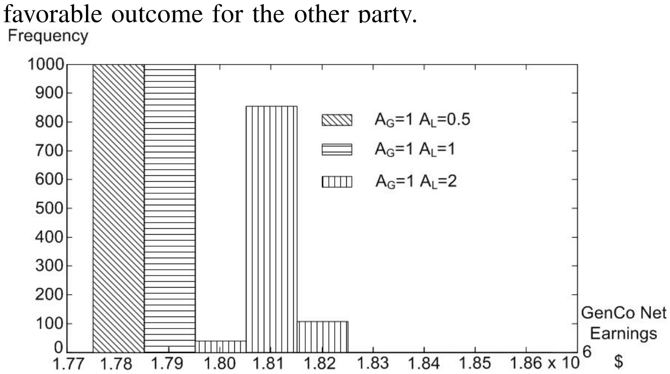

Fig. 6. GenCo net earnings histogram given a fixed GenCo risk-aversion factor $A_G = 1$ and varying values for the LSE risk-aversion factor A_L

2) LMP Bias Treatment: Experiments were conducted to determine the effects of changes in the price bias factors K_G and K_L for each risk-aversion treatment (A_G, A_L) in Table III. It follows from Theorem 1 in Section II-D that a higher value for K_G (or K_L) implies G6 (or LSE 2) expects higher LMP outcomes. Due to space limitations, only the price bias results for $A_G = A_L = 1$ are reported below.[6]

Table IV reports Nash bargaining outcomes for the contract amount M and strike price S as the price bias factors K_G

[6]The price bias results for the other risk-aversion treatments are qualitatively similar.

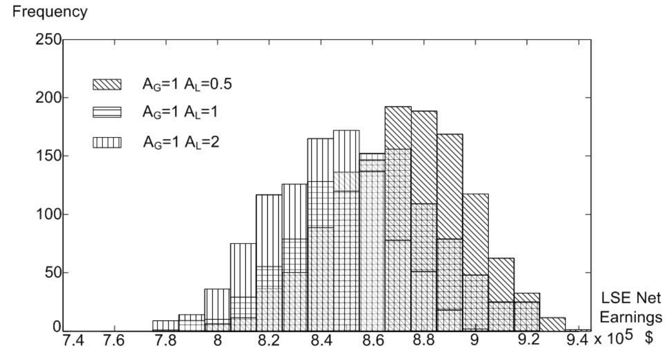

Fig. 7. LSE net earnings histogram given a fixed GenCo risk-aversion factor $A_G = 1$ and varying values for the LSE risk-aversion factor A_L

and K_L are each systematically varied from $-0.01E^P(\lambda_\Sigma)$ to $0.01E^P(\lambda_\Sigma)$. The no-bias case $K_L = 0$ and $K_G = 0$ provides a useful benchmark of comparison. Relative to this benchmark, if LSE 2 underestimates λ_Σ, then the strike price S decreases; and if LSE overestimates λ_Σ, then S increases. Conversely, relative to this benchmark, if G6 underestimates λ_Σ, then S decreases; and if G6 overestimates λ_Σ, then S increases. Also, moving from the lower-left to the upper-right cell of Table IV—that is, letting K_G increase and K_L decrease together—the contract amount M is seen to either remain the same or decrease.

TABLE IV
EFFECTS OF BIASES IN LMP ESTIMATES ON THE CONTRACT AMOUNT M AND STRIKE PRICE S DETERMINED THROUGH NASH BARGAINING.

K_L \ K_G	$-0.01E^P(\lambda_\Sigma)$	0	$0.01E^P(\lambda_\Sigma)$
$-0.01E^P(\lambda_\Sigma)$	\$19.60/MWh 300.0 MW	\$19.73/MWh 282.8 MW	\$19.85/MWh 263.4 MW
0	\$19.71/MWh 300.0 MW	\$19.81/MWh 300.0 MW	\$19.93/MWh 284.4 MW
$0.01E^P(\lambda_\Sigma)$	\$19.81/MWh 300.0 MW	\$19.91/MWh 300.0 MW	\$20.01/MWh 300.0 MW

Moreover, for each given price bias level for one negotiation participant (either G6 or LSE 2), S increases with increases in the price bias of the other participant. As noted above, LSE 2 will prefer a smaller M and G6 a larger M as S increases, all else equal. However, it is then unclear which way the negotiated contract amount M would move if the negotiated strike price S increases due to some change in price bias. Interestingly, Table IV reveals that M is always either unchanged or *lower* when S increases due to an increase in the price bias of G6 conditional on a given price bias for LSE 2. Conversely, M is always either unchanged or *higher* when S increases due to an increase in the price bias of LSE 2 conditional on a given price bias for G6.

Additional simulations were also conducted to search for combinations of the normalized price-bias factors $K_G/E^P(\lambda_\Sigma)$ and $K_L/E^P(\lambda_\Sigma)$ such that the negotiated contract amount M was zero, implying a no-contract outcome. These no-contract regions are depicted in Fig. 8 for three alternative specifications for the risk-aversion factors. As seen, for each risk-aversion case the boundary of the no-contract region in the $(K_L/E^P(\lambda_\Sigma), K_G/E^P(\lambda_\Sigma))$ plane is a line, and the no-contract region is the half-plane bounded below by this no-contract line. An important observation from Fig. 8 is that the no-contract region shrinks in size as the traders become more risk averse and hence more anxious to successfully agree on a contract.

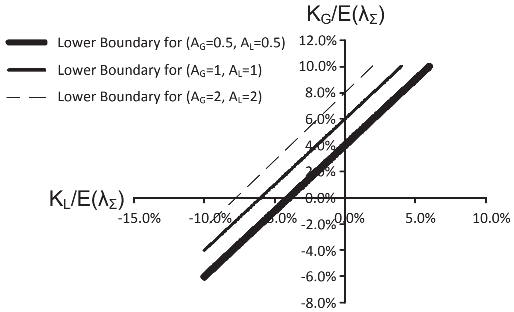

Fig. 8. No-contract boundaries and regions under three combinations of risk-aversion factors

V. Conclusion

This study analyzes Nash bargaining settlement outcomes for a contract-for-difference (CFD) negotiation between a GenCo and an LSE facing price risk from uncertain LMP outcomes at a common bus location. Using both analysis and computational experiments, it is shown that differing levels of risk aversion and biases in LMP estimations have systematic effects on the negotiated contract amount and strike price, hence also on the post-contract net earnings distributions for the GenCo and LSE. In addition, circumstances in which the two parties can fail to reach an agreement are identified.

Future studies will consider more general contract negotiation problems involving both financial and physical energy contracts between wholesale power market traders located at possibly different buses. In this case full hedging of price risk can require traders to combine CFDs with additional instruments, such as financial transmission rights, to take into account LMP separation across buses due to transmission congestion. Another important topic for future studies is the extension of the current model to handle multilateral contract negotiation problems to better capture medium-term contracting opportunities.

Appendix A
Proof of Theorem 1 in Section II-D

The proof of Theorem 1 follows directly from Propositions 1–3, below. For expositional simplicity, the assumptions of Theorem 1 are not repeated in the statement of each proposition but are instead tacitly assumed to hold.

Proposition 1: The expected values for λ_Σ derived under the three probability measures P, Q_G, and Q_L satisfy (18) and (20).

