THE HOUSEHOLD SECTOR IN THE SMETS-WOUTERS DSGE MODEL: COMPARISONS WITH THE STANDARD OPTIMAL GROWTH MODEL

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Basic References:

[**1**] ** L. Tesfatsion, "A Simple Illustrative Optimal Growth Model"
www.econ.iastate.edu/classes/econ502/tesfatsion/OptGrow.pdf

[**2**] * Frank Smets and Raf Wouters, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area", National Bank of Belgium Working Paper, 2003, 71pp. **On-Line**
www.econ.iastate.edu/classes/econ502/tesfatsion/SmetsWoutersNotes.LT.pdf

Note: The Smets-Wouters DSGE model has been used by the European Central Bank over the past several years to analyze macroeconomic policy for the Euro area, i.e., the particular set of European countries that have commonly adopted the Euro as their currency.

QUESTION 1: Compare and contrast the DSGE modeling *of the household sector* in Section 2.1 of Smets-Wouters [2] with the standard optimal growth modeling of the household (consumer) sector described in [1].

NOTES FOR Q1:

The closest comparison is obtained if in [1] one adopts the "single representative household (consumer)" interpretation for the optimizing agent in the optimal growth model rather than thinking of this optimizing agent as a social planner. This interpretation will be assumed throughout the remaining parts of these notes without further comment.

The household sector in [1] is developed in continuous time for a continuum of time points t, whereas the household sector in [2] is developed in discrete time for a countable sequence of time periods $t = [t, t+1)$.

In [1], households are assumed to supply labor inelastically in each time period t, so there is no labor/leisure "choice"; the only argument of the household utility function at each time point t is amounts of a single produced consumption good (or a scalar index for "real" consumption). In contrast, in [2] the household utility function has two arguments in each time period t: (i) (produced) goods; and (ii) leisure. Moreover, labor is differentiated over households so that there is some monopoly power over wages.

Model [1] assumes the existence of a single "representative" optimizing household. In contrast, model [2] permits the existence of a limited degree of heterogeneity across households.

More precisely, in model [2] there is a continuum of representative optimizing households, one representative optimizing household for each type of labor supplied. That is, as explained in [2, p. 4], households are indexed by an index parameter ι ranging over an interval $[0, l]$, implying there are uncountably many households. Households with distinct indices are assumed to supply a distinct type of labor. In particular, note that households are structurally identical apart from the fact that they supply distinct types of labor with differing disutilities. In particular, the household instantaneous (period t) utility functions (2) are structurally identical except that different households supply different types of labor with different coefficients σ_ι representing the inverse of their elasticity of work effort with respect to the real wage.

In both models the households own the capital stock, a homogeneous factor of production. In model [1] there are no explicitly modeled external "firms". Rather, the one representative household is also the one representative firm. (This is why model [1] is often referred to as a "Robinson Crusoe" model of an economy – think of a single agent shipwrecked on an island who produces breadfruit and who is also the sole consumer of this produced breadfruit.) In contrast, as explained on pages 8-9, model [2] has an explicitly modeled continuum of intermediate-good firms/producers to whom the households supply both capital and labor services. The intermediate goods produced by the intermediate-good firms are then used to produce a final good that is used for both consumption and investment by the households.

In both models [1] and [2] the households decide how much of their current income to allocate towards investment (production of new capital). In model [1] the remainder is entirely consumed (capital investment is the only form of saving); there are no financial assets in model [1]. However, in model [2] the households can also save part of their income in the form of bond holdings. Note, however, that model [2] does not include "outside" (government-issued) money; see [2, footnote 7].

In model [1] there is no modeled source of uncertainty. Hence the representative household in model [1] at the initial time point 0 maximizes its lifetime utility with respect to a choice of consumption and investment paths over its lifetime $[0, T]$ subject to macro technological feasibility constraints (see below), where T can be either finite or infinite.

