

## ANSWER OUTLINE

**FINAL EXAM: 100 POINTS TOTAL**

**L. Tesfatsion/Econ 502/Fall 2008**

**\*\*\*Important Note:\*\*\*** Please answer **ALL FOUR** questions in this packet.

In answering these four questions, please do the following:

- (a) Use the provided answer packet for all answers (include all scratch work).
- (b) Read each question carefully before you begin your answer.
- (c) Define terms and concepts clearly.
- (d) Carefully label all graphs.
- (e) Justify your assertions as carefully as time permits.
- (f) **Watch the time – plan, roughly, to allocate 1 minute for each point. This should give you about 20 remaining minutes to check back over your answers.**

## QUESTION 1: SHORT QUESTIONS [12 Points Total, About 12 Minutes]

**Part Q1.A (4 Points):** The required readings for *Discussion Group 2 (DG2)* discussed two starkly different approaches to human development that have been attempted in the past, referred to as the “direct” and “indirect” approaches. Briefly but carefully explain the distinction between these two approaches.

### Answer Outline for Part Q1.A:

As explained in the handed-out assignment for DG2, the *direct approach* to human development stresses the direct improvement of human welfare through increased spending on social services (medical services, education,...), technical support (e.g., job training), and direct cash transfers. The *indirect approach* to human development (associated with the World Bank) is a growth-oriented approach that targets investment in infrastructure improvements (e.g., dams, airports, power plants, etc.) as well as institutional reforms (e.g., privatization, deregulation, and trade liberalization).

**Part Q1.B (4 Points):** Give brief but careful definitions for the following key concepts that appeared in the required reading materials for *Discussion Group 3 (DG3)* and that were defined by the DG3 moderators in their hand-out: (i) mortgage-backed security; and (ii) moral hazard.

### Answer Outline for Part Q1.B:

The following definitions from Wikipedia were given by the DG3 moderators in their hand-out:

A *mortgage-backed security (MBS)* is an asset-backed security whose cash flows are backed by the principal and interest payments of a set of mortgage loans.

*Moral hazard* is the prospect that a party insulated from risk may behave differently from the way it would behave if it were fully exposed to the risk. Moral hazard arises because an individual or institution does not bear the full consequences of its actions, and therefore has a tendency to act less carefully than it otherwise would, leaving another party to bear some responsibility for the consequences of those actions. For example, an individual with insurance against automobile theft may be less vigilant about locking his or her car, because the negative consequences of automobile theft are (partially) borne by the insurance company.

**Part Q1.C (4 Points):** As asked as part of the *Discussion Group 4 (DG4)* handed-out assignment, give a concrete example of a “Taylor Rule” with all its elements explained, and briefly discuss what the use of a Taylor Rule by a central bank is supposed to accomplish for an economy.

**Answer Outline for Part Q1.C:**

As presented in the DG4 hand-out and the DG4 required readings, the general form of a Taylor Rule (as formulated and used by John B. Taylor) is as follows:

$$[i - i^*] = \theta_\pi \cdot [\pi - \pi^*] + \theta_q \cdot [q - q^*], \quad (1)$$

where:  $\pi$  denotes the inflation rate measured as the rate of change in some appropriate index for the general price level (e.g., the GDP deflator);  $\pi^*$  denotes a target inflation rate;  $i$  denotes a short-term nominal interest rate;  $i^*$  denotes the “natural” short-term nominal interest rate; and  $[q - q^*]$  denotes the GDP gap measured in terms of the deviation of real output  $q$  (log form) from potential real output  $q^*$  (log form) measured as a linear growth trend line for real output  $q$  (log form).

The use of a Taylor rule by a central bank is supposed to achieve a satisfactory balance between price stability (relative to a target inflation rate) and output stability (relative to a potential output level).

