ANSWER OUTLINE

MIDTERM EXAM: 70 POINTS TOTAL **L. Tesfatsion/Econ 502/Fall 2012**

NOTE: Please answer **all three** questions included on this test. In answering these three questions, be sure to:

(a) use the provided answer packet for all answers (include all scratch work);

(b) read each question carefully before you begin your answer;

(c) define terms and concepts clearly;

(d) carefully label all graphs;

(e) justify carefully all assertions that you make;

(f) watch the time – plan to leave yourself some extra time at the end to check over your answers.

QUESTION 1: [20 Points Total]
All parts below refer to the Walrasian general equilibrium model as presented in Packet 4.

Part A (7 Points) Consider a Walrasian general equilibrium model with n consumers and with two firms X and Y producing two distinct goods. Provide a *carefully labeled graphical depiction* of this model that clearly displays the type of "circular flow" assumed for this model, i.e., the manner in which goods/services, payments, and information are passed between firms and consumers.

Answer Outline for Q1:Part A (See Figure 1 in Packet 4.)

Key aspects that should be included in the carefully labeled graphical depiction are:

- constrained utility/profit maximization by individual consumers/firms, taking (expected) prices/wages/dividends as given;
- consumer/firm derivation of optimal price/wage/dividend-conditioned demand and supply curves for goods/services;
- role of the fictitious centralized clearing house ("Walrasian Auctioneer") in ensuring that a price/wage/dividend vector is found under which supply is at least as great as demand in each market.
- Circular flow lines indicating the direction of flow for physical final goods/services, physical intermediate goods/services, financial payments, and information on prices/wages/dividends.

Part B (8 Points) Explain *in words* what is meant by a *Walrasian equilibrium* for the particular Walrasian general equilibrium model graphically depicted in Part A. That is, explain verbally the basic types of conditions that must hold within this model in order for the definition of a Walrasian equilibrium to be satisfied.

Note: Here you are NOT being asked to discuss the nine assumptions that provide the defining structural characteristics of a Walrasian general equilibrium model – you are being asked to discuss the *additional* conditions characterizing a Walrasian *equilibrium* within this model structure.

Answer Outline for Q1:Part B (See Packet 4.) A Walrasian equilibrium is a specification of values for consumer and firm supplies and demands, expected prices and actual prices, and expected dividend payments, such that the following conditions hold for these values: *Individual Optimization:* all firms and consumers are on their supply and demand curves; *Fulfilled Expectations:* specifically, agents have correct expectations for prices and dividend payments;*Market Clearing:* supply is at least as great as demand in each market; and *Walras Law:* the total value of excess supply is zero.

Part C (5 Points) Explain briefly but carefully why Walrasian equilibrium has traditionally been viewed within economics as a benchmark of coordination success for decentralized market economies.

Answer Outline for Q1:Part C As seen in Q1:Part B, by definition, in every Walrasian equilibrium every agent is optimizing, conditional on their given information, every expectation is fulfilled, and all markets clear.

In addition, assuming consumers have strictly increasing utility (no satiation), the famous *First Welfare Theorem* guarantees that every Walrasian equilibrium is Pareto efficient (i.e., that the allocation of goods and services resulting from this equilibrium is a Pareto efficient allocation).

For these reasons, Walrasian equilibrium has traditionally been viewed within economics as a benchmark of coordination success for decentralized market economies.

QUESTION 2: [15 Points Total]

Q2: Part A (4 Points) Give the analytical form of the *expectations augmented Phillips curve* as developed in class lecture materials.

Answer Outline for Q2:Part A (See Packet 7, Equation (19))

Let $\pi(T, T+1)$ denote the inflation rate from period T to T+1, $\pi^e(T, T+1)$ denote the expected inflation rate from period T to T+1, Y(T) denote real GDP for period T, Y^*(T) denote potential real GDP for period T, and f denote some positive constant. Then the expectations augmented Phillips Curve for period T takes the form

$$(EAPC) \qquad \pi(T+1) \ = \pi^e(T, T+1) + f \cdot [Y(T) - Y^*(T)]/Y^*(T)$$

Q2: Part B (8 Points) Explain briefly but clearly, using a carefully labeled graphical depiction for illustration, how the expectations augmented Phillips curve implies the existence of a *family of short-run Phillips curves* together with a *long-run Phillips curve*.