Proof of Proposition 1: The expected value of λ_Σ derived under Q_G (with pdf f_{Q_G}) is given by

$$\begin{aligned} E^G(\lambda_\Sigma) &= \int_{-\infty}^{+\infty} \lambda_\Sigma f_{Q_G}(\lambda_\Sigma) d\lambda_\Sigma \\ &= \int_{-\infty}^{+\infty} \lambda_\Sigma f_P(\lambda_\Sigma - K_G) d\lambda_\Sigma \end{aligned} \tag{31}$$

Introducing the change of variables $\lambda'_\Sigma = \lambda_\Sigma - K_G$,

$$\begin{aligned} E^G(\lambda_\Sigma) &= \int_{-\infty}^{+\infty} (\lambda'_\Sigma + K_G) f_P(\lambda'_\Sigma) d\lambda'_\Sigma \\ &= \int_{-\infty}^{+\infty} \lambda'_\Sigma f_P(\lambda'_\Sigma) d\lambda'_\Sigma + K_G \int_{-\infty}^{+\infty} f_P(\lambda'_\Sigma) d\lambda'_\Sigma \\ &= E^P(\lambda_\Sigma) + K_G \end{aligned} \tag{32}$$

It can similarly be shown that $E^L(\lambda_\Sigma) = E^P(\lambda_\Sigma) + K_L$. QED

Proposition 2: The VaR values for λ_Σ derived under P, Q_G, and Q_L satisfy

$$VaR^G_\alpha(\lambda_\Sigma - K_G) = VaR^P_\alpha(\lambda_\Sigma) \tag{33}$$
$$VaR^L_\alpha(\lambda_\Sigma - K_L) = VaR^P_\alpha(\lambda_\Sigma) \tag{34}$$

Proof of Proposition 2: $VaR^G_\alpha(\lambda_\Sigma)$ and $VaR^P_\alpha(\lambda_\Sigma)$ are defined as follows:

$$\begin{aligned} VaR^G_\alpha(\lambda_\Sigma) &\equiv \inf\{\Lambda \in \Re : Q_G(\lambda_\Sigma > \Lambda) \leq 1 - \alpha\} \\ &= \inf\{\Lambda \in \Re : \int_\Lambda^{+\infty} f_{Q_G}(\lambda_\Sigma) d\lambda_\Sigma \leq 1 - \alpha\} \end{aligned} \tag{35}$$

$$\begin{aligned} VaR^P_\alpha(\lambda_\Sigma) &\equiv \inf\{\Lambda \in \Re : P(\lambda_\Sigma > \Lambda) \leq 1 - \alpha\} \\ &= \inf\{\Lambda \in \Re : \int_\Lambda^{+\infty} f_P(\lambda_\Sigma) d\lambda_\Sigma \leq 1 - \alpha\} \end{aligned} \tag{36}$$

It follows from the definition of $VaR^G_\alpha(\lambda_\Sigma)$ that

$$\begin{aligned} &VaR^G_\alpha(\lambda_\Sigma - K_G) \\ &= \inf\{\Lambda \in \Re : Q_G(\lambda_\Sigma - K_G > \Lambda) \leq 1 - \alpha\} \\ &= \inf\{\Lambda \in \Re : Q_G(\lambda_\Sigma > \Lambda + K_G) \leq 1 - \alpha\} \\ &= \inf\{\Lambda \in \Re : \int_{\Lambda + K_G}^{+\infty} f_{Q_G}(\lambda_\Sigma) d\lambda_\Sigma \leq 1 - \alpha\} \\ &= \inf\{\Lambda \in \Re : \int_{\Lambda + K_G}^{+\infty} f_P(\lambda_\Sigma - K_G) d\lambda_\Sigma \leq 1 - \alpha\} \end{aligned} \tag{37}$$

Introducing the change of variables $\lambda'_\Sigma = \lambda_\Sigma - K_G$,

$$\begin{aligned} &VaR^G_\alpha(\lambda_\Sigma - K_G) \\ &= \inf\{\Lambda \in \Re : \int_\Lambda^{+\infty} f_P(\lambda'_\Sigma) d\lambda'_\Sigma \leq 1 - \alpha\} \\ &= VaR^P_\alpha(\lambda_\Sigma) \end{aligned} \tag{38}$$

It can similarly be shown that $VaR^L_\alpha(\lambda_\Sigma - K_L) = VaR^P_\alpha(\lambda_\Sigma)$. QED

Proposition 3: The CVaR values for λ_Σ derived under P, Q_G, and Q_L satisfy (19) and (21).

Proof of Proposition 3: Let Y denote any real-valued random variable measurable with respect to a probability space $(\Omega, \mathcal{F}, \mu)$. Let $\alpha \in (0,1)$, and let A denote the measurable subset of points $\omega \in \Omega$ such that $Y(\omega) \geq VaR^{\mu}_{\alpha}(Y)$, which implies (by definition of VaR) that $\mu(A) = [1-\alpha]$. Then CVaR$^{\mu}_{\alpha}$(Y) is defined as follows:

$$CVaR^{\mu}_{\alpha}(Y) \quad \equiv \quad \frac{1}{1-\alpha}\int_A Y d\mu(Y) \tag{39}$$

Recall that f_{Q_G} is the pdf corresponding to the probability measure Q_G. It follows that

$$\begin{aligned} CVaR^{G}_{\alpha}(\lambda_\Sigma) &= \frac{1}{1-\alpha}\int_{VaR^G_\alpha(\lambda_\Sigma)}^{+\infty} \lambda_\Sigma f_{Q_G}(\lambda_\Sigma) d\lambda_\Sigma \\ &= \frac{1}{1-\alpha}\int_{VaR^P_\alpha(\lambda_\Sigma+K_G)}^{+\infty} \lambda_\Sigma f_P(\lambda_\Sigma - K_G) d\lambda_\Sigma \end{aligned} \tag{40}$$

Introducing the change of variables $\lambda'_\Sigma = \lambda_\Sigma - K_G$,

$$\begin{aligned} &CVaR^{G}_{\alpha}(\lambda_\Sigma) \\ &= \frac{1}{1-\alpha}\int_{VaR^P_\alpha(\lambda_\Sigma+K_G)-K_G}^{+\infty} (\lambda'_\Sigma + K_G) f_P(\lambda'_\Sigma) d\lambda'_\Sigma \\ &= \frac{1}{1-\alpha}\int_{VaR^P_\alpha(\lambda_\Sigma)}^{+\infty} \lambda'_\Sigma f_P(\lambda'_\Sigma) d\lambda'_\Sigma \\ &\quad + \frac{1}{1-\alpha} K_G \int_{VaR^P_\alpha(\lambda_\Sigma)}^{+\infty} f_P(\lambda'_\Sigma) d\lambda'_\Sigma \\ &= CVaR^P_\alpha(\lambda_\Sigma) + K_G \end{aligned} \tag{41}$$

It can similarly be shown that $CVaR^L_\alpha(\lambda_\Sigma) = CVaR^P_\alpha(\lambda_\Sigma) + K_L$. QED

Appendix B
Proof of Theorem 2 in Section III-B

This section provides a proof for Theorem 2 making use of four lemmas. For expositional simplicity, the assumptions of Theorem 2 are not repeated in the statement of each lemma but are instead tacitly assumed to hold. Also, throughout this appendix the α subscripts on all VaR and CVaR expressions are omitted, as are the P-superscripts for all expectations, VaR, and CVaR expressions calculated using the true probability measure P.

Lemma 1: $CVaR^L(-\pi_L(M,S))$ is convex in M for any $S \in [S^R, S^U]$.