**Remark:** Taylor also presented a more specific rule, known as the *classic Taylor Rule*, which assumes that  $i^*$  equals  $r^* + \pi$ , where the “natural” real interest rate  $r^*$  is taken to be 2, and each of the weight factors  $\theta_\pi$  and  $\theta_q$  is set equal to 0.5. The result is

$$i = 2 + \pi + 0.5 \cdot [\pi - 2] + 0.5 \cdot [q - q^*], \quad (2)$$

Any depiction of a Taylor rule that illustrates the general form (1), including the classic Taylor rule (2), is acceptable as an answer for Part Q1.C.

## QUESTION 2: [25 Points, About 25 Minutes]

**Part Q2.A (6 Points):** Provide an *economic interpretation* for the following optimal growth model  $M^*$  taken from Packet Reading 20 (“A Simple Illustrative Optimal Growth Model”):

$$\max_{\mathbf{c}, \mathbf{k}} \int_0^T u(c(t)) \exp(-\rho t) dt \quad (3)$$

subject to

$$\begin{aligned} c(t) &= f(k(t)) - \theta k(t) - D_+ k(t), 0 \leq t \leq T; \\ k(0) &= k_0; \\ k(T) &= k_T. \end{aligned}$$

**Caution:** Several different economic interpretations can be given for model  $M^*$ . You are only asked to provide *one* of these economic interpretations.

### Answer Outline for Part Q2.A:

See Packet Reading 20, Section A. There are two basic interpretations that can be given for Model  $M^*$ : (i) a social welfare planning problem subject to macro feasibility constraints and exogenously given initial and final boundary conditions on per-capita capital; or (ii) a representative consumer maximizing his intertemporal lifetime utility subject to macro feasibility constraints and exogenously given initial and final boundary conditions on per-capita capital.

**Part Q2.B(15 Points):** Let  $\mathbf{K}$  denote the collection of all continuously differentiable sequences  $\mathbf{k} = (k(t) : t \in [0, T])$  satisfying the boundary conditions  $k(0) = k_0$  and  $k(T) = k_T$ , as in (3). The following theorem is presented in Packet Reading 20:

**THEOREM 1:** Consider the optimal growth model  $M^*$  described by (3). Suppose the utility function  $u: R \rightarrow R$  is twice continuously differentiable with  $u' > 0$  and  $u'' < 0$ , and the production function  $f: R \rightarrow R$  is twice continuously differentiable with  $f' > 0$  and  $f'' < 0$ . Then, in order for a trajectory  $\mathbf{k}$  in  $\mathbf{K}$  to be the unique solution for the optimal growth problem (3), it is necessary and sufficient that  $\mathbf{k}$  solve the following differential system:

$$Dk(t) = f(k(t)) - \theta k(t) - c(t), \quad t \in [0, T]; \quad (4)$$

$$Dc(t) = -\frac{u'(c(t))}{u''(c(t))} [f'(k(t)) - \theta - \rho], \quad t \in [0, T]. \quad (5)$$

Provide a carefully labeled *phase diagram* that shows the general dynamic form of the solutions to the Euler-Lagrange system of equations (4) and (5) under the assumptions of Theorem 1 together with the *additional* assumption that  $\rho > 0$ . Be sure to identify on this phase diagram the following special features:

- (i) the unique stationary solution  $(\bar{k}, \bar{c})$  for this system of equations (together with a verbal definition for a *stationary solution* for Model M\*);
- (ii) the direction of motion in  $k$  and  $c$  starting at any point  $(k, c)$  in the phase diagram;
- (iii) the golden-rule point  $(\hat{k}, \hat{c})$  for the economy (together with a verbal definition of a *golden rule point* for Model M\*).

### Answer Outline for Part Q2.B:

See Packet Reading 20, Section D (in particular Figure 3).

**Part Q2.C(4 Points):** Explain briefly but carefully how a *decrease* in  $\rho$  to some smaller but still positive value affects the phase diagram you presented in Part Q2.B. Provide an *economic interpretation* of your findings.

### Answer Outline for Part Q2.C:

The key observation is that the discount parameter  $\rho$  enters equation (5) but not equation (4). Thus, on the phase diagram, only the locus of points corresponding to  $Dc = 0$  is affected. This locus of points is characterized by the following equation:

$$0 = [f'(k) - \theta - \rho] \quad (6)$$

By the Theorem 1 conditions on  $f(k)$ , a *smaller* value of  $\rho$  corresponds to a *larger* solution  $\bar{k}'$  for equation (6).