Answer Outline for Q2:Part B (See Packet 7, Section D.)

The bracketed term on the right-hand side of (EAPC) is called the *real GDP gap*. As explained in Packet 7, graphed in a plane with the real GDP gap on the horizontal axis and the inflation rate π on the vertical axis, relation (EAPC) is an upward sloping line with its π-intercept equal to the expected inflation rate. This is interpreted as a *short-run Phillips curve* because it predicts that π and the real GDP gap will vary directly with each other "in the short run" for a given expected inflation rate.

However, in the longer run, the idea is that the expected inflation rate will tend to converge to the actual inflation rate as people adjust their expectations to their observations. Consequently, the only possible long-run position is where $\pi = \pi^e$. However, $\pi = \pi^e$ only holds along the vertical axis corresponding to a zero real GDP gap, hence this vertical axis is called the *long-run Phillips curve*.

A carefully labeled graphical depiction of these relationships should be provided.

Q2: Part C (3 Points) Discuss, briefly, one key macroeconomic policy implication that follows from the existence of these short-run/long-run Phillips curves that was not implied by the traditional (original) Phillips curve relation.

Answer Outline for Q2:Part C (See Packet Reading 7, Section D.)

Packet 7, Section D, highlights a number of different key macroeconomic policy implications of (EACP) that could be discussed here.

For example, one key implication is that a zero real GDP gap is consistent with *any* inflation rate as long as this inflation rate is correctly expected. The traditional Phillips curve construction implied that government could control the inflation rate through the real GDP gap – in particular, that government could ensure a zero inflation rate by keeping the real GDP gap at zero.

Another key implication is that stagflation (rising inflation together with rising unemployment) is not ruled out, as it was by the original Phillips curve construction. Hence, (EACP) is more consistent with empirical episodes (e.g., the U.S. in the 1970s) during which stagflation was observed.

QUESTION 3: [35 Points Total] Appended at the end of Q3 is a version of the basic Solow-Swan descriptive growth model with labor-augmenting technological change, hereafter referred to as **Model M**. Also appended is the per-capita version of this model, hereafter referred to as **Model M***.

*****Important:*** All parts of Q3 should be answered using Model M and Model M* appended at the end of this packet.**

Q3: Part A (6 Points) Carefully show, step by step, how the three equations for Model M* can be derived from the six equations for Model M using the following definitions that link the two models: $\hat{k}(t) = K(t)/N(t)$; $\hat{y}(t) = Y(t)/N(t)$; $\hat{s}(t) = S(t)/N(t)$; and $f(\hat{k}) = F(\hat{k}, 1)$.

Important Note: You are not being asked to derive the admissibility conditions for Model M*, just the three equations for Model M*.

Answer Outline for Q3:Part A (See Packet Reading 17, Section 4)

First note that the admissibility conditions for Model M imply that $B(0) > 0$ and $L(0) > 0$, hence $N(0) = B(0)L(0) > 0$. It can then be shown either by direct solution[1] of equations (25) and (26), or by a forward recursion argument, that $N(t) = B(t)L(t) > 0$ for all $t \geq 0$.

Now let $t \geq 0$ by given. Divide the first three equations of Model M by $N(t)$, and use constant returns to scale for $F(K, N)$, to get

$$\begin{aligned} \hat{y}(t) &= Af(\hat{k}(t)) && (1) \\ \hat{s}(t) &= s[Af(\hat{k}(t)) - \delta\hat{k}(t)] && (2) \\ D_+K(t)/N(t) &= \hat{s(t)} && (3) \end{aligned}$$

Using equations (25), (26), and (27) of Model M, it follows that

$$\begin{aligned} \frac{D_+N(t)}{N(t)} &= \frac{D_+[B(t)L(t)]}{B(t)L(t)} && (4) \\ &= \frac{D_+B(t)}{B(t)} + \frac{D_+L(t)}{L(t)} \\ &= \mu + g \ . \end{aligned}$$

Thus,

$$\begin{aligned} \frac{D_+\hat{k}(t)}{\hat{k}(t)} &= \frac{D_+K(t)}{K(t)} - \mu - g && (5) \\ &= \frac{[\frac{D_+K(t)}{N(t)} - \mu\hat{k}(t) - g\hat{k}(t)]}{\hat{k}(t)} \end{aligned}$$

[1] From Packet 15 (p. 7) on differential equations, for any linear homogenous differential equation of the form $D_+x(t) = ax(t)$ with initial condition $x(0) = u$, the solution is given by $x(t) = ue^{at}$. Consequently, if $u > 0$, it follows that $x(t) > 0$ for all $t \geq 0$.