Proof of Lemma 1: Let $S \in [S^R, S^U]$ be given. To prove that $CVaR^L(-\pi_L(M,S))$ is convex in M, we need to show that, for arbitrary M_1, M_2, and $0 < \lambda < 1$, the following inequality holds,

$$\begin{aligned} &CVaR^L(-\pi_L(\lambda M_1 + [1-\lambda M_2], S)) \\ &\leq \lambda CVaR^L(-\pi_L(M_1,S)) + (1-\lambda) CVaR^L(-\pi_L(M_2,S)) \end{aligned} \tag{42}$$

Using the convexity of CVaR we have,

$$\begin{aligned} \text{right} =& \lambda CVaR^L(-\pi^0_L - M_1(\lambda_\Sigma - TS)) \\ &+ (1-\lambda) CVaR^L(-\pi^0_L - M_2(\lambda_\Sigma - TS)) \\ \geq& CVaR^L(-\lambda\pi^0_L - \lambda M_1(\lambda_\Sigma - TS) \\ &- (1-\lambda)\pi^0_L - (1-\lambda) M_2(\lambda_\Sigma - TS)) \\ =& CVaR^L(-\pi^0_L - [\lambda M_1 + (1-\lambda)M_2](\lambda_\Sigma - TS)) \\ =& \text{ left} \end{aligned} \tag{43}$$

QED

Lemma 2: Given any contract amount $M \in [M^R, M^U]$, varying the strike price S from S^R to S^U maps under (6) and (10) into a straight line in U with slope $-[1+A_L]/[1+A_G]$.

Proof of Lemma 2: Using (5) and (9), we have

$$\pi_G(M, S+\Delta S) - \pi_G(M,S) = TM\Delta S \tag{44}$$

$$\pi_L(M, S+\Delta S) - \pi_L(M,S) = -TM\Delta S \tag{45}$$

Taking expectations on each side of equations (44) and (45),

$$E^G\pi_G(M, S+\Delta S) - E^G\pi_G(M,S) = TM\Delta S \tag{46}$$

$$E^L\pi_L(M, S+\Delta S) - E^L\pi_L(M,S) = -TM\Delta S \tag{47}$$

It follows immediately from the definition of CVaR that CVaR is translation-equivariant, i.e. $CVaR(Y+c) = CVaR(Y)+c$. Thus

$$\begin{aligned} &CVaR^G(-\pi_G(M,S+\Delta S)) \\ &\qquad = CVaR^G(-\pi_G(M,S) - TM\Delta S) \\ &\qquad = CVaR^G(-\pi_G(M,S)) - TM\Delta S \end{aligned} \tag{48}$$

Rearranging the terms in the above equation, we have

$$\begin{aligned} &CVaR^G(-\pi_G(M,S+\Delta S)) - CVaR^G(-\pi_G(M,S)) \\ &\qquad = -TM\Delta S \end{aligned} \tag{49}$$

Similarly, we have

$$\begin{aligned} &CVaR^L(-\pi_L(M,S+\Delta S)) - CVaR^L(-\pi_L(M,S)) \\ &\qquad = TM\Delta S \end{aligned} \tag{50}$$

The utility functions for GenCo G and LSE L are defined in (6) and (10). Using these definitions, together with relationships (46), (47), (49), and (50), we have

$$\begin{aligned} &u_G(E^G(\pi_G(M,S+\Delta S)), CVaR^G(-\pi_G(M,S+\Delta S))) \\ &\quad - u_G(E^G(\pi_G(M,S)), CVaR^G(-\pi_G(M,S))) \\ &\quad = TM\Delta S[1+A_G] \end{aligned} \tag{51}$$

and

$$\begin{aligned} &u_L(E^L(\pi_L(M,S+\Delta S)), CVaR^L(-\pi_L(M,S+\Delta S))) \\ &\quad - u_L(E^L(\pi_L(M,S)), CVaR^L(-\pi_L(M,S))) \\ &\quad = -TM\Delta S[1+A_L] \end{aligned} \tag{52}$$

It follows that

$$\frac{du_G}{dS} = TM[1+A_G] \tag{53}$$

$$\frac{du_L}{dS} = -TM[1+A_L] \tag{54}$$

Integrating both sides of equations (53) and (54) with respect to S, we have

$$u_G + C_1 = TM[1 + A_G]S + C_2 \quad (55)$$
$$u_L + C_3 = -TM[1 + A_L]S + C_4 \quad (56)$$

Multiply equations (55) and (56) by $[1+A_L]$ and $[1+A_G]$, respectively, and add the resulting expressions. After rearranging terms,

$$[1 + A_L]u_G + [1 + A_G]u_L$$
$$= -[1 + A_L]C_1 + [1 + A_L]C_2 - [1 + A_G]C_3 + [1 + A_G]C_4 \quad (57)$$

Totally differentiating this expression, it follows that

$$\frac{du_L}{du_G} = -\frac{1 + A_L}{1 + A_G} \quad (58)$$

QED

To better understand the proof of the next lemma, the reader might wish to view Figs. 10 through 12 used below for the main proof of Theorem 2.

Lemma 3: If the strike price S is fixed at its lowest possible level S^R, then the locus of points (u_G, u_L) traced out in the utility possibility set U under (6) and (10) as the contract amount M varies from M^R to M^U determines a concave curve V_1 in U. Conversely, if the strike price S is fixed at its highest possible level S^U, then the locus of points (u_G, u_L) traced out in U under (6) and (10) as M varies from M^R to M^U determines a concave curve V_2 in U.

Proof of Lemma 3: Suppose the strike price S is fixed at its lowest possible level S^R. Using (5),

$$\begin{aligned} E^G\pi_G(M + \Delta M, S) - E^G\pi_G(M, S) &= \Delta M[TS - E^G(\lambda_\Sigma)] \\ &= \Delta M[TS - E(\lambda_\Sigma)] - \Delta M K_G \\ &\equiv -\Delta\delta - \Delta M K_G \end{aligned} \quad (59)$$

Similarly,

$$\begin{aligned} E^L\pi_L(M + \Delta M, S) - E^L\pi_L(M, S) &= \Delta M[E^L(\lambda_\Sigma) - TS] \\ &= \Delta M[E(\lambda_\Sigma) - TS] + \Delta M K_L \\ &= \Delta\delta + \Delta M K_L \end{aligned} \quad (60)$$

The rest of the proof for $S = S^R$ will be presented under two conditions that cover all possibilities.

Condition 1: $M > P_G$

$$\begin{aligned} &CVaR^G(-\pi_G(M, S)) \\ &= CVaR^G(-P_G\lambda_\Sigma + COST - M[TS - \lambda_\Sigma]) \\ &= CVaR^G((M - P_G)\lambda_\Sigma + COST - TM\ S]) \\ &= (M - P_G)CVaR^G(\lambda_\Sigma) + COST - TM\ S \end{aligned} \quad (61)$$

Therefore, we have

$$\begin{aligned} &CVaR^G(-\pi_G(M + \Delta M, S)) - CVaR^G(-\pi_G(M, S)) \\ &= \Delta M\ CVaR^G(\lambda_\Sigma) - \Delta MTS \\ &= \Delta M[CVaR(\lambda_\Sigma) - TS] + \Delta M K_G \equiv \Delta\varepsilon_1 + \Delta M K_G \end{aligned} \quad (62)$$

Now,

$$\begin{aligned} &\Delta u_G \equiv \\ &u_G(E^G(\pi_G(M + \Delta M, S)), CVaR^G(-\pi_G(M + \Delta M, S))) \\ &-u_G(E^G(\pi_G(M, S)), CVaR^G(-\pi_G(M, S))) \\ &= -\Delta\delta - \Delta M K_G - A_G(\Delta\varepsilon_1 + \Delta M K_G) \end{aligned} \quad (63)$$