However, as long as  $\rho$  is positive,  $\bar{k}'$  will still be smaller than the golden-rule level of per-capita capital  $\hat{k}$ . Note that the golden-rule point  $(\hat{k}, \hat{c})$  is determined solely by equation (4), hence it is not affected by a change in  $\rho$ .

Consequently, the decrease in  $\rho$  leads to a new stationary solution  $(\bar{k}', \bar{c}')$  at higher levels of  $k$  and  $c$  than the previous stationary solution  $(\bar{k}, \bar{c})$ . This makes sense because the smaller discount implies that consumption in future periods is not being discounted relative to the present as strongly as before. Consequently, there is less pressure to allocate more consumption to earlier periods.

**QUESTION 3: [38 Points, About 38 Minutes]** Consider the following Model M of an economy over periods  $t \geq 1$ . Model M incorporates the expectations of both *informed* firms and *uninformed* firms. Each firm sets its output production in accordance with its expected consumption sales in an attempt to keep its inventories as close to zero as possible:

**Model Equations:**

$$(1) \quad Q_t = [1 - \lambda] \cdot C_{t-1} + \lambda \cdot E[C_t | I_{t-1}]$$

$$(2) \quad Y_t = C_t + INV_t$$

$$(3) \quad C_t = a + bY_t + u_t$$

$$(4) \quad INV_t = [Q_t - C_t]$$

**Classification of Variables:**

- *Period- $t$  Endogenous* ( $t \geq 1$ ):  $Q_t$ ,  $Y_t$ ,  $C_t$ ,  $INV_t$  where
  - $Q_t$  denotes produced real output in period  $t$ ;
  - $Y_t$  denotes actual real sales (real GDP) in period  $t$ ;
  - $C_t$  denotes real consumption in period  $t$ ;
  - $INV_t$  denotes the change in real inventories in period  $t$ .
- *Period- $t$  Predetermined* ( $t > 1$ ):  $C_{t-1}$  and  $E[C_t | I_{t-1}]$ , where: (i)  $C_{t-1}$  denotes the expectation for  $C_t$  held at the beginning of period  $t$  by each of the **uninformed** firms; (ii)  $I_{t-1}$  denotes a period- $t$  predetermined information set that is available to each of the **informed** firms at the beginning of period  $t$  (equivalently, at the end of period  $t - 1$ ); and (iii)  $E[C_t | I_{t-1}]$  denotes the objectively true  $I_{t-1}$ -conditional expectation for  $C_t$  formed by each of the **informed** firms as a function of  $I_{t-1}$  at the beginning of period  $t$ .
- *Deterministic Exogenous Variables*:  $C_0$ ,  $E[C_1 | I_0]$ ,  $\lambda$ ,  $a$ , and  $b$ , with  $0 < C_0$ ,  $0 < E[C_1 | I_0]$ ,  $0 < a$ ,  $0 < b < 1$ , and  $0 \leq \lambda \leq 1$ . Here  $I_0$  represents the information set of the **informed** firms at the beginning of period 1, and the parameter  $\lambda$  denotes the proportion of firms in the Model M economy who are **informed**.
- *Stochastic Exogenous Variables*: The shock term  $u_t$ , which is a mean-zero serially-independent sequence of random variables with stationary variance  $\sigma_u^2$ . *Serial independence* means, roughly, that the realized value of  $u_t$  in any period  $t$  is independent of the realized value for  $u_s$  in any time period  $s \neq t$ .

**IMPORTANT NOTE:** Assume the modeler knows the structure of the Model M equations and the classification of variables for Model M, and that the modeler observes all past realizations of endogenous variables and all past realizations of stochastic exogenous variables.

