DISCOVERING ARTIFICIAL ECONOMICS

How Agents Learn and Economies Evolve

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Coevolving Markets

"One with another, soul with soul
They kindle fire with fire."
ARTHUR PIGOU

{A} Are Stock Markets Efficient?{/A}

We're ready to return to that baffling puzzle, mentioned briefly in the opening chapter. Why is it that academics, by and large, see markets quite differently from the way that actual traders see them? By now you'll have a pretty good idea of how classical economic theorists see the financial world. If you thought of equilibria and deductive rationality, you'd be right. There's simply no place for those inductive explorers we met earlier. It's sheepish, risk-aversion all the way. And for good reasons. If all investors possess identical, perfect foresight, then markets should behave efficiently. All the available information gets discounted into current prices. If the sole driving force behind price changes for any stock or commodity is assumed to be new information, then we can assume that traders are able to process this information so efficiently that prices will adjust instantaneously to the news. Because the news itself is assumed to appear randomly, so the argument goes, prices must move in a random fashion as well.

As mentioned earlier, the credit for this idea goes to the French mathematician, Louis Bachelier. In his doctoral dissertation addressing price fluctations on the Paris bond market, the seeds of the efficient markets hypothesis were sown.[1] He concluded that the current price of a commodity was also an unbiased estimate of its future price. Bachelier's viewpoint is a long-standing equilibrium theory. Price changes become unpredictable and technical trading using price charts is regarded as a waste of time.

[1] See Bachelier (1900).

Today economists make use of *martingales*, a sort of random process that Bachelier introduced in passing. In fact, his notion of efficiency has proved to be extremely influential. The vast majority of academic economists accept that this is the way real markets work.

That's the explanation in theory. But what about in practice. What's meant by an *efficient* market in practical terms? The stock exchange provides one answer. Common stocks are traded on well-organized exchanges like the New York Stock Exchange, or in dealer markets called over-the-counter markets. This allows a rapid execution of buy and sell orders. The price response to any change in demand caused by new information can be almost instantaneous. Such stock markets are also competitive due to the large number of participating individuals, institutions, corporations, and others. Competitive forces also tend to cause prices to reflect available information quickly. A market that quickly and accurately reflects available information is thought of as an efficient market. Those that adjust more rapidly and accurately are considered more efficient.

Are markets efficient? Yes, according to many economists. Like rationality, however, this efficiency is simply *assumed*. There's no existence proof. It's virtually impossible to test for market efficiency since the "correct" prices can't be observed. To get over this hurdle, most tests examine the ability of information-based trading strategies to make above-normal returns.[2] But the results of such tests don't really prove anything, least of all whether markets are efficient. Therein lies the basic dilemma. Given that stock markets have certain characteristics that are thought to make them more efficient than other markets, they seem like a reasonable place to start our investigation in earnest. Let's take a brief look at what the efficient market hypothesis posits in this setting.

Eugene Fama coined the term "efficient market" and suggested three levels of efficiency.[3] Studies of *weak-form* market efficiency began with Bachelier, and concluded that stock prices follow a random walk. The *random walk* hypothesis means that, at a

[2] Traditionally, most tests of market efficiency have been based on empirical derivatives of the *capital asset pricing model* (CAPM).

[3] This summary of the efficient market hypothesis has been drawn largely from Fama (1970), and Dyckman and Morse (1986). Fama's article contains a comprehensive discussion of the different categories of market efficiency.

given point in time, the size and direction of the next price change is random with respect to the knowledge available at that point in time. This implies that charting and all other forms of technical analysis practiced by various investors, amateur and professional alike, are doomed to fail. Market efficiency can also take a *semistrong-form* or *strong-form*, but these two classes needn't concern us here.[4] It'll be enough to take a critical look at weak-form efficiency. If this form's credibility tends to unravel, then so will the others.

Market efficiency also seems to have its roots in the idea of *intrinsic value*. Although the value of most goods is acknowledged to be a function of consumer beliefs, preferences and endowments, securities have often been treated as having a value independent of these consumer charcteristics. Their value is based on the characteristics of the firm behind the security. This is a supply-side approach. The price of any security, however, depends not only on the characteristics of the firm or commodity involved, but also on the demand for the security. In other words, it depends on the characteristics of the *investor*.

To date, the most commonly used model to relate investors' current price expectations with future price distributions is one that we've met earlier: the *rational expectations* equilibrium model. A fully-revealing, rational expectations equilibrium occurs when prices reveal all the information held by individual investors. In other words, when price expectations are realized in a future period. But *whose* expectations? If investors possess *homogeneous* beliefs, the choice of whose expectations to use is greatly simplified. As Rubinstein states: "In a perfect and competitive economy composed of rational individuals with homogeneous beliefs about future prices, by any meaningful definition present security prices must fully reflect all available infrmation about future prices."[5]

Now the real problem of *defining* market efficiency becomes clear. Overlooking the fact that investors might not have access to the same information, what happens if

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[4] The semistrong-form of market efficiency implies that markets adjust rapidly and in an unbiased manner to public information. Under the strong-form of market efficiency, both public and private information are quickly impounded in the security price. Strong-form market efficiency implies semistrong-form market efficiency, and semistrong-form market efficiency in turn implies weak-form market efficiency.

[5] The quote is from Rubinstein (1975), page 812. It's also worth noting that prices in a market in which the participants have bounded rationality, and have access to different information sets, may still converge to the level predicted by rational expectations theory. For an example, see Sargent (1993).

these investors happen to be different psychologically? From earlier chapters, we know that individuals possess different expectations in everday situations. We know, for instance, that sheep and explorers coexist in traffic. When it comes to choosing alternative strategies, some drivers are risk-averse while others are willing to experiment. Similar variability exists among the strategies of fishermen or technological imitators and innovators. Some search in familiar zones, others are willing to risk unchartered waters. The truth of the matter is that any population possesses a rich spectrum of different beliefs, hypotheses and expectations. We need look no further than the electoral boxes for proof of that! Why should it be any different in stock markets?

The basic problem with the efficient market hypothesis, and the theory of random walks, is that they concentrate exclusively on the security itself and the information relating to it. The demand side of the market is trivialized. All the idiosyncrasies of human nature are ignored. Furthermore, all these homogeneous investors are locked up in a static world. Expectations aren't allowed to vary. Yet real marketplaces are incredibly *dynamic* and *interactive*. Just ask any trader on the floor or in the pits. Different investors attempt to maximize their returns over different time horizons. Each has a different personality. What each investor does individually affects what the market does collectively and, in turn, what the market does collectively affects each investor individually. There are plenty of positive feedback loops at work. In other words, markets are coevolutionary in character and learning is the engine of change.

Perhaps this explains why the newspapers and financial tabloids are full of graphs and advertisements by self-professed "chartists" claiming insights into future price movements? Could it be that these traders "see" something in those market gyrations that the academics have missed? Perhaps they feel that the geometry of price histories is important. Maybe it's just seasonal variations? Or does the position of the stars matter most? In any event, traders and academics view markets differently. Many traders believe that technical trading can be consistently profitable. They also believe that factors such as market "psychology" and "herd" effects do affect price changes.

Which group should hold sway? Markets do appear to be reasonably efficient in a limited sense of the word. Stock prices seem to reflect available information, despite trader's different information sets, beliefs, attitude to risk, and trading horizons. As stressed in Chapter 1, however, statistical tests have shown that technical trading can produce consistent profits over time.[6] The widespread use of technical trading rules continues to be a puzzle in academic finance. Yet other studies have also shown that trading volume and price movements are more volatile in real markets than the standard theory predicts.[7] Temporary bubbles and crashes, like the major crash in 1987, are well beyond the scope of rational adjustments to market news. Although some economists have looked for signs that prices are being generated by chaotic mechanisms, we'll not dwell on these tests here. It suffices to say that the evidence implicating chaos as a factor influencing price fluctuations in financial markets is mixed.[8] To learn more about how the market evolves over time, let's take a closer look at a favourite tool of the technical traders: the patterns formed by price gyrations.