Next calculate the right derivative of u_G with respect to M:

$$\begin{aligned} \frac{du_G}{dM}\,|_+ &= \lim_{\Delta M \to 0^+} \frac{\Delta u_G}{\Delta M} \\ &= \lim_{\Delta M \to 0^+} \{\frac{\Delta M[TS - E(\lambda_\Sigma) - K_G]}{\Delta M} \\ &\quad + \frac{-A_G\Delta M[CVaR(\lambda_\Sigma) + K_G - TS]}{\Delta M}\} \\ &= TS - E(\lambda_\Sigma) - K_G - A_G\ CVaR(\lambda_\Sigma) - A_G K_G + TA_G S \end{aligned} \quad (64)$$

Integrate both sides of the above equation and rearrange the terms we have,

$$\begin{aligned} u_G =& [TS - E(\lambda_\Sigma) - A_G CVaR(\lambda_\Sigma) \\ &+ TA_G S - (1 + A_G)K_G]M + C_5 \\ =& C_6 M + C_5 \end{aligned} \quad (65)$$

From (65), M can be viewed as a function of u_G. We can thus calculate the derivative of u_L with respect to u_G as follows:

$$\begin{aligned} \frac{du_L}{du_G} &= \frac{du_L}{dM} \cdot \frac{dM}{du_G} \\ &= [\frac{dE^L(\pi_L(M, S))}{dM} - A_L \frac{dCVaR^L(-\pi_L(M, S))}{dM}]\frac{1}{C_6} \end{aligned} \quad (66)$$

Taking the derivative of each side of (66) with respect to u_G, we have

$$\begin{aligned} \frac{d^2u_L}{du_G^2} &= \frac{1}{C_6}[\frac{d^2E^L(\pi_L(M, S))}{dM^2}\frac{dM}{du_G} \\ &\quad - A_L\ \frac{d^2CVaRL(-\pi_L(M, S))}{dM^2}\frac{dM}{du_G}] \\ &= \frac{1}{C_6^2}[\frac{d^2E^L(\pi_L(M, S))}{dM^2} \\ &\quad - A_L\ \frac{d^2CVaR^L(-\pi_L(M, S))}{dM^2}] \end{aligned} \quad (67)$$

Taking the expectation and then the derivative with respect to M on each side of equation (7), we get

$$\frac{dE^L(\pi_L(M, S))}{dM} = E(\lambda_\Sigma) + K_L - TS \quad (68)$$

Then obviously we have

$$\frac{d^2E^L(\pi_L(M, S))}{dM} = 0 \quad (69)$$

Now equation (67) can be reduced to the following:

$$\frac{d^2u_L}{du_G^2} = -A_L\ \frac{1}{C_6^2}\frac{d^2CVaR^L(-\pi_L(M, S))}{dM^2} \quad (70)$$

As shown in Lemma 1, $CVaR^L(-\pi_L(M,S))$ is convex in M. Consequently,

$$\frac{d^2CVaR^L(-\pi_L(M,S))}{dM^2} \geq 0 \quad (71)$$

It follows that

$$\frac{d^2u_L}{du_G^2} = -A_L\frac{1}{C_6^2}\frac{d^2CVaR(-\pi_L(M,S))}{dM^2} \leq 0 \quad (72)$$

Therefore, given Condition 1, the curve of points (u_G, u_L) traced out in U space as M varies from P_G to M^U is concave.

Condition 2: $\boldsymbol{M \leq P_G}$

$$\begin{aligned}&CVaR^G(-\pi_G(M,S))\\ &= CVaR^G(-P_G\lambda_\Sigma + COST - M(TS - \lambda_\Sigma))\\ &= CVaR^G(-\lambda_\Sigma(P_G - M) + COST - TM\ S])\\ &= (P_G - M)CVaR^G(-\lambda_\Sigma) + COST - TM\ S \quad (73)\end{aligned}$$

Therefore, we have

$$\begin{aligned}&CVaR^G(-\pi_G(M+\Delta M,S)) - CVaR^G(-\pi_G(M,S))\\ &= -\Delta M\ CVaR^G(-\lambda_\Sigma) - \Delta MTS\\ &= -\Delta M[CVaR(-\lambda_\Sigma) + TS] + \Delta MK_G\\ &\equiv \Delta\varepsilon_2 + \Delta MK_G \quad (74)\end{aligned}$$

Now,

$$\begin{aligned}&\Delta u_G \equiv\\ &u_G(E^G(\pi_G(M+\Delta M,S)), CVaR^G(-\pi_G(M+\Delta M,S)))\\ &-u_G(E^G(\pi_G(M,S)), CVaR^G(-\pi_G(M,S)))\\ &= -\Delta\delta - \Delta MK_G - A_G(\Delta\varepsilon_2 + \Delta MK_G) \quad (75)\end{aligned}$$

Next calculate the left derivative of u_G with respect to M:

$$\begin{aligned}\frac{du_G}{dM}|_- &= \lim_{\Delta M\to 0^-}\frac{\Delta u_G}{\Delta M}\\ &= \lim_{\Delta M\to 0^-}\{\frac{\Delta M[TS - E(\lambda_\Sigma) - K_G]}{\Delta M}\\ &\quad + \frac{A_G\Delta M[CVaR(-\lambda_\Sigma - K_G) + TS]}{\Delta M}\}\\ &= TS - E(\lambda_\Sigma) + A_G\ CVaR(-\lambda_\Sigma) + TA_G\ S - (1+A_G)K_G \quad (76)\end{aligned}$$

Integrate both sides of the above equation and rearrange the terms we have,

$$\begin{aligned}u_G &= [TS - E(\lambda_\Sigma) + A_G\ CVaR(-\lambda_\Sigma)\\ &\quad + TA_G\ S - (1+A_G)K_G]M + C_7\\ &= C_8M + C_7 \quad (77)\end{aligned}$$

Similar to the derivation in Condition 1, the second derivative of u_L with respect to u_G can be calculated as

$$\frac{d^2u_L}{du_G^2} = -A_L\ \frac{1}{C_8^2}\frac{d^2CVaR^L(-\pi_L(M,S))}{dM^2} \quad (78)$$

Given the inequality relationship in (71), we have

$$\frac{d^2u_L}{du_G^2} = -A_L\frac{1}{C_8^2}\frac{d^2CVaR^L(-\pi_L(M,S))}{dM^2} \leq 0 \quad (79)$$

Therefore, given Condition 2, the curve of points (u_G, u_L) traced out in U space as M varies from M^R to P_G is once again concave.

In summary, when the strike price S is fixed at the lowest possible level, S^R, it has been shown that the contract amount interval from M^R to P_G and the contract amount interval from P_G to M^U each map under (6) and (10) into a concave curve in the utility possibility set U traced out by $u_L(u_G)$ in U as M varies from M^R to P_G and from P_G to M^U, respectively. It remains to show that the entire curve V_1 traced out by $u_L(u_G)$ in U as M varies from M^R to M^U is concave in U.

It follows easily from previous results above that $u_L(u_G)$ is a continuous function of u_G at the meeting point $M = P_G$. Therefore, to prove the concavity of V_1 in U, it suffices to show that the following inequality holds at the meeting point $M = P_G$:

$$\frac{du_L}{du_G}|_- \quad \geq \quad \frac{du_L}{du_G}|_+ \quad (80)$$

As will be established formally in Lemma 4 below, when the contract strike price S is fixed at S^R, the GenCo's utility level u_G decreases as the contract amount M increases, i.e., u_G and M move in opposite directions. Consequently, the left (right) derivative of u_L with respect u_G in (80) can be reexpressed in terms of right (left) derivatives with respect to M.