**Part Q3.A (6 Points):** According to Packet Reading 25: “Introduction to Rational Expectations”, what does it mean in general to say that an agent in a model of an economy has a *strong-form rational expectation (RE)* at the beginning of period  $t$  [equivalently, at the end of period  $(t - 1)$ ] regarding the value that a variable  $v$  will take on in some period  $t + k$  with  $k \geq 0$ ?

**CAUTION:** Here you are being asked for a *general definition*, not a specific example. Parts Q3.B and Q3.C, below, will ask you to relate this general definition specifically to Model M.

#### Answer Outline for Part Q3.A:

From Packet Reading 25: An agent  $i$  in a model of an economy has a *weak-form rational expectation (RE)* at the beginning of period  $t$  regarding the value that a variable  $v$  will take on in some period  $t + k$  with  $k \geq 0$  if agent  $i$ ’s subjective expectation  $E_{t-1,i}[v_{t+k}]$  for  $v_{t+k}$  at the beginning of period  $t$ , conditional on his information set  $I_{t-1,i}$  at the beginning of period  $t$ , coincides (up to an unsystematic error term) with  $E[v_{t+k} | I_{t-1,i}]$ , the *objectively true* expectation of  $v_{t+k}$  conditional on  $I_{t-1,i}$ ; that is, if

$$E_{t-1,i}v_{t+k} = E[v_{t+k}|I_{t-1,i}] + \mu_{t,i}, \quad (7)$$

where  $\mu_{t,i}$  is a forecasting error satisfying  $E[\mu_{t,i}|I_{t-1,i}] = 0$ . Moreover, agent  $i$ ’s expectation  $E_{t-1,i}[v_{t+k}]$  is a *strong-form RE* if in addition to being a weak-form RE the information set  $I_{t-1}$  for agent  $i$  at the beginning of period  $t$  contains the following information:

- (a) The true structural equations and classification of variables for the model, including the actual decision rules used by any other agent (private or public) in the model to generate their actions and/or expectations;
- (b) The true values for all deterministic exogenous variables for the model.
- (c) The true probability distributions governing all stochastic exogenous variables.
- (d) All past realized values for variables as observed by the modeler through the beginning of period  $t$  (equivalently, through the end of period  $t - 1$ ). **NOTE: Packet 25 as distributed at the beginning of the course only mentions past “endogenous” variable realizations here, inadvertently omitting**

mention of past realizations for stochastic exogenous variables, so no credit should be taken off here if students only mention past endogenous variable realizations.

**Part Q3.B (4 Points):** Focusing specifically on Model M, above, explain carefully and precisely what information must be included in the information set  $I_{t-1}$  appearing in the Model M equation (1) for any period  $t \geq 1$  in order for this information set to be consistent with the definition of strong-form RE for Model M.

**Answer Outline for Part Q3.B:**

From Part Q3.A, the information that must be included in the information set  $I_{t-1}$  is as follows.

- (a) The true structural equations (1) through (4) for Model (M), along with the classification of variables for Model M, as set out in the statement of Question 1.
- (b) The true values for all of the deterministic exogenous variables for Model M: namely,  $C_0$ ,  $E[C_1 | I_0]$ ,  $a$ ,  $b$ , and  $\lambda$ .
- (c) The true probability distribution governing the stochastic exogenous variables  $\{u_t | t = 1, 2, \dots\}$  – more precisely, the particular aspects of this probability distribution as set out in the statement of Model M’s classification of variables (mean zero, serially independent, with stationary variance  $\sigma_u^2$ ).
- (d) All values for endogenous and stochastic exogenous variables for Model M realized prior to period  $t$ , as observed by the modeler, i.e.,  $\{Q_s, Y_s, C_s, INV_s, u_s | s < t\}$ .