{A}Pattern Recognizers{/A}

Many technical traders believe that patterns of price movements in the marketplace tend to repeat themselves as human nature weaves its collective spell. There's plenty of evidence to support their view. Records of historical price changes show countless configurations that seem too striking to be attributable to mere chance. One of the

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[6] In addition to the work of William Brock (cited in Chapter 1), de la Maza and Yuret (1995), have conducted simulation experiments which show that in simple markets with heterogeneous investors, opportunities exist for making consistent profits over extended periods of time. Some of the results stemming from de la Maza and Yuret's research were used to manage a small options account since September 1993. When this pair of authors participated in the options division of the 1993 U.S. Investing Championship using this account, they finished fifth with a 43.9% return; see de la Maza and Yuret (1995), page 330.

[7] Evidence of excess volatility in financial markets, beyond what's justified by the fundamentals, can be found in dozens of publications in the finance and economics journals. Three examples are LeRoy and Porter (1981), Shiller (1981), and de Bondt and Thaler (1985). These levels of volatility are inconsistent with the efficient markets model.

[8] For interesting discussions of the evidence for and against the presence of chaos in economic and financial data, see Brock, Hsieh and LeBaron (1991) or Benhabib (1992).

earliest observers to find repetition in the market's price gyrations was Charles Dow. Dow started the Dow Jones Chemical Company and was the founding editor of the Wall Street Journal until his death in 1902. During the last few years of his life, he wrote a few editorials dealing with stock price movements which are the only personal record we have of his recognition of recurring patterns in price histories.[9] His theory is arguably the oldest and most famous technical trading approach in existence, there being many versions of it still alive today.

Dow realized that the market did not resemble a balloon bobbing about aimlessly in the wind. Rather than bouncing along in a random fashion, he surmised that it moved through discernable sequences. As Dow stated: "The market is always considered as having three movements, all going at the same time. The first is the narrow movement from day to day. The second is the short swing, running from two weeks to a month or more; the third is the main movement, covering at least four years in duration."[10] Dow theory practitioners refer to these three components as daily fluctuations, secondary movements, and primary trends. They're really *time horizons*, extending over the short, medium, and long term. The longer horizons, or primary trends, are commonly called *bull* or *bear* markets. To search for patterns in these trends over time, technical analysts use various charts - such as line, bar, and point-and-figure charts. Some of the price patterns formed by market action, and recognized by technical traders, are shown in Figure 7.1.

[Fig. 7.1 near here]

Two of Dow's less-discussed principles are of special interest. In its primary uptrend, he argued that the market was characterized by three upward swings. The first swing he attributed to a rebound from the "over-pessimism" of the preceding primary downswing; the second upward swing geared into the improving business and earnings

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[9] William Hamilton, who served under Dow, carried on the study and interpretation of Dow's theory through the editorial pages of *The Wall Street Journal* until 1929. He made up for Dow's paucity of written work by publishing a book on the topic in 1922, which was applauded in England by his election as a Fellow of the Royal Statistical Society in the following year. For an introduction to the Dow-Hamilton theories, see Rhea (1932).

picture; the third and last swing was an overdiscounting of value. Dow's second principle was geometrical. This asserted that, at some point in every market swing, whether up or down, there would be a reverse movement (or reaction) cancelling 40% to 60% of that swing. It's hard to know if he thought of such geometrical regularities as being shaped by the human factor, but such repetition could hardly be judged as purely accidental.

More than 80 years after Dow's death, the Options Division of an annual tournament conducted by the Financial Traders Association in the USA was won by a former drummer in a rock band, one Robert Prechter. Prechter, who also holds a psychology degree from Yale University, managed to increase the value of his portfolio by a whopping 444.4 percent in the allotted four months![11] By 1989, the Financial News Network had named him "Guru of the Decade." One could be forgiven for thinking that Prechter's approach was novel. But the truth is that it was based on a more sophisticated form of Dow's geometrical principles. Let's take a quick look at this intriguing pattern recognizer, known as the *Elliott wave principle*.[12]

Prechter's mentor, Ralph N. Elliott, was a Los Angeles accountant and an expert on cafeteria management. He was also a keen student of all the gyrations in the Dow Jones averages. Having lost his job and part of his savings on Wall Street in 1929, he had plenty of time on his hands to search for a better way to play the markets. Like Dow, Elliott discerned repetitive patterns, but his discoveries went beyond Dow theory in comprehensiveness and exactitude. What Dow outlined with broad strokes of his brush, Elliott painted in careful detail. The *wave principle* is Elliott's discovery that investor behaviour trends and reverses collectively in recognizable patterns. The basic pattern is shown below.

[Fig. 7.2 near here]

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[10] See *The Wall Street Journal*, 19 December 1900.

[11] The second-highest gain in this part of the tournament was only 84 percent, and over eighty percent of the competitors actually *lost* money.

[12] See Frost and Prechter (1990).

Market action unfolds according to a basic rhythm of five waves up and three waves down, to form a complete cycle of eight waves. Note that in its primary uptrend, there are *three* rising waves or upswings - just as Dow observed. What Dow called primary trend upswings or downswings, Elliott called *impulse* waves. In Elliott's jargon, waves numbered 2 and 4 are *corrective* waves. A complete Elliott cycle consists of eight waves: a primary uptrend of five waves (1-2-3-4-5) being corrected by a secondary downtrend of three waves (6-7-8).

Following completion of this cycle, a second cycle of similar form begins. Once again, it's five upward waves and three downward waves. A third then follows, but this time it's only five waves up. This completes a major five-wave-up movement over a longer time horizon. Then follows a major three-wave-down movement, correcting the preceding major five-wave-up movement. Each of these "phases" is actually a wave in its own right, but is one degree larger (or longer) than the waves of which it's composed.[13] The complete 34-wave pattern is shown in the lower part of Figure 7.3.

[Fig. 7.3 near here]

Note how closely the geometrical form of this *major* wave pattern resembles that of its component *minor* wave pattern. According to Elliott, two waves of a particular degree can be broken into eight waves of the next lower degree; then those eight waves can be subdivided in exactly the same manner to reveal thirty-four waves of the *next* lower degree. The wave principle recognizes that waves of any degree fulfill a dual role. They can be subdivided into waves of lesser degree, but they're also components of waves of higher degree. For example, the corrective pattern shown in the major wave illustrated above subdivides into a 5-3-5 pattern. If we could place this corrective pattern under a "microscope," it would also reveal a 5-3-5 pattern. Waves (1) and (2) in the 34-wave movement shown in Figure 7.3 take on the same form as waves {1} and {2}, confirming

[13] Our short discussion of the Wave Principle is taken from Frost and Prechter (1990). For a comprehensive introduction to this principle, see Chapters 1-3 of that book.

the phenomenon of constant form within ever-changing scale. This suggests that Elliott waves at different levels may be self-similar.

Self-similarity, or invariance against changes in scale or size, is a familiar attribute of many natural phenomena in the world around us. But who would have thought it might apply to financial markets. Because more than one scale factor is involved, strictly speaking these markets don't exhibit self-similarity. Instead they're said to be *self-affine*, which turns out to be a close relative of self-similarity. Both these concepts will be explained in the next section.

{A}Scaling the Market's Peaks{/A}

What does self-similarity of form really mean? Underlying the wave principle is the idea that financial markets exhibit a very special kind of symmetry: nature's symmetry. In effect, price gyrations display fractal geometry. The science of fractals is a relatively new one, which is gradually commanding the recognition that it deserves. Much of nature conforms to specific patterns and relationships, some of which are identical to those that Elliott recognized and described in the stock market. But there's a practical difficulty with Elliott's wave principle. It's virtually impossible to apply the technique succesfully in an objective and repetitive manner.[14] In other words, it fails to provide a "descriptive phenomenology" that is organized tightly enough to ensure a degree of understanding and consistent application. Fortunately, the science of fractals features the statistical notion of "scaling", which helps to restore this objectivity.

Scaling is a morphological term. Starting from the rules that govern the variability of price on one particular timescale, higher-frequency and lower-frequency variation is found to be governed by the same rules, but acting faster or more slowly.[15] The founder of the fractal concept, Benoit Mandelbrot, suggests that a wealth of features beloved by chartists (and Elliott wave theorists) need not be judged subjectively, but may

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[14] Elliott is quick to point out that considerable experience is required to interpret his principle correctly. No interpretation is valid unless made by an expert such as himself; see Elliott (1946).