Specifically, making use of (66),

$$\begin{aligned}\frac{du_L}{du_G}|_- &= \frac{du_L}{dM}|_{M=P_G^+}\frac{dM}{du_G}|_{M=P_G^+}\\ &= [\frac{dE^L(\pi_L(M,S))}{dM}|_{M=P_G^+}\\ &\quad - A_L\frac{dCVaR^L(-\pi_L(M,S))}{dM}|_{M=P_G^+}]\frac{1}{C_6} \quad (81)\end{aligned}$$

Similarly, making use of (77)

$$\begin{aligned}\frac{du_L}{du_G}|_+ &= \frac{du_L}{dM}|_{M=P_G^-}\frac{dM}{du_G}|_{M=P_G^-}\\ &= [\frac{dE^L(\pi_L(M,S))}{dM}|_{M=P_G^-}\\ &\quad - A_L\frac{dCVaR^L(-\pi_L(M,S))}{dM}|_{M=P_G^-}]\frac{1}{C_8} \quad (82)\end{aligned}$$

Using the definition of C_6 and C_8 from (65) and (77), we have

$$\begin{aligned}C_8 - C_6 &= A_G[CVaR(-\lambda_\Sigma) + CVaR(\lambda_\Sigma)]\\ &= 2A_G[\frac{1}{2}CVaR(-\lambda_\Sigma) + (1-\frac{1}{2})CVaR(\lambda_\Sigma)]\\ &\geq CVaR(\frac{1}{2}(-\lambda_\Sigma) + \frac{1}{2}\lambda_\Sigma) = 0 \quad (83)\end{aligned}$$

Since, $S^R < S^{R^*} \leq \frac{E(\lambda_\Sigma) - A_GCVaR(-\lambda_\Sigma) + (1+A_G)K_G}{T(1+A_G)}$, we have $C_8 < 0$. Similarly, since $S^R < S^{R^*} \leq \frac{E(\lambda_\Sigma) + A_GCVaR(\lambda_\Sigma) + (1+A_G)K_G}{T(1+A_G)}$, we have $C_6 < 0$. Therefore, together with (83), we have $-\frac{1}{C_8} \geq -\frac{1}{C_6} > 0$.

From (68) we have,

$$\begin{aligned}&\frac{dE^L(\pi_L(M,S))}{dM}|_{M=P_G^+} = \frac{dE^L(\pi_L(M,S))}{dM}|_{M=P_G^-}\\ &= E(\lambda_\Sigma) + K_L - TS \quad (84)\end{aligned}$$

From the definition of left and right derivative we have,

$$\frac{dCVaR^L(-\pi_L(M,S))}{dM}\mid_{M=P_G^-} = \lim_{\Delta M\to 0^-} \frac{CVaR^L(-\pi_L(P_G+\Delta M,S))-CVaR^L(-\pi_L(P_G,S))}{\Delta M} = \lim_{\Delta M\to 0^+} \frac{CVaR^L(-\pi_L(P_G-\Delta M,S))-CVaR^L(-\pi_L(P_G,S))}{-\Delta M} = \lim_{\Delta M\to 0^+} \frac{CVaR^L(-\pi_L(P_G))-CVaR^L(-\pi_L(P_G-\Delta M,S,S))}{-\Delta M} \tag{85}$$

and,

$$\frac{dCVaR^L(-\pi_L(M,S))}{dM}\mid_{M=P_G^+} = \lim_{\Delta M\to 0^+} \frac{CVaR^L(-\pi_L(P_G+\Delta M,S))-CVaR^L(-\pi_L(P_G,S))}{\Delta M} \tag{86}$$

Using the convexity of CVaR, we have

$$\begin{aligned} &2\ [\frac{1}{2}CVaR^L(-\pi_L(P_G+\Delta M,S))+ \\ &\frac{1}{2}CVaR^L(-\pi_L(P_G-\Delta M,S))] \\ &\geq 2\ (CVaR^L(-\pi_L(\frac{1}{2}(P_G+\Delta M)+\frac{1}{2}(P_G-\Delta M),S))) \\ =&2\ CVaR^L(-\pi_L(P_G,S)) \end{aligned} \tag{87}$$

Moving the terms around in the above inequality, we have

$$\begin{aligned} &CVaR^L(-\pi_L(P_G+\Delta M,S))-CVaR^L(-\pi_L(P_G,S)) \\ \geq\ &CVaR^L(-\pi_L(P_G,S))-CVaR^L(-\pi_L(P_G-\Delta M,S)) \end{aligned} \tag{88}$$

Combining, (88), (85) and (86), we have

$$\frac{dCVaR^L(-\pi_L(M,S))}{dM}\mid_{M=P_G^+} \geq \frac{dCVaR^L(-\pi_L(M,S))}{dM}\mid_{M=P_G^-} \tag{89}$$

Therefore, from (81) and (82), and that if $S^R < S^{R^*}$, as M increase, u_L increases we have

$$0 < \frac{du_L}{dM}\mid_{M=P_G^+} \leq \frac{du_L}{dM}\mid_{M=P_G^-} \tag{90}$$

Combining (90) with (81), (82), and the previously derived condition $-\frac{1}{C_8} \geq -\frac{1}{C_6} > 0$, one obtains the desired condition (80).

The proof of the second statement in Lemma 3 pertaining to the case in which the strike price S is fixed at its highest possible level S^U is entirely analogous to the proof above for the first statement in Lemma 3.

QED

Before moving onto Lemma 4, additional derivations are provided with regard to Δu_L, which will be used in the following Lemma.

As is well known, $CVaR$ is convex in the following sense: For arbitrary (possibly dependent) random variables Y_1, Y_2 and λ with $0<\lambda<1$, $CVaR(\lambda Y_1+(1-\lambda)Y_2) \leq \lambda CVaR(Y_1)+(1-\lambda)CVaR(Y_2)$. Hence we have,

$$\begin{aligned} &CVaR^L(-\pi_L(M,S))-\Delta\varepsilon_2-\Delta M K_L \\ =&CVaR^L(-\pi_L^0-M(\lambda_\Sigma-TS) \\ &+\Delta M(CVaR(-\lambda_\Sigma-K_L)+TS)) \\ =&CVaR^L(-\pi_L^0-M(\lambda_\Sigma-TS) \\ &+CVaR^L(-\lambda_\Sigma\Delta M+TS\Delta M) \\ =&2\{\ \frac{1}{2}CVaR^L(-\pi_L^0-M(\lambda_\Sigma-TS)) \\ &+\frac{1}{2}CVaR^L(-\lambda_\Sigma\Delta M+TS\Delta M)\} \\ \geq&2[CVaR^L(\frac{1}{2}(-\pi_L^0-M(\lambda_\Sigma-TS)) \\ &+\frac{1}{2}(-\lambda_\Sigma\Delta M+TS\Delta M) \\ =&CVaR^L(-\pi_L^0-(M+\Delta M)(\lambda_\Sigma-TS)) \\ =&CVaR^L(-\pi_L(M+\Delta M,S)) \end{aligned} \tag{91}$$

Rearranging the terms in the above equation, we have

$$\begin{aligned} &CVaR^L(-\pi_L(M+\Delta M,S))-CVaR^L(-\pi_L(M,S)) \\ &+\Delta M K_L \equiv -\Delta\varepsilon_2'+\Delta M K_L \\ &\leq -\Delta\varepsilon_2=\Delta M[CVaR(-\lambda_\Sigma)+TS] \end{aligned} \tag{92}$$