**Part Q3.C (16 Points):** Suppose the information set  $I_{t-1}$  for each period  $t \geq 1$  contains the information that makes it consistent with the definition of strong-form RE for Model M. Using your results from Part Q3.B, determine *explicit* analytical expressions for the objectively true  $I_{t-1}$ -conditional expectations  $E[Q_t | I_{t-1}]$ ,  $E[Y_t | I_{t-1}]$ ,  $E[C_t | I_{t-1}]$ , and  $E[INV_t | I_{t-1}]$  for any arbitrary time period  $t \geq 1$  that represent these expectations as functions solely of the information in  $I_{t-1}$ . **Be sure to show your derivations step by step, and be sure to justify carefully each of these steps.**

### Answer Outline for Part Q3.C:

As explained in Section 3 of Packet Reading 25, linear models such as Model M can be solved for strong-form RE expressions using four steps. The first step is to determine the explicit type of information to be included in the information sets – this was done in Part Q3.B. The second step is to replace any subjective expectations by true conditional expectations, but this has already been done in Model M.

The third step is to take  $I_{t-1}$ -conditional expectations of each side of each of the four equations (1)-(4) of model M, *explaining carefully* how the specific information in the information set  $I_{t-1}$  has been used to simplify the form of these expectations. In particular, note that the values for all of the deterministic exogenous variables are in  $I_{t-1}$  for each  $t \geq 1$ , so these terms can be pulled out of any expectation conditional on  $I_{t-1}$ . A similar comment holds for all past endogenous variable realizations up to period  $t$ . In addition,  $I_{t-1}$  contains the information that  $u_t$  is a mean-zero serially independent process, hence  $E[u_t | I_{t-1}] = 0$  for all  $t \geq 1$ . The fourth step is then to use the resulting equations to solve for the required strong-form rational expectations.

Before undertaking the last two steps, let equations (2) and (4) be used to substitute out for  $Y_t = Q_t$  and  $INV_t = [Q_t - C_t]$  in equations (1) and (3), resulting in the following reduced-form Model M\*:

$$(1)^* \quad Q_t = [1 - \lambda] \cdot C_{t-1} + \lambda \cdot E[C_t | I_{t-1}]$$

$$(2)^* \quad C_t = a + bQ_t + u_t$$

Now taking the  $I_{t-1}$ -conditional expectation of each side of each of these two equations, and using the linearity of the expectations operator, one obtains:

$$(1)^* \quad E[Q_t | I_{t-1}] = [1 - \lambda] \cdot C_{t-1} + \lambda \cdot E[C_t | I_{t-1}]$$

$$(2)^* \quad E[C_t | I_{t-1}] = a + bE[Q_t | I_{t-1}]$$

A further manipulation of terms yields:

$$E[Q_t | I_{t-1}] - \lambda \cdot E[C_t | I_{t-1}] = [1 - \lambda] \cdot C_{t-1} ; \quad (8)$$

$$-b \cdot E[Q_t | I_{t-1}] + E[C_t | I_{t-1}] = a . \quad (9)$$

Equations (8) and (9) can be expressed more compactly in matrix form as follows:

$$A \cdot x_t = v_{t-1} ; \quad (10)$$

where  $x_t$  denotes the  $2 \times 1$  column vector  $(E[Q_t | I_{t-1}], E[C_t | I_{t-1}])'$ ,  $v_{t-1}$  denotes the  $2 \times 1$  column vector  $([1 - \lambda] \cdot C_{t-1}, a)'$ , and  $A$  is the following  $2 \times 2$  matrix of exogenous variables:

$$A = \begin{bmatrix} 1 & -\lambda \\ -b & 1 \end{bmatrix}_{2 \times 2} \quad (11)$$

with determinant given by  $\Delta \equiv (1 - \lambda \cdot b)$ . Since  $\Delta \neq 0$  by admissibility,  $A$  is nonsingular. It follows that the unique solution of (10) is

$$x_t = A^{-1} v_{t-1} \quad (12)$$

where the right-hand side of (12) depends only on information in  $I_{t-1}$ . By construction, then, (12) gives the unique explicit analytical form of the strong-form RE expressions  $E[Q_t | I_{t-1}]$  and  $E[C_t | I_{t-1}]$ . Written out in more explicit form,

$$E[Q_t | I_{t-1}] = \frac{([1 - \lambda] \cdot C_{t-1} + \lambda \cdot a)}{[1 - \lambda \cdot b]} \equiv F_{t-1}(\lambda) \quad (13)$$

$$E[C_t | I_{t-1}] = a + b \frac{([1 - \lambda] C_{t-1} + \lambda \cdot a)}{[1 - \lambda \cdot b]} = a + b \cdot F_{t-1}(\lambda) \quad (14)$$