[15] See Mandelbrot (1997), page 2.

follow inevitably from suitable forms of random variability. In other words, we shouldn't be surprised by the fact that the market seems to trace out characteristic patterns at all levels - such as charts of similar general shape on different timescales. Even major market corrections, like the "October crashes" of 1929 and 1987, may simply be larger versions of what's happening all the time on smaller timescales.

Mandelbrot's scaling principle is more objective than Elliott's wave principle. His key idea is that much in economics is *self-affine*. This almost visual notion allows us to test the idea that "all charts look similar." Consider what happens if you inspect a financial chart from up close, then far away. Often you can "see" a pattern, like the basic Elliott wave pattern of five waves up then three waves down. Many smaller and larger patterns often look similar. Look what happens if we take a complete pattern, then diverse pieces of it, and resize each to the same horizontal format. Two such renormalized charts are never perfectly identical, of course, but they're often remarkably similar. Resizing in this way is known technically as "renormalizing by performing an affinity." This motivated Mandelbrot to coin the term "self-affinity."[16]

Self-affinity designates a property that's closely related to self-similarity, since it also involves a transformation from a whole to its parts. But it's not a similarity that reduces both coordinates in the same ratio. Instead it's an affinity which reduces time in one ratio and the other coordinate in a different but related ratio. Thus if two price charts, or two parts of one chart, happen to look very much alike, technically speaking they could be *self-affine* - statistically invariant by dilation or reduction. Two sequences of price gyrations which appear to be self-affine are shown in Figure 7.4. Far from being a rarity, such resemblances are rife throughout all financial markets. These fascinating discoveries have important implications for much of economics and finance. To date, they remain unexplained.

[Fig. 7.4 near here]

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[16] Scaling and renormalization are terms which originated in physics rather than economics. As noted earlier, scaling can also be *self-similar*.

Mandelbrot posed a key question: Is the mathematical notion of chance powerful enough to bring about the strong degree of irregularity and variability in financial charts as well as in coastlines? The answer to that question came as a surprise. Not only is it powerful enough, but there's a tendency to underestimate the ability of chance to generate ordered structures that have not been anticipated in advance.[17] Chance remains important over a wide range of levels, including the macroscopic one. Several decades after Elliott, Mandelbrot's pioneering studies of fractals have confirmed that nature and markets abound with this special kind of symmetry.

In Chapter 1, we mentioned that Mandelbrot collected daily and monthly price data for various commodities. Logarithmic plots of the resulting size classes of price variations revealed that the distribution of price variations did not change over fifty year periods or longer, except for scale. All of his curves could be superposed on each other by horizontal translation, confirming a strong quantitative symptom of scaling. Once again, a set of economic outcomes seems to be under the spell of a power law distribution.

But Mandelbrot went much further than this. To achieve a workable description of price changes, of firm sizes, or of income distribution, he argued that we must use random variables that have an infinite population variance. Thus he expected a revival of interest in the family of statistical distributions that adhere to a power law. These are exemplified by Pareto's law for the distribution of personal income, Zipf's law, and the work of the probability theorist, Paul Lévy. Mandelbrot's work recognized a kinship between the various empirical laws and the theoretical power laws that occur in probability theory, and to interpret these power laws in terms of scaling. Sadly, this revival of interest has not yet materialized.

More recently, studies of multifractals have revealed that the price variations recorded by Mandelbrot and others exhibit self-affinity.[18] Such price changes have no typical or preferred size of variations. They're "scale-free", just like the sandpile

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[17] See Mandelbrot (1997), page 15.

[18] For a brief and easily readable summary of the geometry that describes the shape of coastlines, the patterns of the galaxies, and how stock prices soar and plummet, see Mandelbrot (1999).

avalanches that we discussed earlier. It would seem that prices and sandpiles do have something in common after all. They're both capable of evolving to a self-organized critical state.

To create a multifractal from a unifractal, you must lengthen or shorten the horizontal time axis so that the pieces of the generator are stretched or squeezed. Meanwhile, the vertical price axis may remain untouched. Market activity may speed up in the interval of time represented by the first piece of the generator, and slow down in the interval that corresponds to the second piece. Such simple alterations can produce a full replication of price fluctuations over a given period, including the periods of very high or very low volatility. On a more practical level, these findings suggest that fractal generators could be developed based on historical market data. Such generators would help to introduce some much-needed order to the seemingly chaotic gyrations of financial markets.

What's most disturbing is that much of Mandelbrot's important work has largely been ignored . Nobody seems to know why. Is it because his ideas don't fit into the traditional picture or because he's a physicist rather than an economist? Perhaps his notions are too esoteric for economists to fully comprehend. Most classical economists attribute large events -- like the stock market crashes of 1929 and 1987 -- to once-off, abnormal circumstances, such as depressions or the automated responses of computer trading programs. They look to econometric models for the explanation, paying scant attention to the statistical distributions underlying the actual geometry of price histories. Mandelbrot's results suggest otherwise. Eventually they will change the statistical underpinnings of economics in a fundamental way.

{A}Fibonacci Magic{/A}

The emerging pattern of market evolution involves many interrelated dynamic principles - Elliott waves, fractals, self-affinity, and power laws, to name just a few. We're tempted to ask if these dynamic perspectives have anything more in common? By the time you reach the end of this chapter, you may like to decide for yourself. For some readers, the material in this section will appear to be nothing more than an amusing mathematical diversion from our mainstream discussion of financial markets. Others, however, may feel it deserves to be taken more seriously!

The ancient world was full of outstanding mathematicians. When Elliott wrote *Nature's Law*, he referred specifically to a sequence of numbers discovered in the thirteenth century by the mathematician, Leonardo da Pisa. Better known by his nickname *Fibonacci,* this remarkable mathematician was taught the Arabic system of numbers by the Mohammedans of Barbary.[19] In 1202, he published a voluminous book entitled, *Liber Abaci*, in which he introduced Europeans to the Arabic system and to nearly all the arithmetic and algebraic knowledge of those times. Among the many mathematical examples to be found in this "Book of the Abacus", Fibonacci discussed a breeding problem of the following kind: "*How many pairs of rabbits can be produced in a single year from one pair of baby rabbits, if a pair of baby rabbits requires one month to grow to adulthood and each pair of adult rabbits gives birth to a new pair of baby rabbits after one month?"*

For the first two months, obviously there will only be one pair of rabbits. The sequence of numbers defining the population of rabbit pairs thus begins with the digits 1, 1. This population doubles by the end of the second month, so that there are two pairs at the start of the third month. Of these two, only the older pair begets a third pair the following month, so that at the beginning of the fourth month, the sequence is 1, 1, 2, 3. Of these three, the two older pairs reproduce, so the number of rabbit pairs expands to five. Of these five, the three older pairs reproduce, so that the next entry in the sequence is eight.

In the comparatively short period of twelve months, Mr. and Mrs. Rabbit would have a family of 144 rabbit pairs. Their monthly breeding program gives rise to the following sequence of rabbit pairs:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.

This justly famous sequence of numbers is known today as the Fibonacci sequence. Should they opt to continue their breeding habits for several years, the number of rabbit pairs would grow to astronomical proportions. After 100 months, for example, we would be facing a rabbit population of 354,224,848,179,261,915,075 pairs!

No doubt you're wondering what this rabbit breeding problem can possibly have to do with price histories in financial markets? One thing to note is that the Fibonacci sequence has many interesting properties in itself. For example, the sum of any two numbers in the sequence equals the next number in the sequence. 1 plus 1 equals 2, 1 plus 2 equals 3, 2 plus 3 equals 5, 3 plus 5 equals 8, and so on to infinity. Secondly, and more importantly, the ratio of any two numbers in the sequence approaches 1.618, or its inverse, 0.618, after the first few pairs of numbers. The ratio of any number to the next higher number, called *phi*, is about 0.618 to 1 and to the next lower number is about 1.618. The higher the numbers in the sequence, the closer to 0.618 and 1.618 are the ratios between the numbers.