Hence, Δu_L can be derived as,

$$\begin{aligned} &\Delta u_L \equiv \\ &u_L(E^L(\pi_L(M+\Delta M,S)),CVaR^L(-\pi_L(M+\Delta M,S))) \\ &-u_L(E^L(\pi_L(M,S)),CVaR^L(-\pi_L(M,S))) \\ &=\Delta\delta+\Delta M K_L+A_L\Delta\varepsilon_2' \end{aligned} \tag{93}$$

Similar to inequality (91) we have,

$$\begin{aligned} &CVaR^L(-\pi_L(M+\Delta M,S))+\Delta\varepsilon_1+\Delta M K_L \\ =&\ CVaR^L(-\pi_L(M+\Delta M,S)) \\ &+[CVaR(\lambda_\Sigma)+K_L-TS]\Delta M \\ =&\ 2\{\ \frac{1}{2}CVaR^L(-\pi_L(M+\Delta M,S)) \\ &+\frac{1}{2}CVaR^L(\Delta M(\lambda_\Sigma-TS))\} \\ \geq&2CVaR^L(\frac{1}{2}(-\pi_L^0-(M+\Delta M)(\lambda_\Sigma-TS) \\ &+\Delta M\lambda_\Sigma-TS\Delta M)) \\ =&\ CVaR^L(-\pi_L^0-M(\lambda_\Sigma-TS)) \\ =&\ CVaR^L(-\pi_L(M,S)) \end{aligned} \tag{94}$$

Rearranging the terms in the above equation, we have

$$\begin{aligned} &CVaR^L(-\pi_L(M+\Delta M,S))-CVaR^L(-\pi_L(M,S)) \\ &+\Delta M K_L \equiv -\Delta\varepsilon_1'+\Delta M K_L \\ &\geq -\Delta\varepsilon_1=-\Delta M[CVaR(\lambda_\Sigma)-TS] \end{aligned} \tag{95}$$

Hence, Δu_L can be derived as,

$$\begin{aligned}\Delta u_L \equiv\\ u_L(E^L(\pi_L(M+\Delta M,S)),CVaR^L(-\pi_L(M+\Delta M,S)))\\ -u_L(E^L(\pi_L(M,S)),CVaR^L(-\pi_L(M,S)))\\ =\Delta\delta+\Delta M K_L+A_L\Delta\varepsilon_1' \end{aligned} \tag{96}$$

Lemma 4: If S^R is less than S^{R^*} as defined in (27), then with the strike price S fixed at S^R, as the contract amount M increases, u_G decreases and u_L increases. If S^U is greater than S^{U^*} as defined in (28), then with the strike price S fixed at S^U, as the contract amount M increases, u_G increases and u_L decreases.

Proof of Lemma 4:

Part 1: Proof that if $\mathbf{S}^R$ is less than $\mathbf{S}^{R^*}$ as defined in (27), then with the strike price fixed at $\mathbf{S}^R$, as the contract amount M increases, $\mathbf{u}_G$ decreases and $\mathbf{u}_L$ increases.

As shown in equation (93), $\Delta u_L=\Delta\delta+\Delta M K_L+A_L\Delta\varepsilon_2'$. As given in equation (92), $\Delta\varepsilon_2'\geq\Delta\varepsilon_2+\Delta M K_L$. Hence, we have

$$\Delta u_L\geq\Delta\delta+\Delta M K_L+A_L(\Delta\varepsilon_2+\Delta M K_L) \tag{97}$$

After substituting $\Delta\delta$ and $\Delta\varepsilon_2$ into the above equation, we see that inequality (97) is equivalent to

$$\begin{aligned}\Delta u_L\geq&\Delta M[E(\lambda_\Sigma)-\\ &TS^R+K_L-A_L CVaR(-\lambda_\Sigma)-A_L TS^R+A_L K_L]\end{aligned} \tag{98}$$

Since, S^R is less than $\frac{E(\lambda_\Sigma)+(1+A_L)K_L-A_L CVaR(-\lambda_\Sigma)}{T(1+A_L)}$, and $A_L>-1$, it can be shown that the right hand side of the above inequality is greater than 0. Therefore, with the strike price fixed at S^R, as the contract amount M increases, u_L increases.

The rest of the proof will be presented under two conditions that cover all possibilities.

Condition 1: $\boldsymbol{M>P}_G$

As shown in equation (65), $u_G=C_6M+C_5$. Since $S^R<\frac{E(\lambda_\Sigma)+A_G CVaR(\lambda_\Sigma)+(1+A_G)K_G}{T(1+A_G)}$, and $A_G>-1$, $C_6<0$. Hence, given Condition 1, with the strike price fixed at S^R, when M increases, u_G decreases.

Condition 2: $\boldsymbol{M\leq P}_G$

As shown in equation (77), $u_G=C_8M+C_7$. Since $S^R<\frac{E(\lambda_\Sigma)-A_G CVaR(-\lambda_\Sigma)+(1+A_G)K_G}{T(1+A_G)}$, and $A_G>-1$, $C_8<0$. Hence, given Condition 2, with the strike price fixed at S^R, when M increases, u_G decreases.

Part 2: Proof that if S^U is greater than S^{U^*} as defined in (28), then with the strike price S fixed at S^U, as the contract amount M increases, u_G increases and u_L decreases.

As shown in equation (96), $\Delta u_L=\Delta\delta+\Delta M K_L+A_L\Delta\varepsilon_1'$. As given in equation (95), $\Delta\varepsilon_1'\leq\Delta\varepsilon_1+\Delta M K_L$. Hence, we have

$$\Delta u_L\leq\Delta\delta+\Delta M K_L+A_L(\Delta\varepsilon_1+\Delta M K_L) \tag{99}$$

After substituting $\Delta\delta$ and $\Delta\varepsilon_1$ into the above equation, we see that inequality (99) is equivalent to

$$\begin{aligned}\Delta u_L\leq&\Delta M[E(\lambda_\Sigma)-\\ &TS^U+K_L+A_L CVaR(\lambda_\Sigma)-A_L TS^U+A_L K_L]\end{aligned} \tag{100}$$

Since, S^U is greater than $\frac{E(\lambda_\Sigma)+(1+A_L)K_L+A_L CVaR(\lambda_\Sigma)}{T(1+A_L)}$, and $A_L>-1$, it can be shown that the right hand side of the above inequality is smaller than 0. Therefore, with the strike price fixed at S^U, as the contract amount M increases, u_L decreases.

The rest of the proof will be presented under two conditions that cover all possibilities.

Condition 1: $\boldsymbol{M>P}_G$

As shown in equation (65), $u_G=C_6M+C_5$. Since $S^U>\frac{E(\lambda_\Sigma)+A_G CVaR(\lambda_\Sigma)+(1+A_G)K_G}{T(1+A_G)}$, and $A_G>-1$, $C_6>0$. Hence, given Condition 1, with the strike price fixed at S^U, when M increases, u_G increases.

Condition 2: $\boldsymbol{M\leq P}_G$

As shown in equation (77), $u_G=C_8M+C_7$. Since $S^R>\frac{E(\lambda_\Sigma)-A_G CVaR(-\lambda_\Sigma)+(1+A_G)K_G}{T(1+A_G)}$, and $A_G>-1$, $C_8>0$. Hence, given Condition 2, with the strike price fixed at S^U, when M increases, u_G increases.