Hence,

$$D_+ K(t)/N(t) \;=\; D_+\hat{k}(t) + [\mu + g]\hat{k}(t) \tag{6}$$

Substituting equation (6) into equation (3), using equation (2), and manipulating terms, one gets

$$D_+\hat{k}(t) \;=\; sAf(\hat{k}(t)) - \theta\hat{k}(t) \tag{7}$$

where

$$\theta \;=\; [\mu + g + s\delta] \tag{8}$$

Equations (1), (2), and (7) are the three equations of Model M*, as desired.

Q3: Part B (10 Points) Establish that Model M* has a unique admissible stationary solution $\hat{k}^* > 0$, given any admissible specification for $(A, s, \delta, \theta, f(\hat{k}))$. Justify your assertions with care, and illustrate your assertions using a carefully labeled graph.

Answer Outline for Q3:Part B (See Packet Reading 17, Section 3)

Let an admissible specification for $(A, s, \delta, \theta, f(\hat{k}))$ in Model M* be given. Given this specification, $\hat{k}^* > 0$ is an admissible stationary solution for Model M* if and only if $\hat{k}^*$ satisfies

$$0 \;=\; sAf(\hat{k}^*) - \theta\hat{k}^* \tag{9}$$

Define two functions of $\hat{k}$ as follows: $x(\hat{k}) = sAf(\hat{k})$ and $z(\hat{k}) = \theta\hat{k}$. Then $\hat{k}^* > 0$ is an admissible stationary solution for Model M* if and only if $x(\hat{k})$ and $z(\hat{k})$ intersect at $\hat{k} = \hat{k}^*$.

The admissibility conditions for Model M* imply that $s > 0$, $A > 0$, and $\theta > 0$. They also imply that $x(0) = 0$, $x'(\hat{k}) = sAf'(\hat{k}) > 0$, $x''(\hat{k}) = sAf''(\hat{k}) < 0$, $x'(\hat{k})$ approaches $+\infty$ as $\hat{k}$ approaches 0, and $x'(\hat{k})$ approaches 0 as $\hat{k}$ approaches $+\infty$. It follows that $x(\hat{k})$ and $z(\hat{k})$ have one and only one intersection point $\hat{k}^*$ over the admissible range $\hat{k} > 0$. This should be illustrated using a carefully labeled graph; see, for example, Figure 4 in Packet 17.

Q3: Part C (10 Points) Let an admissible specification $(A, s, \delta, \theta, f(\hat{k}))$ for Model M* be given. Let $\hat{k}^* > 0$ denote the unique admissible stationary solution for Model M* corresponding to this admissible specification, whose existence you established in Q3:Part B. Use a clearly explained *graphical* analysis to establish the *global stability* of $\hat{k}^*$ relative to the family of all possible admissible solutions for Model M* conditional on the admissible specification $(A, s, \delta, \theta, f(\hat{k}))$. Justify your assertions with care.

Answer Outline for Q3:Part C (See Packet Reading 17, Section 3)

Define a function $\psi{:}R \to R$ by $\psi(\hat{k}) = sAf(\hat{k}) - \theta\hat{k}$. The admissibility conditions imposed on $f(\hat{k})$ in Model M* then imply that

$$\psi(0) \;=\; \psi(\hat{k}^*) \;=\; 0 \tag{10}$$

$$\psi'(\hat{k}) \;=\; sAf'(\hat{k}) \;-\; \theta \tag{11}$$

$$
\begin{aligned}
\psi'(0) &= +\infty && (12)\\
\psi''(\hat{k}) &= sAf''(\hat{k}) < 0 && (13)\\
\psi'(k^*) &< \frac{[\psi(\hat{k}^*) - \psi(0)]}{[\hat{k} - 0]} = 0 && (14)
\end{aligned}
$$

The graph of $\psi(\hat{k})$ in the R^2 plane should be graphically depicted, and the "phase diagram" arrows indicating the direction of motion in $\hat{k}$ at each admissible point $(\hat{k}, \psi(\hat{k}))$ should be indicated; see, for example, Figure 6 in Packet 17.

