ANSWER OUTLINE

MIDTERM EXAM: 70 POINTS   **L. Tesfatsion/Econ 502/Fall 2014**

IMPORTANT: Please answer **all three** questions included on this exam in accordance with the following instructions:

(a) ***Read each question with care BEFORE you begin your answer***.

(b) *Answer Packet:* Use the provided paper packet for all answers and scratch work.

(c) *Clarity:* Define terms and concepts clearly, and carefully label all graphs.

(d) *Accuracy:* Carefully check computations for calculational errors.

(e) *Support:* Justify carefully all assertions that you make.

(f) *Completeness:* Be sure you answer all required parts for each question.

(g) *Time Management:* Allocate your time with care so you can finish the exam.

QUESTION 1: [24 Points Total]

Consider a one-period economy E consisting of a potato-producing corporate firm F^{PO}, a ham-producing corporate farm F^H, a ketchup-producing corporate firm F^K, and two consumers $C1$ and $C2$. Each firm uses labor services supplied by the consumers to produce its output by means of a production function.

The two consumers have identical endowments at the beginning of the period as well as identical preferences for the consumption of goods. More precisely, each consumer C1 and C2 has: a money endowment $M^* > 0$; a labor endowment $L^* > 0$, measured in person-hours; 50% of the stock shares of each firm F^{PO}, F^H, and F^K; and a utility function of the form $U(po, h, k, le)$, where po denotes potato consumption, h denotes ham consumption, k denotes ketchup consumption, and le denotes leisure (measured in person-hours).

- **Q1.A (12 Points)** Provide a carefully labeled and carefully explained figure that depicts in a clear manner how economy E can be structurally represented as a Walrasian General Equilibrium (WGE) model. Call your resulting WGE model the *WGE Hash Economy Model*.

- **Q1.B (12 Points)** The standard definition of a *Walrasian equilibrium* consists of various properties that must be satisfied by a solution for a WGE model in order for this solution to be called an equilibrium. Explain carefully what form these properties would take *specifically for the WGE Hash Economy Model in Q1.A*. In other words, provide a definition for a Walrasian equilibrium *specifically tailored to the WGE Hash Economy Model*; do *not* simply given the definition of a Walrasian equilibrium stated in general terms.

Answer Outline for Q1.A:

See Section 3 and Figure 1 in the required Packet 4 for Section I.C of Econ 502.

A satisfactory answer will include a figure along the lines of Figure 1 (Packet 4) that is *tailored specifically to the WGE Hash Economy Model*, together with an accompanying verbal explanation of this figure at the level of detail seen in the WGE illustration given in Section 3 (Packet 4).

Answer Outline for Q1.B:

See Section 3 in the required Packet 4 for Section I.C of Econ 502.

A satisfactory answer will include a definition for a Walrasian equilibrium that includes a description of the following four defining properties (individual optimality, fulfilled expectations, market clearing, and Walras' Law) at the level of detail provided in Section 3 (Packet 4) *but tailored specifically to the WGE Hash Economy Model.*

In particular, instead of detailing the four properties for a general consumer i and two firms X and Y, the properties should instead be stated for the WGE Hash Economy Model consisting of the two consumers C1 and C2 and the three firms F^{PO}, F^H, and F^K.

QUESTION 2: [18 Points Total]

The assumptions underlying the standard Walrasian General Equilibrium (WGE) Model (Packet 4) and the Dynamic Sticky-Price IS-LM Model (Packet 7) have both important similarities and important differences.

- **Q2.A (9 points)** Describe *three important assumptions* that are *shared* by these two models.

- **Q2.B (9 points)** Describe *three important ways* in which the assumptions of these two models *differ*.

Answer Outline for Q2.A:

All clearly expressed answers describing at least three important common assumptions will receive full credit for Q2.A.

Some Important Common Assumptions:

(1) Both models assume the implicit presence of a central "clearing house" that determines prices so that all product markets are continuously cleared (supply=demand). In particular, no out-of-equilibrium trades are allowed in product markets.

(2) Both models assume that consumers and firms take prices as given – they are "price takers" rather than "price setters."

(3) Both models assume that there are no strategic (game) interactions among consumers and firms. Consumers and firms make supply and demand decisions with no concern for the actions that other agents either have already taken or will take in the future.

(4) Both models assume that consumers own all capital and provide both labor and capital services to firms as inputs to production.

(5) Neither model explains the process by which agents form their expectations.

Answer Outline for Q2.B:

All clearly expressed answers describing at least three important differing assumptions will receive full credit for Q2.B.