For some unknown reason, the ratio 1.618 (or 0.618) to 1 seems to be pleasing to the senses. The Greeks based much of their art and architecture upon this proportion, calling it the *Golden Mean*. Among mathematicians, it's commonly known as the *Golden Ratio*, an irrational number defined to be (1+ 5)/2.[20] It's the mathematical basis for the shape of Greek vases and the Parthenon, sunflowers and snail shells, the logarithmic spiral and the spiral galaxies of outer space. It seems to imply a natural harmony that feels good, looks good, and even sounds good. Music, for instance, is based on the 8-note octave. On a piano, this is represented by 5 black keys and 8 white ones - 13 in all. Perhaps it's no accident that the musical harmony that seems to give us the greatest

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[19] Because his father, Bonaccio, was a customs inspector in the city of Bugia (called Bougie today) on the north coast of Africa, Fibonacci was effectively educated by the Mohammedans. His nickname is an abbreviated form of *filius Bonaccio* (son of Bonaccio).

[20] An irrational number is one which cannot be expressed as a ratio of finite integers. There are an infinite set of such numbers. Some, like pi (the ratio of the circumference to the diameter of a circle) and e (the base of natural logarithms) are well known because they have obvious applications in many fields. For a discussion of some speculative hypotheses linking pi to the Golden Ratio, see Dunlap (1997, pages 5-6.)

satisfaction is the major sixth. The note E vibrates at a ratio of 0.625 to the note C, just a skerrick above the Golden Ratio. Note that the ear is also an organ that happens to be shaped in the form of a logarithmic spiral.

Nature seems to have adopted the Golden Ratio as a geometrical rule in its magical handiwork.[21] From miniscule forms, like atomic structure and DNA molecules, to systems as large as planetary orbits and galaxies. It's also involved in many diverse phenomena such as quasi crystal arrangements, relections of light beams on glass surfaces, the brain and nervous system, and the structure of many plants and animals. Some have even suggested that the Golden Ratio is a basic proportional principle of nature. Could it be an *emergent* property of certain classes of natural systems?

Some of the greatest surprises of nonlinear dynamics and chaos theory have been the discovery of emergent simplicities, deep universal patterns concealed within the erratic behaviour of dynamical systems. One of the first of these unexpected simplicities was found by Mitchell Feigenbaum, and is known as the Feigenbaum number. Virtually any mathematical equation with a period-doubling bifurcation produces the same universal ratio: 4.669 and a bit! This was a totally unexpected new number in mathematics, emerging from some of the most complex behaviours known to mathematicians.[22] The period-doubling cascade (depicted in Figure 7.5) is important because it's one of the most common routes from order to chaos. Despite the fact that the Feigenbaum number is an emergent feature of period-doubling dynamical systems, we've only known about it for the last twenty years. Such emergent simplicities may be viewed as peaks in the landscapes of the possible.

[Fig. 7.5 near here]

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[21] The Golden Ratio has also been called the Golden Section, the Golden Cut, the Divine Proportion, the Fibonacci number and the Mean of Phidias. For a focused discussion of this ratio and the Fibonacci sequence of numbers, see Frost and Prechter (1990, Chapter 3), Schroeder (1991), or Dunlap (1997).

[22] For a deeper discussion of period-doubling cascades and the Feigenbaum number, see Cohen and Stewart (1988, pages 228-230).

Different kinds of simplicities can emerge from underlying chaos - numbers, shapes, patterns of repetitive behaviour. Some of these features have their own internal structure. Another fascinating example is Mandelbrot's fractal set. It's one of the most intricate geometric objects ever to have decorated a child's bedroom wall (see Figure 7.6). On viewing it, we might believe that it's extremely complex. Yet the computer program that generates it is just a few instructions long. As Murray Gell-Mann suggests, it has *logical depth* rather than *effective complexity*.[23] Putting it more bluntly, Mandelbrot's set is as simple as the rule that generates it. It only looks complicated because you don't know what the rule is. It's another case of simple rules producing seemingly complex results.

[Fig. 7.6 near here]

Perhaps the Golden Ratio is like the Feigenbaum number or Mandelbrot's set. After all, iteration is one of the richest sources of self-similarity. Given a proper start, any repeated application of some self-same operation, be it geometric, arithmetic, or symbolic, leads almost invariably to self-similarity. Take the Fibonacci sequence of numbers. If we multiply each number by the Golden Ratio and round to the nearest integer, we get

0, 2, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,……..

which is the Fibonacci sequence again, except for a few initial terms (and perhaps some later ones). The Golden Ratio reveals its own self-similarity if it's written down as a *continued fraction*. Like so many self-similar objects, the Fibonacci sequence of numbers contains within it the seeds of chaos.

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[23] Murray Gell-Mann contends that something entirely random, with practically no regularities, has effective complexity near zero. So does something completely regular, such as a bit string consisting only of zeros. Effective complexity can be high only in a region intermediate between total order and complete disorder. Logical depth is a crude measure of the difficulty of making predictions from theories. It's often hard to tell whether something possesses a great deal of effective complexity or reflects instead underlying simplicity and some logical depth. For futher elaboration, see Gell-Mann (1995).

If natural law permeates the universe, might it not permeate the world of people as well. How different from nature's laws are the laws of human nature? Nothing in nature suggests that life is disorderly or formless. We mustn't reject the possibility that human progress, which is a byproduct of human nature, also possesses order and form. If we examine the plentiful data on price gyrations of the stock market, the unmistakable self-affinity of these gyrations over different timescales suggests that they're sustained by the Golden Ratio. This was the basis for Elliott's wave principle. Two waves of a particular size can be broken into eight waves of a smaller size; then those eight waves can be subdivided in exactly the same manner to reveal thirty-four waves of an even smaller size (as depicted in Figure 7.3).[24] Both fractals and market action discern constant form within ever-changing scale.

We can generate the complete Fibonacci sequence by using Elliott's concept of the progression of the market.[25] The same basic pattern of movement which shows up in minor waves, using hourly plots, also shows up in what Elliott calls Supercycles and Grand Supercycles, using yearly plots. Take a look at the two graphs in Figure 7.7. They trace out extraordinarily similar patterns of movement despite a difference in the time horizon of over 1000 to 1. No preference is shown for any particular timescale. Instead the evolving pattern reflects the properties of the Fibonacci sequence. Waves may sometimes appear to be stretched or compressed, but underlying patterns never change. This is consistent with Mandelbrot's notion of self-affinity. The spiral-like form of market action conforms repeatedly to the Golden Ratio.

[Fig. 7.7 near here]

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[24] The cycles shown in Figures 7.2 and 7.3 are idealized in the sense that perfect wave symmetry is rarely observed in real markets. Although most five-wave formations have definite wavelike characteristics, many contain what Elliott called "extensions." Extensions are exaggerated or elongated movements which generally appear in one of the three impulse waves. Because these extensions can be of a similar amplitude and duration to the other four main waves, they give the impression that the total count is nine waves instead of the normal five. This makes the application of Elliott's wave principle more difficult in practice. For a comprehensive discussion of extensions, and other irregularities like "truncated fifths" and "diagonal triangles," see Frost and Prechter (1990).

[25] A discussion of the market's progression, and the many links between the Fibonacci sequence and Elliott's wave principle, may be found in Frost and Prechter (1990).

From the working of the Golden Ratio as a "five up, three down" movement of the stock market cycle, the astute reader might anticipate that the ensuing correction after the completion of any bull phase would be three-fifths of the previous rise in time or amplitude. Sadly, such simplicity is rarely seen within individual waves. However, time and amplitude ratios do play their part over longer timescales. For example, one of the great Dow theorists, Robert Rhea, found that over a thirty-six year time period (1896 to 1932), bear markets ran 61.1 percent of the time assigned to bull markets. He later corrected this figure to 62.1 percent. Thus Rhea discovered, without knowing it, that the Golden Ratio relates bull phases to bear phases in both time and amplitude.