It is then seen that, starting at any admissible initial state value $\hat{k} > 0$, the direction of motion in $\hat{k}$ is always towards the unique stationary solution $\hat{k}^*$. It follows that $\hat{k}^*$ is globally stable relative to the family of all admissible solutions for Model M*, conditional on the admissible specification $(A, s, \delta, \theta, f(\hat{k}))$.

Q3: Part D (9 Points) Using your findings from Parts Q.3:A through Q3:C, explain how each of the following three stylized facts is satisfied *in the long run* by any admissible solution for Model M.

[**SF1**] The ratio K/Y of physical capital K to output Y is nearly constant over time.

[**SF2**] Per capita output $y = Y/L$ grows over time without a tendency to converge to a constant value.

[**SF3**] Per-capita physical capital $k = K/L$ grows over time without a tendency to converge to a constant value.

Answer Outline for Q3:Part D (See Packet Reading 17, Section 4)

Let an admissible specification $(K(0), B(0), L(0), A, s, \delta, \mu, g, F(K, N)$ for Model M be given, which implies an admissible specification $(\hat{k}(0), A, s, \delta, \mu, g, f(\hat{k}))$ for Model M*. By Q3:Part C,

$$\frac{K(t)}{N(t)} = \hat{k}(t) \rightarrow \hat{k}^* \text{ as } t \rightarrow +\infty \tag{15}$$

It follows that

$$\frac{K(t)}{Y(t)} = \frac{\hat{k}(t)}{\hat{y}(t)} = \frac{\hat{k}(t)}{f(\hat{k}(t))} \rightarrow \frac{\hat{k}^*}{f(\hat{k}^*)} \text{ as } t \rightarrow +\infty \tag{16}$$

Thus, [SF1] holds in the long run for any Model M admissible solution.

Another implication of (15) is that

$$\hat{y}(t) = \frac{Y(t)}{N(t)} = \frac{AF(K(t), N(t))}{N(t)} = Af(\hat{k}) \rightarrow Af(\hat{k}^*) \text{ as } t \rightarrow +\infty \tag{17}$$

Using (17) together with (4), it follows that

$$\begin{aligned}\frac{D_+\hat{y}(t)}{\hat{y}(t)} &= [\tfrac{D_+Y(t)}{Y(t)} - \tfrac{D_+N(t)}{N(t)}] \\ &= [\tfrac{D_+Y(t)}{Y(t)} - \mu - g] \\ &\to 0 \text{ as } t \to +\infty\end{aligned} \tag{18}$$

Note also that (15), together with (4), implies

$$\begin{aligned}\frac{D_+\hat{k}(t)}{\hat{k}(t)} &= [\tfrac{D_+K(t)}{K(t)} - \tfrac{D_+N(t)}{N(t)}] \\ &= [\tfrac{D_+K(t)}{K(t)} - \mu - g] \\ &\to 0 \text{ as } t \to +\infty\end{aligned} \tag{19}$$

Let $y(t) = Y(t)/L(t)$ and $k(t) = K(t)/L(t)$ denote per-capita output and per-capita capital calculated in terms of *raw* labor $L(t)$. It then follows from (18) that

$$\lim_{t\to\infty} D_+y(t)/y(t) \ = \ [\lim_{t\to\infty} D_+Y(t)/Y(t) - \lim_{t\to\infty} D_+L(t)/L(t)] \ = \ \mu \ > \ 0 \tag{20}$$

Similarly, it follows from (19) that

$$\lim_{t\to\infty} D_+k(t)/k(t) \ = \ [\lim_{t\to\infty} D_+K(t)/K(t) - \lim_{t\to\infty} D_+L(t)/L(t)] \ = \ \mu \ > \ 0 \tag{21}$$

Consequently, [SF2] and [SF3] also hold in the long run for any Model M admissible solution.