Some Important Differing Assumptions:

(1) The Dynamic Sticky-Price IS-LM Model directly specifies demand and supply functions for goods, services, and assets, whereas the standard WGE Model derives demand/supply functions as the "first order conditions" for explicitly expressed optimization problems. In particular, consumers in the WGE model are assumed to maximize utility subject to budget constraints and firms in the WGE model are assumed

to maximize profits subject to technological feasibility restrictions. In this sense, the standard WGE model provides a more detailed "microfoundations" for postulated economic relationships.

(2) Related to (1), the Dynamic Sticky-Price IS-LM Model does not take into account underlying budget constraints that restrict the decision-making of each consumer and hence result in cross-restrictions on a consumer's demand and supply functions. In contrast, since the WGE model derives the demand and supply functions for each consumer from a single logically-coherent optimization problem subject to budget constraints, these cross-restrictions are properly accounted for. (A similar point can be made for firm decision-making in the two models).

(3) The Dynamic Sticky-Price IS-LM Model assumes that the general price level adjusts with a one-period lag. This permits the labor market to be in disequilibrium, in the sense that actual employment can deviate from potential employment (hence actual real GDP can deviate from potential GDP). In contrast, the standard WGE model assumes that the economy is in a Walrasian equilibrium in which all markets clear.

(4) As presented in Packet 4, the standard WGE model is a static one-period model with no government or foreign sector. In contrast, the Dynamic Sticky-Price IS-LM model represents a dynamic economy, changing over time, that includes at least some consideration for the the role of government (through the inclusion of fiscal and monetary policy variables) and the role of foreign trade (through inclusion of net exports).

Remarks (Not Required): In fact, the standard WGE model can be extended to a dynamic form in which consumers maximize intertemporal lifetime utility functions subject to intertemporal budget constraints and firms maximize intertemporal profit functions subject to intertemporal technological feasibility constraints. (Indeed, DSGE modeling to be covered in Section V of the course is a dynamic variant of WGE modeling.) The standard WGE model can also be extended to include a government interested in determining fiscal and/or monetary policies to achieve a social welfare objective.

The one-period WGE model presented in Packet 4 is deliberately given in simplified one-period closed form in order to facilitate understanding of the key assumptions characterizing WGE modeling.

QUESTION 3: [28 Points Total]

Endogenous growth theorists such as Paul Romer began the study of "endogenous growth" in the 1980s using a modification of the basic Solow-Swan descriptive growth model known as the *AK Model.*

In the AK Model, K represents a composite capital stock that combines both human capital (e.g., embodied labor skills) and physical capital. Consider the following simple version of the AK Model:

AK MODEL EQUATIONS: For each $t \geq 0$,

[1] $Y(t) \; = \; AK(t)$

[2] $S(t) \; = \; s[Y(t) - \delta K(t)]$

[3] $D_+K(t) \; = \; S(t)$

AK MODEL CLASSIFICATION OF VARIABLES:

Time-t Endogenous Variables ($t \geq 0$):
   $Y(t), \quad S(t), \quad D_+K(t)$

Time-t Predetermined (State) Variable ($t > 0$):
   $K(t) = \int_0^t D_+K(\tau)d\tau \; + \; K(0)$

Admissible Exogenous Variables and Functional Forms:

   Initial capital $K(0)$, total factor productivity A, savings propensity s, and capital depreciation rate δ satisfying the admissibility conditions $0 < K(0)$, $0 < s < 1$, and $0 < \delta < A$, and a linear production function $AF(K) = AK$.

Q3:A (12 Points) Provide a separate *economic* interpretion for each of the three equations [1] through [3] for the AK Model.

   Answer Outline for Q3:A

   [1] By definition, a production function gives the maximum possible output for each possible combination of inputs. Equation [1] thus postulates that the economy in the aggregate is producing efficiently on its production possibility frontier (no wasted capital inputs). In addition, since only one capital variable $K(t)$ is used in the model, an implicit assumption is being made that all available capital is fully utilized in production, or in other words, the capital input demanded by firms in production is equal to the total available supply of capital input.

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[2] Equation [2] postulates that net savings $S(t)$ is a fixed proportion s of net income $[Y(t) - \delta K(t)]$. In addition, since only one savings variable $S(t)$ is used in the model, an implicit assumption is being made that planned net savings is equal to realized net savings.

[3] Equation [3] sets net investment $D_+ K$ equal to net savings $S(t)$. This reduced-form relation combines both the national income accounting identity (*realized* net investment equals *realized* net savings) and the equilibrium postulate that *planned* net investment equals *planned* net savings.

Remark (Not Required):

If an additional equation [4] were added for planned/realized consumption $C(t)$ in each period t, of the form $C(t) = [1 - s][Y(t) - \delta K(t)]$, and an additional equation [5] were added for planned/realized gross investment $I(t)$ in each period t, of the form $I(t) = D_+ K(t) + \delta K(t)$, it would follow from [1]-[5] that $Y(t) = C(t) + I(t)$.