Robert Prechter, that "Guru of the Decade" we met earlier in this chapter, sees the wave principle as a major breakthrough in *sociology*. He believes that the personality of each wave in the Elliott sequence is an integral part of the mass psychology it embodies. Some waves are powerful and may subdivide or feature extensions. Others are short and abrupt. Nevertheless, the progression of mass emotions from optimism to pessimism and back again tends to trace out a roughly similar wave sequence each time around. These emotions lead to cycles of overvaluation and undervaluation, producing similar circumstances at each corresponding stage in its wavelike structure. The Golden Ratio helps to shape progress overall. But each wave reflects a collective mood or personality of its own.

Because the stock market is one of the finest reflectors of mass psychology available to us, perhaps it's not surprising that it illustrates the scaling principle so vividly. Is such a principle everywhere present? Perhaps it shapes the minds of investors and hence movements of the market in a coevolutionary dance to the tune of the Golden Ratio! The answer to this intriguing question is left to the reader's imagination.

{A}Market Moods{/A}

Elliott and Prechter were not the first to focus on the moods and attitudes of investors *en masse*. In a remarkable book attempting to explain the peaks and troughs in the business cycle in the 1920s, the English economist, Arthur Pigou, placed special emphasis on the human element.[26] He surmised that *changes in expectations* were the proximate causes of variations in the economic marketplace. Although his interest was in the changing demand for labour, his theory helps to explain excessive volatility and other vagaries observed in financial markets.

What was Pigou's theory? He began by classifying the causes of expectations into three groups: (a) *real* causes, namely changes in actual conditions, (b) *psychological* causes, namely changes in men's attitude of mind, and (c) *autonomous monetary* causes, namely events like gold discoveries that affect the money supply. Also he claimed that, in our day-to-day world, real causes and psychological causes exist simultaneously, *and they react on one another*. Once started, these reactions may become reciprocating and continous. A real cause prompts a psychological reaction, this in turn adds further to the real cause, this in turn adds something further to the psychological cause, and so on.

If you're thinking that there's something familiar about this, you'd be right. Pigou was describing a positive feedback loop. We can illustrate his ideas in the familiar financial arena of commodity markets. Imagine that news of major strike action by members of the transport workers' union reaches the marketplace, triggering concern among farmers about livestock and fruit deliveries. Soon they express this concern publicly in the media, prompting further concern by the transport workers that a prolonged dispute may put their jobs at risk. Gradually the mood of the market as a whole begins to sour, exacerbating the importance of the news even further. As the strike lingers on, progressively angrier responses by the farmers serve only to trigger an even more defiant stance by the union. The *real* cause - industrial action - has triggered a *psychological* response which adds further fuel to the gravity of the real cause, which adds further to the worries of all the individuals and collectives involved.[27] Thus undue pessimism develops.

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[26] See Pigou (1927).

[27] Pigou's definition of real causes included crop variations, inventions and technological improvements, industrial disputes, changes of taste or fashion, and changes in foreign demand.

This is a highly self-reinforcing feedback loop (see Figure 7.8). Swings in optimism or pessimism arise as a psychological reflex from the original real cause(s). Pigou emphasized that these swings occur simultaneously over a large number of people because of "psychological interdependence, sympathetic or epidemic excitement, or mutual suggestion."[28] He did not believe in the theory of rational expectations, pointing to an "instability in the facts being assumed." Psychological causes arise because *expected* facts are substituted for accomplished facts as the impulse to action. This leads to errors of undue optimism or undue pessimism.

[Fig. 7.8 near here]

In summary, Pigou felt that the upward and downward swings seen in markets are partly caused by excesses of human optimism followed by excesses of pessimism. It's as if the pendulum swings too far one way and there is glut, then it swings too far the other way and there's scarcity. An excess in one direction breeds an excess in the other, diastole and systole in never-ending succession.[29] There's plenty of evidence of such cycles of overreaction. Psychologists acknowledge the moody, contagious nature of crowds. There's a degree of psychological interdependence which can magnify the initial response. An error of optimism by one person can pump up the optimism of others. It's almost like an epidemic. When prices rise in the stock market, for example, because a few more businessmen become more prosperous, they're apt to look on the brighter side. This serves as a spur to optimistic error among others. Thus the error is magnified.

There's another interesting twist to Pigou's theory. Once they're discovered, errors of optimism can quickly change to errors of pessimism, and vice versa. This keeps the pendulum swinging too far in both directions. The result is a relentless ebb and flow in the tide of emotions affecting investors' stock market decisions. If Pigou happens to be right, then the implication is that human nature doesn't change. Despite the errors in

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[28] See Pigou (1927), page 86.

[29] See Frost and Prechter (1985), page 11.

optimism and pessimism, certain patterns will tend to repeat themselves as human nature weave its spell. Suddenly, those patterns of self-affinity that we've observed in market gyrations take on a new meaning. Could self-affine price histories – those same patterns displaying fractal geometry and conforming to power laws – simply be reflecting the collective moods and vagaries of human nature? Perhaps the marketplace experiences mental phase transitions, transforming it from a simpler to a more complex regime, and later back again?

Pigou was one of the earliest scholars to question the validity of the efficient markets hypothesis from a psychological viewpoint. Others have followed recently in his footsteps. Robert Shiller, Professor of Economics at Yale University's Cowles Foundation, typifies a group of modern scholars exploring the idea that price movements in speculative markets may be due to changes in opinion or psychology. He poses the following basic question:

{EXT}Can we trace the source of movements back in a logical manner to fundamental shocks affecting the economy, the shocks to technology, to consumer preferences, to demographics, to natural resources, to monetary policy or to other instruments of government control? Or are price movements due to changes in opinion or psychology, that is, changes in confidence, speculative enthusiasm, or other aspects of the worldview of investors, shocks that are best thought of as coming ultimately from peoples's minds?{/EXT}[30]

Shiller finds that investor attitudes are of great importance in determining the course of prices of speculative assets. Prices change in substantial measure because the investing public *en masse* capriciously changes its mind. He found clear evidence of price volatility, relative to the predictions of efficient markets theories, particularly in the stock market.[31] This means that the very variability of price movements is too large to be

[30] See Shiller (1989), page 1.

[31] Shiller's research concentrates on the ultimate causes of price volatility in speculative markets, including the influence of fashions, fads, and other social movements. Impressive evidence is amassed in stock, bond and real estate markets; see Shiller (1989).

justified in terms of efficient markets models, given the relatively low variability of fundamentals and given the correlation of price with fundamentals.

Shiller studies various kinds of *popular models* - simple, qualitative hypotheses of what may happen to prices. Many popular models focus on behavioural patterns observed in the marketplace. They bear a striking resemblance to Brian Arthur's temporary mental models associated with the processes of pattern recognition and inductive reasoning (see Chapter 2). They're also reminiscent of the temporary hypotheses which drivers adopt in their attempts to combat traffic jams (see Chapter 6).

A well-known example of a popular model in the stock market is the sequence of price movements surrounding the crash of October 1929. People who adopt this model think that this particular pattern of price movements may happen again at a later date. Because they're easy for the general public to understand, models like these usually get plenty of attention in the press. For example, there was an article advancing the "1929 hypothesis" in the *Wall Street Journal* on October 19, 1987 - the very morning of the day the stock market crashed again!

Singling out patterns like the one in 1929 for so much attention is rather arbitrary. History provides many more episodes that might be used for comparison than ever enter the public's mind. Other dramatic stock market rises, as well as many less-dramatic stock market episodes, are largely forgotten -- because investors mostly fear the major crash. Yet the self-similarity of these gyrations over different timescales may be the potent, pattern-making feature of markets in general. Why concentrate on a very infrequent part of this overall picture? Shiller argues that such popular models may create a vicious circle, or feedback loop in our terminology, whereby people's reaction to price changes causes further price changes, yet more reaction, and so on. We've argued in earlier chapters that economists should examine these mental models directly. The approximation of allowing economic theorists to model human behaviour, without collecting information on the popular models of the world, has serious drawbacks. Nowhere are these limitations more apparent than in the study of speculative markets.

{A}Reading the Market's Mind{/A}

Now let's turn to the ideas of two of the most successful investors to have put their pens to paper for our benefit. One has become a household name, by virtue of his aggressive currency plays which have challenged the stability of nations. The other is largely unknown outside his close-knit circle of enthusiastic disciples. Both can justifiably claim to understand the "mind" of the market. More importantly, their highly profitable records are living proof of the potential fallibility of the efficient markets hypothesis.