QED

Lemma 5: Consider the following two conditions:

$$\frac{du_L}{du_G}\mid_{M^R,S^R} \quad < \quad -\frac{1+A_L}{1+A_G} \tag{101}$$

$$\frac{du_L}{du_G}\mid_{M^U,S^R} \quad > \quad -\frac{1+A_L}{1+A_G} \tag{102}$$

Inequality (101) is equivalent to inequality (30), and inequality (102) is equivalent to inequality (29).

Proof of Lemma 5:

Part 1: Proof that inequality (101) is equivalent to inequality (30)

Inequality (101) implies $M\leq P_G$. Similar to equation (106), we now have

$$\frac{du_L}{du_G}=[E(\lambda_\Sigma)+K_L-TS-A_L\frac{dCVaR^L(-\pi_L(M,S))}{dM}]\frac{1}{C_8} \tag{103}$$

After substituting C_8 into the above equation, we see that inequality (101) is equivalent to

$$-\frac{E(\lambda_\Sigma)+K_L-TS-A_L\frac{dCVaR^L(-\pi_L(M^R,S^R))}{dM}}{E(\lambda_\Sigma)-TS-A_G CVaR(-\lambda_\Sigma)-TA_G S+(1+A_G)K_G} < -\frac{1+A_L}{1+A_G} \tag{104}$$

Rearranging the terms in the above equation, we have

$$\frac{du_L}{du_G}\mid_{M^R,S^R} < -\frac{1+A_L}{1+A_G} \Leftrightarrow$$
$$\frac{dCVaR^L(-\pi_L(M^R,S^R))}{dM} < \frac{A_G-A_L}{A_L(1+A_G)}E(\lambda_\Sigma)$$
$$+\frac{A_G(1+A_L)}{A_L(1+A_G)}CVaR(-\lambda_\Sigma)+\frac{1}{A_L}K_L-\frac{1+A_L}{A_L}K_G+TS \tag{105}$$

Part 2: Proof that inequality (102) is equivalent to inequality (29)

Inequality (102) implies $M > P_G$. Substituting equation (68) into (66), we have

$$\frac{du_L}{du_G} = [E(\lambda_\Sigma)+K_L-TS-A_L\frac{dCVaR^L(-\pi_L(M,S))}{dM}]\frac{1}{C_6} \tag{106}$$

After substituting C_6 into the above equation, we see that inequality (102) is equivalent to

$$-\frac{E(\lambda_\Sigma)+K_L-TS-A_L\frac{dCVaR^L(-\pi_L(M^U,S^R))}{dM}}{E(\lambda_\Sigma)-TS+A_GCVaR(\lambda_\Sigma)-TA_GS+(1+A_G)K_G}$$
$$> -\frac{1+A_L}{1+A_G} \tag{107}$$

Rearranging the terms in the above equation, we have

$$\frac{du_L}{du_G}\mid_{M^U,S^R} > -\frac{1+A_L}{1+A_G} \Leftrightarrow$$
$$\frac{dCVaR^L(-\pi_L(M^U,S^R))}{dM} > \frac{A_G-A_L}{A_L(1+A_G)}E(\lambda_\Sigma)$$
$$-\frac{A_G(1+A_L)}{A_L(1+A_G)}CVaR(\lambda_\Sigma)+\frac{1}{A_L}K_L-\frac{1+A_L}{A_L}K_G+TS \tag{108}$$

QED

Theorem 2: Suppose the stated restrictions on the CFD bargaining problem hold for G and L. Suppose, also, that the lowest strike price S^R is less than S^{R^*} as defined in (27), the highest strike price S^U is greater than S^{U^*} as defined in (28), and $0 \leq M^R < P_G$. Then the Nash barter set B for this problem is a non-empty, compact, convex subset of $\Re^2$, as follows:

Case 1. *The barter set B is a compact right triangle when conditions (29) and (30) both hold, cf. Fig. 2.*

Case 2. *The barter set B reduces to the no-contract threat point when inequality (30) does not hold, cf. Fig. 3.*

Case 3. *The barter set B is a compact right triangle when (29) does not hold but (30) holds, cf. Fig. 4.*

Proof of Theorem 2:

Before considering the shape of the utility possibility set U, first consider the following two curves. The first curve V_1 is the locus of points (u_G, u_L) traced out in U as M varies from M^R to M^U, given a strike price $S = S^R$. The second curve V_2 is the locus of points (u_G, u_L) traced out in U as M varies from M^R to M^U, given a strike price $S = S^U$.

As seen in Lemma 3, the curves V_1 and V_2 are concave in U. Moreover, as proved in Lemma 4, with the strike price fixed at S^R, as M increases, u_G decreases and u_L increases. Similarly, if the strike price is fixed at S^U, as M increases, u_G increases and u_L decreases. Therefore, at each point along V_1 and V_2 the slope is negative. Note, as proved in Lemma 2, given any contract amount $M \in [M^R, M^U]$, varying the strike price S from S^R to S^U maps under (6) and (10) into a straight line in U with slope $-[1+A_L]/[1+A_G]$. Hence, connecting the points on V_1 and V_2 that have the same contract amount M, we have straight lines with a slope of $-[1+A_L]/[1+A_G]$. In addition, every single point on these straight lines belongs to U.

The proof of Theorem 2 will be divided into three parts corresponding to the three possible cases in the statement of the theorem.

Case 1:

When following the proof below, please refer to Fig. 10. As shown in Lemma 5, when conditions (101) and (102) both hold, the slope of V_1 at the threat point is smaller than $-\frac{1+A_L}{1+A_G}$; and, when $M = M^U$, the slope of V_1 is greater than $-\frac{1+A_L}{1+A_G}$. Therefore, since V_1 is concave, the slope of V_1 must steadily increase from below $-\frac{1+A_L}{1+A_G}$ to over $-\frac{1+A_L}{1+A_G}$ as M increases from 0 to M^U, and u_G correspondingly decreases.

As indicated in Lemma 3, V_2 is also concave. Moreover, the slope of V_2 at the threat point is larger than $-\frac{1+A_L}{1+A_G}$. This statement can be proved by contradiction. Assume that, when the slope of V_1 at the threat point is smaller than $-\frac{1+A_L}{1+A_G}$, the slope of V_2 at the threat point is also smaller than $-\frac{1+A_L}{1+A_G}$. This situation is plotted in Fig. 9. Pick a point Z on V_1 above the straight line with a slope of $-\frac{1+A_L}{1+A_G}$ which passes the threat point. By construction, Z takes the form $Z = (u_L\mid_{(M',S^R)}, u_G\mid_{(M',S^R)})$. According to Lemma 2, the point $(u_L\mid_{(M',S^U)}, u_G\mid_{(M',S^U)})$ on V_2 together with Z must be on a straight line with a slope of $-\frac{1+A_L}{1+A_G}$. Therefore, the point $(u_L\mid_{(M',S^U)}, u_G\mid_{(M',S^U)})$ on V_2 must be above the straight line with a slope of $-\frac{1+A_L}{1+A_G}$ that passes through the threat point. However, since the initial slope of V_2 is smaller than $-\frac{1+A_L}{1+A_G}$, and V_2 is concave, no point on V_2 is above this straight line. This contradicts Lemma 2, which completes the proof.