Remark: The household's objective function in model [1] is a standard discounted utility (DU) model (as defined in Course Packet 21). Hence, even if the household were to be allowed to "re-optimize" at a later time, it would not choose to do so.

In model [2], each household maximizes its *expected* lifetime utility (1) with respect to choices of paths for consumption, investment, and bond holdings over its *infinite* lifetime $[0, \infty]$ subject to an intertemporal budget constraint (4) and a capital accumulation equation (14), where the discount rate β is common across households. Household utility is perturbed by two exogenous shocks, commonly experienced across households, as follows: A shock to the discount rate β; and a shock to the labor supply.

Remark: In model [2], each household's lifetime utility function is a discrete-time version of the standard DU model used in model [1], extended to include habit persistence (see below).

In model [1] there is no "habit persistence," i.e., the utility of consumption attained by a household at any time point t is a function only of the household's consumption at t. It does not depend on the amount of consumption that the household consumed in the past or will consume in the future. In contrast, habit persistence is introduced in model [2] in an attempt to obtain a better fit with empirical data. In particular, as seen in relations (2) and (3), a household's utility of consumption in time period t is assumed to depend on a "habit variable" H_t that is a constant h times consumption in the previous period $t - 1$.

In model [1], household income at time t is identified with national income Y_t, which is then divided between consumption C_t and gross investment I_t. In short, the household's "budget constraint" is the national income accounting identity for a closed economy: Y = C + I.

In contrast, in model [2] each household's income (5) is determined from three sources: roughly, labor income; income from sale of capital services (i.e., from renting out capital – this is misstated on page 5); and dividends received from firms. A separate budget constraint (4) is imposed on each separate household because the income (5) for each household is determined in part by the supply of a distinct type of labor at a distinct wage level. However, a "trick" is used on page 5 that effectively eliminates heterogeneity in the budget constraints across households: namely, it is assumed that households can insure against variations in their household-specific labor income by purchasing "state-contingent securities".

Also, in model [2, p. 6] it would appear that the household constraints involve an equilibrium condition; namely, that each household set their wages and labor supply so that their labor supply equals demand (9) for their type of labor, where the latter depends on *aggregator functions* for total labor demand and the the "aggregate nominal wage".

Technical Remark for Q1: Game-theoretic issues are avoided here by the assumption that households form a "continuum." The latter assumption means that the integrals in (10) and (11) are not affected by the "point" decisions of any one household, conditional on given decisions for all other households, because what happens at a single household point is a "measure zero" event from the vantage point of the integrals as a whole.

QUESTION 2: Discuss the motivation and justification provided in Smets-Wouters [2] for the modeling of the household sector in Section 2.1. In particular, to what extent does this modeling appear to be based on theoretical considerations? on empirical considerations?

NOTES FOR Q2:

Based on Section 1 and Section 2.1 in [2], it would appear that the Smets-Wouters modeling of the household sector is motivated by two primary concerns:

1. **Theoretical Considerations:** Stick as closely as possible to the assumptions employed in previous DSGE modelings, in particular in the DSGE modeling by Christiano, Eichenbaum, and Evans (CEE,2001).

2. **Empirical Considerations:** Modify these modeling assumptions only to the extent that the modifications appear to enable a better goodness-of-fit between the resulting DSGE model *outputs* and macroeconomic data for the main euro area of Europe over a given data sample period: namely, 1980:2-1999:4 (p. 16).

Regarding 2., Smets-Wouters note (p. 2) that they measure goodness-of-fit for their estimated DSGE model by its ability "to capture the empirical stochastics and dynamics in the data." As they further clarify on page 15, this involves an ability "to match not only the contemporaneous correlations in the observed data series, but also the serial correlation and cross-covariances."