George Soros runs his own international fund management group. Its flagship vehicle, The Quantum Fund, is a Curacao-based investment firm headquartered in Manhattan. Despite the recent corrections among some hedge funds, typical per-annum gains by Quantum have exceeded 50%.[32] Part of his fame (and notoriety) stems from the fact that he made a billion dollars going up against the British pound. Some say he rescued England from recession. Others are less complimentary. Dubbed by Business Week as "The Man Who Moves Markets," Soros is arguably the most powerful and profitable investor in the world today.[33] He has all the trappings of an intelligent thinker and sponsors major philanthropic efforts. For our purpose, it's enough to concentrate on his philosophical train of thought.

Soros is highly critical of the way in which economists use the concept of an equilibrium. As he views it, the deception lies with their emphasis on the final outcome instead of the process that leads up to it. This endows the equilibrium concept with an aura of empirical certainty. Yet that's not the case in reality. Equilibria have rarely been observed in real life. Market prices have a habit of fluctuating incessantly. We've seen historical examples of price fluctuations in earlier sections. More will appear later in this chapter. If market participants are actually adapting to a constantly moving target, calling their behaviour an adjustment process may be a misnomer and equilibrium theory becomes irrelevant to the real world.

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[32] For example, the Quantum Fund gained 68.6% in 1992 and 61.5% in 1993.

[33] The introductory material in this paragraph has been drawn from a cover of his recent book entitled The Alchemy of Finance: Reading the Mind of the Market. For further details and an exciting read, see Soros (1994).

The assumption that Soros found so unacceptable as a student of economics was that of *perfect knowledge*. How could one's own understanding of a situation, in which one participates and interacts with others, possibly qualify as full knowledge of that situation? Because we're more aware today of the limits to knowledge, modern theories of perfect competition and efficient markets merely postulate perfect *information*. But this merely shirks the real issue. By assigning themselves the task of studying the relationship between supply and demand, and not either by itself, economists disguise a sweeping assumption behind the facade of a methodological device. The sweeping assumption is that each participant knows all that needs to be known to take a correct decision.

As we've argued in earlier chapters, this kind of assumption is untenable. Even the shape of supply and demand curves cannot be taken as independently given, because both are built on the participants' expectations (or hypotheses) about events that are, in turn, shaped collectively by their own expectations. Anyone who trades in markets where prices are changing incessantly knows that participants are strongly influenced by market developments. As Soros suggests: "Buy and sell decisions are based on expectations about future prices, and future prices, in turn, are contingent on present buy and sell decisions."[34] Rising prices, fueled by buyer interest exceeding that of sellers, tend to attract even more buyers. Likewise falling prices tend to attract more sellers.

There's plenty of evidence of positive feedback loops in financial markets of all varieties. How could such self-reinforcing trends persist if supply and demand curves were independent of market prices? In the normal course of events, a speculative price rise provokes counteracting forces: supply is increased and demand reduced. Thus temporary excesses are corrected with the passage of time. But Soros disputes that this always happens. In the stock market, for example, the performance of a stock may affect the performance of the company in question in a variety of ways. He contends that such paradoxical behaviour is typical of all financial markets that serve as a discounting mechanism for future developments: notably stock markets, foreign exchange markets, banking, and all forms of credit.

Soros points to the need to understand the process of change that we can observe all around us. We're both instigators of, and reactors to, change. In his own words: "The presence of thinking participants complicates the structure of events enormously: the participants' thinking affects the course of events and the course of events affects the participants' thinking." From this we can identify the core of Soros' thesis about the dynamics of financial markets. It's a process of coevolutionary learning. Just as we've discussed at length, economic agents must base their decisions on an inherently imperfect understanding of the situation in which they participate. We saw that it baffles music lovers at the El Farol, that it provokes defection in the Traders' Dilemma, and that it frustrates drivers on a congested highway. Thinking always plays a dual role. First, participants seek to understand the situation in which they participate. Second, their imperfect understanding serves as the basis of decisions which influence the actual course of events.

What makes the participants' understanding imperfect is that their thinking affects the very situation to which it applies. They're caught up as participants in the very process that they're trying to understand. Because there's a discrepancy between the expectations (or favoured hypothesis) held by each participant, and the outcome itself, invariably some participants "change their mind" next time around. This also changes future outcomes. Soros gives this discrepancy a special name. He calls it the participants' *bias*. The actual course of events is very likely to differ from the participants' expectations, and this divergence gives an indication of the participants' bias.

It's this bias that forms the centrepiece of what he calls his theory of *reflexivity*. Soros splits the divergence into two components. He calls the participants' efforts to understand the situation, the *cognitive function*, and the impact of their thinking on the real world, the *participating* function.[35] We've used slightly different terms, namely *inductive reasoning* and the *collective outcomes*. When both functions operate simultaneously, they interfere with each other. Instead of a determinate result, we have an interplay in which both the situation and the participants' views are dependent

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[34] See Soros (1994), page 29.

[35] Soros cites "learning from experience" as an obvious example of the cognitive function.

variables, so that an initial change precipitates further changes both in the situation and the participants' views. He calls this particular kind of positive feedback process "reflexivity."[36] Reflexivity doesn't produce an equilibrium. Because the two recursive functions belong to the world of morphogenesis, they produce a never-ending process of change. People are groping to anticipate the future with whatever guideposts they can establish. Outcomes tend to diverge from expectations, leading to constantly changing expectations and constantly changing outcomes.

The idea of a distinction between near-equilibrium and far-from-equilibrium conditions has been emphasized before. Clearly Soros believes that such distinctions are also important in financial markets. But he's quick to add the following rider: "Since far-from-equilibrium conditions arise only intermittently, economic theory is only intermittently false."[37] In other words, his notion of reflexivity operates intermittently. Thus it has strikingly similar properties to the theory of punctuated equilibria and self-organized criticality (introduced in Chapter 1).

Soros claims that it's possible to treat the evolution of prices in all financial markets as a reflexive, historical process. There are long fallow periods when the movements in these markets do not seem to follow a reflexive tune, but resemble the random walks mandated by the efficient markets hypothesis. Because the whole process is open-ended, however, discontinuities arise unexpectedly. These sudden changes are shaped by the misconceptions of the participants. In this respect, Soros' thesis closely resembles that of Pigou. Real causes and psychological causes are reacting upon each other. In both cases, price histories are built on *fertile fallacies*. In both cases, the efficient markets hypothesis is found wanting.

Despite his obvious success as a global investor, Soros is vague when it comes to the crucial question of how to play the markets and win. He shrouds his own methods in a cloak of mystery. In contrast, Charles Lindsay's recipe for trading success is remarkably simple and opaque. Having probed deeply and carefully into the self-affinity

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[36] The word "reflexivity" is used in the sense that the French do when they describe a verb whose object and subject are the same.

[37] See Soros (1994), page 9.

issue in real markets, Lindsay's strategy embodies most of the unconventional concepts discussed in this chapter. He maintains that all events, real or imagined, cause prices to fluctuate as traders and speculators react to these events and rumours. Because rumours abound, he believes that a successful trading system must ignore the rumours themselves, taking only the market's net reaction to them into account. This is the basis of his trading approach.[38]

Lindsay believes that prices are as unstable as waves pounding onto a beach. They reflect the incessant struggle between buyers and sellers. Whenever buying pressure exceeds selling pressure, the price fluctuates upward. Whenever selling pressure exceeds buying pressure, the price fluctuates downward. If buying and selling pressure are equal, the price moves sideways. Lindsay likens such price swings to those associated with the pressure in a hose as water is forced through its nozzle. He defines a *Trident price* as the price at which buying (or selling) pressure is overcome by its opposite. In other words, it's the price at a turning point. Price fluctuation in the same trend direction is called a price swing, joining the lowest price in the trend to the highest price in the trend, or vice versa depending on trend direction. Figure 7.9 illustrates the notion of price swings from one Trident price to the next. Trident prices are nothing more than local maxima or minima in the recent price history. For example, all the depicted swings from P_1 to P_2, and from P_2 to P_3, are swings from a local maximum (or minimum) to a local minimum (or maximum). Such turning points define those occasions when buying or selling pressure is overcome by its opposite.