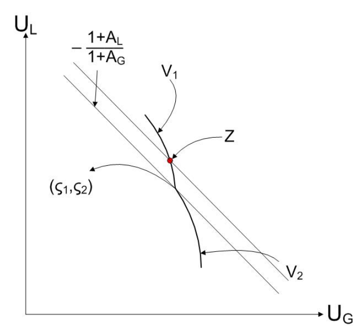

Fig. 9. Supporting graph for proving that the slope of V_2 at the threat point is larger than $-\frac{1+A_L}{1+A_G}$ when the slope of V_1 at the threat point is smaller than $-\frac{1+A_L}{1+A_G}$.

As proved in Lemma 2, all the points that belong to the utility possibility set U are on parallel lines with one end on

V_1 and with a slope of $-[1+A_L]/[1+A_G]$. Hence, the typical utility possibility set U for Case 1 is as shown in Fig. 10.

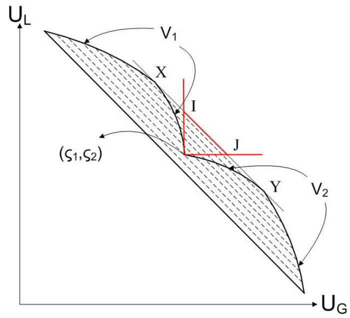

Fig. 10. Illustration of the Case 1 utility possibility set U and barter set B for GenCo G and LSE L. The barter set is a right triangle.

Since the slope of V_1 gradually increases from below $-\frac{1+A_L}{1+A_G}$ to above $-\frac{1+A_L}{1+A_G}$, there exists a contract amount M^* such that $\frac{du_L}{du_G}\mid_{-}\geq -\frac{1+A_L}{1+A_G}$ and $\frac{du_L}{du_G}\mid_{+}\leq -\frac{1+A_L}{1+A_G}$ at $M=M^*$ and $S=S^R$. Using the results proved in Lemma 2, $\frac{du_L}{du_G}\mid_{-}\geq -\frac{1+A_L}{1+A_G}$ and $\frac{du_L}{du_G}\mid_{+}\leq -\frac{1+A_L}{1+A_G}$ will then also hold at $M=M^*$ and $S=S^U$.

Define $X=(u_L\mid_{(M^*,S^R)},u_G\mid_{(M^*,S^R)})$ and $Y=(u_L\mid_{(M^*,S^U)},u_G\mid_{(M^*,S^U)})$. Also define $C^1=[1+A_L]u_G\mid_{(M^*,S^R)}+[1+A_G]u_L\mid_{(M^*,S^R)}$. Since V_1 is concave, it follows from the initial slope and end slope that all the points (u_G,u_L) on V_1 satisfy $[1+A_L]u_G+[1+A_G]u_L\leq C^1$. As proved in Lemma 2, all the points that belongs to U are on parallel lines with one end on V_1 and with a slope of $-[1+A_L]/[1+A_G]$. Hence, all the points in U except the points on the straight line between X and Y satisfy $[1+A_L]u_G+[1+A_G]u_L\leq C^1$.

Now draw a horizonal line and a vertical line from the threat point. As shown in Fig. 10, let I denote the point where the vertical line intersects with the straight line between X and Y, and let J denote the point where the horizonal line intersects with the straight line between X and Y. By definition, the right triangle $I\zeta J$ constitutes the Case-1 barter set, which is clearly non-empty, compact, and convex.

Case 2:

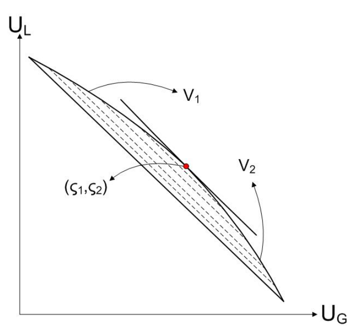

Fig. 11. Illustration of the Case 2 utility possibility set U and barter set B for GenCo G and LSE L. The barter set reduces to the non-contract threat point.

When following the proof below, please refer to Fig. 11. Define $C^2=[1+A_L]\zeta_1+[1+A_G]\zeta_2$. As shown in Lemma 5, when condition (101) fails to hold, $\frac{du_L}{du_G}\mid_{(M^R,S^R)}\geq -\frac{1+A_L}{1+A_G}$.

Because V1 is concave, all the points (u_G,u_L) on V_1 satisfy $[1+A_L]u_G+[1+A_G]u_L\leq C^2$. As proved in Lemma 2, all the points that belong to the utility possibility set U are on parallel lines with one end on V_1 and with a slope of $-[1+A_L]/[1+A_G]$. Hence, all the points in the utility possibility set U satisfy $[1+A_L]u_G+[1+A_G]u_L\leq C^2$.

Therefore, the threat point is the only point in the utility possibility set U that satisfies both $u_G\geq\zeta_1$ and $u_L\geq\zeta_2$. This can be proved by contradiction. Suppose there is another point (u'_G,u'_L) in U apart from the threat point that satisfies both $u'_G\geq\zeta_1$ and $u'_L\geq\zeta_2$. Then, $[1+A_L]u'_G+[1+A_G]u'_L>C^2$. This contradicts our previous conclusion that all points in U satisfy $[1+A_L]u_G+[1+A_G]u_L\leq C^2$. It follows that the Case-2 barter set reduces to the threat point. The typical shapes of the utility possibility set U and the barter set B for Case 2 are thus as shown in figure 11.

Case 3:

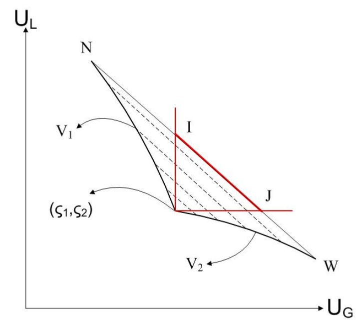

Fig. 12. Illustration of the Case 3 utility possibility set U and barter set B for GenCo G and LSE L. The barter set is a right triangle.

When following the proof below, please refer to Fig. 12. Let $N=(u_G\mid_{(M^U,S^R)},u_L\mid_{(M^U,S^R)})$ denote the endpoint of the curve V_1. As shown in Lemma 5, when condition (101) holds but condition (102) fails to hold, the slope of V_1 at the threat point is smaller than $-\frac{1+A_L}{1+A_G}$ and the slope of V_1 at N is also smaller than $-\frac{1+A_L}{1+A_G}$.

Again, as shown in Lemma 2, all the points that belongs to U are on parallel lines with one end on V_1 and a slope of $-[1+A_L]/[1+A_G]$. Since V_1 is concave, the typical Case-3 shape of U is as shown in Fig. 12.

Let $W=(u_G\mid_{(M^U,S^U)},u_L\mid_{(M^U,S^U)})$ denote the point on curve V_2 corresponding to (M^U,S^U). Let $C^3=[1+A_L]u_G\mid_{(M^U,S^R)}+[1+A_G]u_L\mid_{(M^U,S^R)}$. Given the above findings for the endpoints of V_1, together with the concavity of V_1, it follows that all the points (u_G,u_L) on V_1 satisfy $[1+A_L]u_G+[1+A_G]u_L\leq C^3$. Again, as proved in Lemma 2, all the points that belong to U lie on parallel lines with one end on V_1 and with a slope of $-[1+A_L]/[1+A_G]$. Hence, all the points in U satisfy $[1+A_L]u_G+[1+A_G]u_L\leq C^3$.

Now draw a horizonal line and a vertical line from the threat point. Let the point where the vertical line intersects with the straight line between N and W be denoted by I, and let the point where the horizonal line intersects with the straight line between N and W be denoted by J. As shown in Fig. 12, the right triangle $I\zeta J$ constitutes the Case-3 barter set B by definition. Clearly B is non-empty, compact, and convex.

QED

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