There is no explicit attempt by Smets-Wouters in [2, Section 2.1] to motivate their modeling choices in terms of input empirical validation, i.e., in terms of trying to capture structural characteristics of the "true data generating mechanism" underlying the macroeconomic data for the euro area used for goodness-of-fit checks. However, Smets-Wouters would presumably argue that input empirical validation is implicitly used in their selection of key "representative agents" for their model (households, intermediate-good firms, a final-good firm, and a government implementing a Taylor-like monetary policy rule in an attempt to control bond gross interest rates $\{R_t\}$).

The authors mention the possibility of using predictive performance for empirical model validation in footnote 22 (p. 16). However, the authors also caution that such a validation method "can be very computationally intensive."

Nevertheless, in Section 3.3.1 the authors attempt to calculate the "marginal likelihood" for their estimated DSGE model, roughly defined as the overall likelihood of their model given their data sample. The authors claim (p. 21) that "the marginal likelihood of a model also reflects its predictive performance." However, what they mean by this turns out to be predictions of observables over the same data sample period that they originally used to estimate their model; the authors do not actually use their model for prediction out of sample (i.e., using completely new data). Possibly this is due to the shortness of the data series for the euro area, which was slowly implemented in stages following a treaty in 1992.

Specifically, the authors show (Table 2, p. 34) that "the marginal likelihood of (their) estimated DSGE model is very close to that of the best VAR (vector auto-regressive) model" when used to predict the seven observable variables of their model over their data sample period 1980:2-1999:4. Apparently (footnote 36) it has been very difficult in the past for estimated DSGE models to match the performance of VAR models over data sample periods.

QUESTION 3: Advocates of DSGE models claim it is possible to use DSGE models to evaluate the performance of macroeconomic policies on the basis of Pareto optimality and other social welfare criterion derived from household preferences, an improvement over simple aggregate welfare measures such as the real GDP gap. Carefully explain how monetary policy is represented and evaluated in the Smets-Wouters DSGE model [2]. In particular, is monetary policy evaluated on the basis of Pareto optimality or other social welfare criterion derived from household preferences?

NOTES FOR Q3:

Form of Monetary Policy in [2]:

Smets-Wouters assume [2, pp. 13-14] that government uses a monetary policy *rule* ("reaction function") taking the particular form (36). As the authors note, (36) is a "generalized Taylor rule" that dictates that bond gross interest rates $\{\hat{R}_t\}$ for their DSGE model linearized around the non-stochastic steady state should be set in response to two types of gaps: (i) deviations of lagged inflation from an inflation objective (normalized to zero); and (ii) the lagged output gap, where the "output gap" corresponds to what we have been referring to in class as the "real GDP gap."

Evaluation of Monetary Policy in [2]:

Interestingly, despite the efforts by Smets-Wouters [2] to explicitly include within their DSGE model the preferences of households in the form of lifetime utility functions, and then to estimate the resulting DSGE model, they do not evaluate their monetary policy rule on the basis of Pareto optimality or other social welfare criteria derived from estimated household preferences. See, in particular, [2, footnote 2, p. 2].

Instead, Smets-Wouters [2, p. 14] justify the form of their monetary policy rule by appeal to empirical studies by other authors in which a Taylor rule specification was claimed to be able to capture the behavior of average interest rates in the euro area and the behavior of central banks for Germany, France, and Italy prior to the formation of the European Central Bank in 1998.

In Section 5 (p. 27), the authors note that – in Michael Woodford's "canonical" New Keynesian DSGE model developed in 1999 – "the output gap or the real interest rate gap, both defined as deviations from their flexible price levels, are useful indicators for optimal monetary policy." However, the authors then go on to say how some of the modifications they introduced into their DSGE model (e.g., nominal rigidities in *both* prices *and* wages, and shocks arising from sources other than technology and preferences) invalidate this conclusion for their model.

The authors conclude (p. 32): "In this paper we have not analysed optimal monetary policy. A deeper analysis of the appropriate welfare function and the various trade-offs faced by the monetary authorities in the context of this model would be very welcome."