[Fig. 7.9 near here]

The chart depicting price variations in the market for live hogs highlights a typical trading opportunity using Trident analysis. Note that the drop in price from P_1 to P_2 is about 20 cents per pound. Then the price rises again – from P_2 to P_3 – by about 12 cents per pound. According to Trident theory, the next downward swing – namely P_3 to P_4 – should reach a target price of about 39.5 cents per pound. As the historical chart shows,

[38] A full account of his trading strategy can be found in Lindsay (1991).

the realized price, P_4, dipped slightly below 39 cents per pound. In Trident terms, this trade was fully successful since it exceeded its target price.[39]

Like Elliott Wave theory, Trident analysis is a trading strategy that's based on an investor's ability to recognize various patterns formed by sequential price gyrations in the marketplace. The model itself consists of a collection of formulae deduced from analyses of price swings and price action. It rests on the idea that a future price depends sensitively on the cumulative sequence of historical price movements. In practical terms, the strategy allows the investor to calculate an "ideal" target price and profit for each "tradeable" price swing. The price swing P_3 to P_4 – shown in Figure 7.9 – depicts one and only one best trade: buy at P_3 and sell at P_4. On this basis, the Trident model assumes that the next price swing will resemble the previous swing in the same direction. In other words, it assumes the same kind of symmetry that underlies Elliott wave theory and the principles of fractal geometry: self-affinity.

The target price is ideal in the sense that it serves as an optimistic forecast rather than a hard-and-fast prediction. Targets aren't always reached, but the one algorithm holds true for calculating swing targets at all levels. Lindsay's simple algorithm is built on the notion that price action is not random, but sequential and often close-to-symmetrical. A host of different markets contain numerous examples of Trident formation and target completion. In practice, ideal targets are reached about 40% of the time. Potentially profitable trades occur far more frequently, because trades can be terminated early if their ideal targets turn out to be unattainable. The proof of the Trident is in the bank. Many of Lindsay's devout students have accumulated impressive fortunes.

Like Elliott, the key to Lindsay's success lies in the recognition that prices fluctuate over several levels of time increment continuously and simultaneously. He distinguishes between five levels, claiming that his Trident analysis is effective at each level: (1) Microswings - these are swings that occur during a daily trading session (e.g. at 15 minute intervals); (2) Minor Swings - which are measured from the highest daily high to the lowest daily low in sequence, then from the lowest daily low to the highest daily high in sequence; (3) Intermediate Swings - which are measured from the highest minor high to the lowest minor low in sequence and from the lowest minor low to the highest minor high in sequence; (4) Major Swings - which are measured from the highest intermediate high to the lowest intermediate low in sequence and from the lowest intermediate low to the highest intermediate high in sequence; and (5) Master Swings - these take several years to form and are defined by sequential life of contract highs and lows.

Full details of Lindsay's trading method will not be discussed here.[40] That would breach an oath of commercial confidentiality. Our modest aim was to show that relatively simple trading strategies can be founded on multifractal scaling principles. Such methods are easier to apply than Elliott's wave principle, yet the underlying forces shaping price fluctuations are identical. Importantly, each method recognizes that the market is always right. The market coevolves and you, as a trader, must coevolve with it. One fascinating feature of the Trident algorithm is that success hinges upon the single ability to recognize a tradeable situation. Two key criteria defining "tradeability" are what Lindsay calls "determinate" and "trend reversal" prices. Both of these are defined in relative terms, and are specified as a single fraction of earlier price swings. That fraction just happens to be 0.625, remarkably close to the Golden Ratio!

{A}How Markets Learn{/A}

We've seen how price histories trace out remarkably symmetrical geometrical patterns over different timescales. It's as if markets possess a collective mind of their own, a *fractal* mind. We've also learnt that some of the most successful players in the investment game – people like Robert Prechter, George Soros and Charles Lindsay – believe that individual decisions can "move" markets and, in turn, that the market's collective mind affects individual investors. Once again, the self-reinforcing engine behind all of these observations is the unfolding process of coevolutionary learning.

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[39] Trident analysis functions just as well in bear markets as it does in bull markets.

[40] To get hold of Lindsay's trading strategy and start playing the markets, it's suggested that the interested reader writes directly to his publisher: Windsor Books, Brightwaters, N.Y., USA.

Is there a way of relating these two sets of observations? Can we test whether fractal geometry is consistent with Soros' theory of reflexivity or Pigou's thesis on the excesses of human optimism and pessimism? This would appear to be impossible if we choose to resort to traditional modelling techniques. Closed-form models cannot handle a diverse population of investors harbouring literally hundreds of different hypotheses about market behaviour. But a possible way out of this dilemma has been mentioned in earlier chapters. Agent-based simulations may be able to accommodate the vastly heterogeneous beliefs held by market participants, thereby uncovering some of their emergent features.

Instead of confining ourselves to vehicles, in Chapter 6 we looked at traffic behaviour in terms of drivers' *psychology*. We saw that the beliefs and expectations of drivers are constantly being tested in a world that forms from their and others' actions and subjective beliefs. Perhaps the same may be true of the stock market. After all, the typical investor is not so different from the typical driver! For both, prediction usually means a short-term, beat-the-crowd anticipation of tomorrow's situation (i.e. prices or travel times). Why not view the stock market as a diverse collection of beliefs, expectations and mental models?

Brian Arthur is one Sante Fe Institute economist who opted to test this approach to financial markets. Together with John Holland, Blake LeBaron, Richard Palmer and Paul Tayler, Arthur created an artificial stock market on the computer, inhabited by "investors" who are individual, artificially-intelligent programs that can reason inductively.[41] In this *market-within-a-machine*, artificial investors act like those economic statisticians we described in Chapter 6. They're constantly testing and discarding expectational hypotheses of how the market works and which way prices will move. These subjective, expectational models are a bit like the ones used by Arthur's "silicon patrons" at the El Farol bar. Just as there's no way of telling how many devotees of Irish music plan to come to the El Farol next Thursday evening, or how many drivers

[41] The Sante Fe Artificial Stock Market has existed in various forms since 1989. Like most artificial markets, it can be modified, tested and studied in a variety of ways. For glimpses into this new silicon world, and its methods of mimicking the marketplace and its gyrations, see Arthur (1995) or Arthur, Holland, LeBaron, Palmer and Tayler (1997).

plan to take the same expressway home after work tonight, there's no way that investors can tell what tomorrow's prices will be in the stock market.

There are plenty of clues around, of course. For example, a popular guide to the state of prices the next day is the value of tomorrow's stock index in the futures market. If that value is above today's closing value, it means that the bulk of investors expect tomorrow's prices to rise. But there are literally hundreds of different hypotheses about tomorrow's state of play. Here's a couple of other possibilities:

> IF today's price is higher than its average in the last 100 days,
> THEN predict that tomorrow's price will be 3% higher than today's.

or

> IF today's price breaks the latest trendline upwards,
> THEN predict that next week's price will be 5% lower than this week's.

Some investors may keep many such models in mind, others may retain only at a time. In the Prediction Company's artificial stock market, each agent adopts his "most reliable" model – the one that performs best in the market's current state. Naturally enough, different expectational models may perform better than others at different times. Thus investors must retain and adopt a suite of models for their buy and sell decisions. Eventually, the poorer performing models are discarded. Agents use a genetic algorithm to produce new forecasting models from time to time.

The learning process in this silicon world comes from two sources: discovering "new" expectational models and identifying the ones that perform best from among the current set.[42] Prices form endogenously from the bids and offers of the silicon agents, and thus ultimately from their beliefs. Such expectational models are akin to Pigou's "changes in men's attitudes of mind," and display some feedback effects inherent in his theory. For example, if enough traders in the market happen to adopt similar expectational models, positive feedback can turn such models into self-fulfilling

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[42] "New" expectational models are mostly recombinations of existing hypotheses that work better.

prophecies.[43] The agent-based experiments conducted by the Prediction Company have typically involved about 100 artificial investors each armed with 60 expectational models. As this pool of 6,000 expectational models coevolves over time, expectations turn out to be mutually reinforcing or mutually negating. Temporary price bubbles and crashes arise, of the very kind that Pigou attributed to excesses of human optimism or pessimism. These more volatile states may be attributed to the spontaneous emergence of self-fulfilling prophecies.