MODEL M
SOLOW-SWAN DESCRIPTIVE GROWTH MODEL WITH LABOR-AUGMENTING TECHNOLOGICAL CHANGE

MODEL M EQUATIONS: $t \geq 0$:

$$\begin{aligned} Y(t) &= AF(K(t), N(t)) && (22) \\ S(t) &= s \cdot [Y(t) - \delta K(t)] && (23) \\ D_+K(t) &= S(t) && (24) \\ D_+B(t) &= \mu \cdot B(t) && (25) \\ D_+L(t) &= g \cdot L(t) && (26) \\ N(t) &= B(t)L(t) && (27) \end{aligned}$$

MODEL M CLASSIFICATION OF VARIABLES:

Time-t Endogenous Variables $(t \geq 0): \ Y(t),\ S(t),\ D_+K(t),\ D_+B(t),\ D_+L(t),\ N(t)$

Time-t Predetermined Variables ($t > 0$):

$$\begin{aligned} K(t) &= \int_0^t D_+K(\tau)d\tau \ + \ K(0) && (28) \\ B(t) &= \int_0^t D_+B(\tau)d\tau \ + \ B(0) && (29) \\ L(t) &= \int_0^t D_+L(\tau)d\tau \ + \ L(0) && (30) \end{aligned}$$

Admissible Exogenous Variables and Functional Forms:

$K(0)$, $B(0)$, $L(0)$, A, s, δ, μ, and g, satisfying $0 < K(0)$, $0 < B(0)$, $0 < L(0)$, $0 < A$, $0 < s < 1$, $0 \leq \delta$, $0 < \mu$, and $0 < g$, plus a function $F(K, N)$ that satisfies the following *Standard Neoclassical Production Function Assumptions in Level Form*:

a. $F(K, N)$ exhibits constant returns to scale;

b. $F(K, N)$ is continuous over $(K, N) \geq 0$;

c. $F(K, N)$ is twice continuously differentiable and concave, with $F_{KK}(K, N) < 0$, over all $(K, N) > 0$;

d. $F_K(K, N) > 0$ and $F_L(K, N) > 0$ for all $(K, N) > 0$;

e. $F(0, N) = 0$ for all $N \geq 0$;

f. [*Inada Conditions*] For each $N > 0$, $F_K(K, N) \to +\infty$ as $K \to 0$ and $F_K(K, N) \to 0$ as $K \to +\infty$.

MODEL M*
PER-CAPITA VERSION OF MODEL M

MODEL M* EQUATIONS: For each time $t \geq 0$,

$$\hat{y}(t) = Af(\hat{k}(t)) \; ; \tag{31}$$

$$\hat{s}(t) = s[\hat{y}(t) - \delta\hat{k}(t)] \; ; \tag{32}$$

$$D_+\hat{k}(t) = sAf(\hat{k}(t)) - \theta\hat{k}(t) \; . \tag{33}$$

MODEL M* CLASSIFICATION OF VARIABLES:

Time-t Endogenous Variables ($t \geq 0$): $\hat{y}(t)$, $\hat{s}(t)$, $D_+\hat{k}(t)$;

Time-t Predetermined Variable ($t > 0$):

$$\hat{k}(t) = \int_0^t D_+\hat{k}(\tau)d\tau + \hat{k}(0) \tag{34}$$

Admissible Exogenous Variables and Functional Forms:

$\hat{k}(0)$, A, s, δ, and $\theta = [\mu + g + s\delta]$, where $0 < \hat{k}(0)$, $0 < A$, $0 < s < 1$, $0 \leq \delta$, $0 < \mu$, and $0 < g$.

Also, $f(\hat{k}) \equiv F(\hat{k}, 1)$ satisfies the following *Standard Neoclassical Production Function Assumptions in Per-Capita Form*: $f(\hat{k})$ is continuous over $\hat{k} \geq 0$, and $f(\hat{k})$ is twice continuously differentiable with $f'(\hat{k}) > 0$ and $f''(\hat{k}) < 0$ over $\hat{k} > 0$; $f(0) = 0$; and $f'(\hat{k}) \to +\infty$ as $\hat{k} \to 0$ and $f'(\hat{k}) \to 0$ as $\hat{k} \to +\infty$.