Q3:B (4 Points) Derive explicit mathematical expressions for the growth rates of capital $K(t)$ and output $Y(t)$ in the AK Model at an arbitrary time point t. Show all steps of your derivations.

Answer Outline for Q3:B Using [1] through [3], for any $t > 0$,

$$D_+ K(t) \;=\; s[AK(t) - \delta K(t)]; \tag{1}$$

$$D_+ Y(t) \;=\; A \cdot D_+ K(t) \ . \tag{2}$$

Consequently, letting g_K denote the growth rate of capital and g_Y denote the growth rate of output, it follows from (1) and (2) that

$$g_K(A, s, \delta) \;\equiv\; \frac{D_+ K(t)}{K(t)} \;=\; \frac{s[AK(t) - \delta K(t)]}{K(t)} \;=\; s[A - \delta] \ ; \tag{3}$$

$$g_Y(A, s, \delta) \;\equiv\; \frac{D_+ Y(t)}{Y(t)} \;=\; \frac{D_+ K(t)}{K(t)} \;=\; s[A - \delta] \ . \tag{4}$$

Q3:C (2 Points) Using your results from Q3:B, show that a change in the savings rate s affects the growth rates for $K(t)$ and $Y(t)$ in the AK Model at any time point t, and that this effect does *not* vanish "in the long run" (i.e., as time t goes to infinity).

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Answer Outline for Q3:C

It follows from (3) and (4) that, in the AK Model, the growth rates g_K and g_Y for capital and output are each equal to $s[A - \delta]$ for each $t > 0$. Since $A > \delta$ by admissibility, this constant growth rate depends non-trivially on the savings rate s:

$$\frac{\partial g_K}{\partial s}(A, s, \delta) = \frac{\partial g_Y}{\partial s}(A, s, \delta) = [A - \delta] > 0 \ . \tag{5}$$

Consequently, the savings rate s is an important determinant of the common growth rate for capital and output even in the "long run" as t approaches infinity.

Q3:D (10 Points) An important finding for the basic Solow-Swan descriptive growth model presented in Packet 16 — hereafter referred to as the SS Model — is that the long-run growth rates for capital $K(t)$ and output $Y(t)$ do *not* depend on the savings rate s. This SS Model savings result is in striking contrast to the AK Model savings result derived in Q3:C.

What differences in the assumptions underlying the SS Model and the AK Model explain these different savings results? Justify your assertions carefully, making use of any needed results from Packet 16 as appropriate.

Important Caution: You do *not* need to re-prove any of the results established in Packet 16 for the SS Model; you can simply use these results without re-proving them.

Answer Outline for Q3:D

The AK Model incorporates three major differences in assumptions relative to the SS Model in Packet 16:

(i) Capital K(t) in the SS Model includes only physical capital, whereas K(t) in the AK Model combines physical and human capital;

(ii) The SS Model production function $AF(K, L)$ exhibits diminishing returns to capital, i.e., a diminishing marginal product of capital ($AF_K > 0$ but $AF_{KK} < 0$) whereas the AK Model production function $AF(K) = AK$ is linear in K and hence does *not* exhibit diminishing returns to capital ($AF_K = A > 0$, $AF_{KK} = 0$).

(iii) Labor $L(t)$ in the SS Model grows at an exogenously given rate $g > 0$, which puts a limit on the rate at which capital can productively grow because of diminishing returns to capital. In contrast, the composite capital good $K(t)$ in the AK Model can grow without bounds with no diminishing returns to capital.

More precisely, property (ii) implies for the SS Model that $f''(k) < 0$, where $k = K/L$ and $f(k) \equiv AF(k, 1) = AF(K, L)/L$. If physical capital grows faster than labor, the capital/labor ratio k is increasing and the productivity of physical capital is falling; if physical capital grows more slowly than labor, the capital/labor ratio k is decreasing the the productivity of physical capital is increasing. SS Model admissible solutions

are thus all driven in the long-run to a constant capital/labor ratio $\bar{k}$, implying that both capital $K(t)$ and output $Y(t)$ must grow at the exogenously determined growth rate g for labor in the long run.

The AK Model breaks diminishing returns to physical capital by assuming a "composite" capital stock $K(t)$ that combines both physical and human capital, and by assuming that the growth rate of this composite capital stock is endogenously determined by the amount of investment (savings). That is, human capital is now presumed to grow along with physical capital through investments in job training, education, and so forth. There is no longer an exogenously determined growth rate g for labor that forces diminishing returns to (physical) capital to kick in when the growth of (physical) capital exceeds g.