A key aspect of agent-based simulations are their internal dynamics. Expectations come and go in an ocean of beliefs which form a coevolving ecology. How do the beliefs of fundamentalists fare in this silicon world? Do technical trading beliefs ever gain a firm footing? The results so far suggest that both views are upheld, but under different conditions.[44] If a majority of investors believe the fundamentalist model, the resulting prices will validate it; and deviant predictions that arise by mutation in the population of expectational models will be rendered inaccurate. Thus they can never get a solid foothold in the market. Necessity prevails. But if the initial expectations happen to be randomly distributed uniformly about the fundamentalist ones, trend-following beliefs that appear by chance have enough density to become self-reinforcing in the ecology of beliefs. Chance shatters the conventional wisdom. Then the use of past prices to forecast future ones becomes an emergent property.

In this mutated regime, no stationary equilibrium seems to be reached. The market keeps evolving continuously. If initially successful agents are "frozen" for a while, then injected back into the market much later, they do no better than average. The market seems to be impatient, moving on and discovering new strategies that replace earlier ones. There's no evidence yet of market "moods," but there is evidence of

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[43] In a series of interesting studies – typified by De Long, Shleifer, Summers and Waldmann (1990) or Farmer (1993) – it has been shown analytically that expectations can be self-fulfilling. Thus we may conclude that positive feedback loops, or Pigovian herd effects, do have a significant role in shaping the market's coevolutionary patterns.

[44] This is reminiscent of our earlier discussion on punctuated equilibria. In proposing his general theory of reflexivity, George Soros suggested that "Since far-from-equilibrium conditions arise only intermittently, economic theory is only intermittently false.........There are long fallow periods when the movements in

GARCH.[45] The presence of GARCH means that there are periods of persistent high volatility in the price series, followed randomly by periods of persistent low volatility. Such phenomena make no sense under an efficient market hypothesis. But in an evolutionary marketplace, prices might continue in a stable pattern for quite some time, until new expectations are discovered that exploit that pattern. Then there'll be very rapid expectational changes. These transform the market itself, causing avalanches of further change. Once again, there's evidence of punctuated equilibria and self-organized criticality. Perhaps that see-sawing action we observe in markets is symptomatic of a system driving itself towards then away from the edge of chaos![46]

If it does, this would be further evidence that markets undergo phase transitions. Observable states look like they're poised between necessity and chance, between the deterministic and the seemingly-chaotic, between the simple and the complex. In summing up, Arthur states: "We can conclude that given sufficient homogeneity of beliefs, the standard equilibrium of the literature is upheld. The market in a sense in this regime is essentially "dead." As the dial of heterogeneity of initial beliefs is turned up, the market undergoes a phase transition and "comes to life." It develops a rich psychology and displays phenomena regarded as anomalies in the standard theory but observed in real markets. The inductive, ecology-of-expectations model we have outlined is by definition an *adaptive linear network*.[47] In its heterogeneous mode it displays complex, pattern-forming, non-stationary behaviour. We could therefore rename the two regimes or phases *simple* and *complex*. There's growing evidence suggesting that actual financial markets live within the complex regime."[48]

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financial markets do not seem to follow a reflexive tune but rather resemble the random walks mandated by the efficient market theory;" see Soros (1994, page 9).

[45] GARCH = Generalized AutoRegressive Conditional Hederoscedastic behaviour.

[46] Peter Allen has pointed out that an *adaptive* trading strategy is one that can give good results despite the fact that we cannot know the future, because there are different possible futures. When discernable trends become apparent, the strategy must be able to react to this. By taking such actions, however, the strategy will change what subsequently occurs in reality. This coevolutionary behaviour implies that markets will always drive themselves to the "edge of predictability;" in other words, to the *edge of chaos*.

[47] For a precise definition of an adaptive linear network, see Holland (1988).

[48] See Arthur (1995), page 25.

It seems that market participants are involved in an incessant game of coevolutionary learning. Agent-based simulation experiments like the Sante Fe Artificial Stock Market offer a keyhole through which we can gain useful insights into adaptive behaviour. Similar studies by others have also shown that heterogeneous behaviour on the part of participants can provide opportunities for making consistent profits, that participants with stable bankrolls appear to have an advantage over those who don't, and that small perturbations can sometimes drastically alter the behaviour of the participants.[49] As empirical evidence mounts against the view that markets are efficient, new explanatory approaches like that of the adaptive, boundedly-rational investor will gain more credibility. Scaling principles and computer simulation experiments will play an increasingly important part in this new behavioural revolution. Behavioural experiments in such silicon worlds may even herald a new kind of economics, an experimental economics which relies heavily on agent-based simulation. This new approach to social science is the subject of the final chapter.

[49] Such a set of simulation experiments can be found in de la Maza and Yuret (1995).

TABLE 1.1:
Two Economic Worlds - The Simple and the Complex

NECESSITY	CHANCE
Stasis	Morphogenesis
Resource-Based	Knowledge-Based
Unique Outcome	Multiple Outcomes
Equilibrium	Path-Dependent
Mechanistic	Organic
Predictable	Unpredictable
Diminishing Returns	Increasing Returns
Convex	Nonconvex
Easy to Model	Difficult to Model
A SIMPLE WORLD	A COMPLEX WORLD

TABLE 2.1:
Information and Knowledge

Characteristic	Information	Knowledge
Source	External	Internal
Nature	Weakly-interactive	Strongly-interactive
Primary exchange mode	Interface	Face-to-face
Learning rate	Fast	Slow
Usefulness	Temporary	Longlasting
Exchange process	Simple	Complex
Unit of measurement	Quantitative (e.g. bits)	Qualitative (e.g. deep)

TABLE 4.1
Population Growth in Europe

Date	European Population	Margin of Error (%)
200	48	35
500	36	30
800	32	30
1000	39	20
1300	75	20
1500	76	10
1700	102	8

TABLE 4.2:
The Ten Largest Cities in Europe by Population, 1000-1400

1000	1100	1200	1300	1400
Cordova	Constantinople	Constantinople	Paris	Paris
Constantinople	Fez	Palermo	Granada	Bruges
Seville	Seville	Seville	Constantinople	Milan
Palermo	Palermo	Paris	Venice	Venice
Kiev	Cordova	Venice	Genoa	Genoa
Venice	Granada	Cordova	Milan	Granada
Thessalonika	Venice	Granada	Sarai	Prague
Ratisbon	Kiev	Milan	Seville	Constantinople
Amalfi	Salerno	Cologne	Florence	Rouen
Rome	Milan	London	Cologne	Seville

Table 5.1: Changes in Rank of Selected American Cities, 1810-1910

City	Rank in 1810	Rank in 1860	Rank in 1910
New York	1	1	1
Philadelphia	2	2	3
Baltimore	3	3	7
Boston	4	4	5
New Orleans	6	5	14
Cincinatti	42	6	13
St. Louis	-	7	4
Chicago	-	8	2
Buffalo	-	9	10
Louisville	-	10	22
Albany	17	11	44
Washington	12	12	16
San Francisco	-	13	11
Providence	8	14	21
Pittsburgh	28	15	8
Rochester	-	16	23
Detroit	-	17	9
Milwaukee	-	18	12
Cleveland	-	19	6
Charleston	4	20	77

Table 5.2: Similarities Between CAs and Socio-Economic Dynamics

	Cellular Automata	Socio-Economic Dynamics
Basic elements	Cells are the basic units or “atoms” of a CA	Individual agents are the basic units of an economy
Possible states	Cells assume one of a set of alternative states	Agents form mental models which enable them to make choices from alternatives
Interdependence	The state of a cell affects the state of its closest neighbors	The choices made by agents affect the choices made by other agents
Applications and tasks	Modeling the emergence of order, macro outcomes explained by micro rules, and the path dependence of dynamic processes	Important tasks include: understanding the emergence of order, macro to micro relationships, and economic dynamics

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