It then follows from (13) and (14), plus the original Model M equations (2) and (4), that

$$E[Y_t | I_{t-1}] = F_{t-1}(\lambda) \quad (15)$$

$$E[INV_t | I_{t-1}] = [1 - b] \cdot F_{t-1}(\lambda) - a \quad (16)$$

**Part Q3.D (6 Points):** Using your results from Part Q3.C, and the original equations (1)-(4) for Model M, express in explicit analytical form the solutions  $Q_t^*$ ,  $C_t^*$ , and  $INV_t^*$  for period- $t$  output  $Q_t$ , consumption  $C_t$ , and change-in-inventories  $INV_t$  as a function of  $\lambda$  and other exogenous and period- $t$  predetermined variables.

**Answer Outline for Part Q3.D:**

$$\begin{aligned} Q_t^* &= (1 - \lambda)C_{t-1} + \lambda(a + b \cdot F_{t-1}(\lambda)) \\ &= ((1 - \lambda) \cdot C_{t-1} + \lambda \cdot a) + \lambda \cdot b \cdot F_{t-1}(\lambda) \\ &= (1 - \lambda \cdot b)F_{t-1}(\lambda) + \lambda \cdot b \cdot F_{t-1}(\lambda) \\ &= F_{t-1}(\lambda) \quad ; \end{aligned} \quad (17)$$

$$C_t^* = a + b \cdot F_{t-1}(\lambda) + u_t ; \quad (18)$$

$$\begin{aligned} INV_t^* &= Q_t - C_t \\ &= F_{t-1}(\lambda) - a - b \cdot F_{t-1}(\lambda) - u_t \\ &= [1 - b] \cdot F_{t-1}(\lambda) - a - u_t . \end{aligned} \quad (19)$$

**Part Q3.E (6 Points):** Using your results from Part Q3.D, briefly compare the properties of the inventory solution processes  $\{INV_t^* \mid t = 1, 2, \dots\}$  for Model M under each of the following two special cases: (a)  $\lambda = 0$  (all firms uninformed); and (b)  $\lambda = 1$  (all firms informed). Discuss the economic implications of your findings.

**Answer Outline for Part Q3.E:**

Using the definition of  $F_{t-1}(\lambda)$  in (13), it is seen that

$$F_{t-1}(\lambda) = \begin{cases} C_{t-1} & \text{if } \lambda = 0 \text{ (uninformed firms) ;} \\ \frac{a}{[1-b]} & \text{if } \lambda = 1 \text{ (informed firms) .} \end{cases} \quad (20)$$

Examining the solutions in Part Q3.D, if all firms are uninformed, it follows from (20) that

$$INV_t^* = [1 - b]C_{t-1} - a - u_t . \quad (21)$$

Consequently, the stochastic solution process for change-in-inventories for Model M is serially dependent because its period- $t$  realization depends on the change-in-inventory realization  $INV_{t-1}^* = [Q_{t-1} - C_{t-1}]$  in period  $t - 1$  through the period- $t$  predetermined variable  $C_{t-1}$ . Note, also, that the  $I_{t-1}$ -conditional mean for  $INV_t^*$  is time varying, through its dependence on  $C_{t-1}$ .

Conversely, if all firms are informed, it follows from (20) that

$$INV_t^* = -u_t . \quad (22)$$

Consequently, the stochastic solution process for change-in-inventories is a *serially independent* process because its period- $t$  realization does not depend on realizations in any previous periods.

Once again, here is an example where the precise form of agent expectations matters. When  $\lambda = 0$ , all firms have simple adaptive expectations for consumption sales and this induces serial dependence in the resulting time series for the Model M change-in-inventories process (actually, in the resulting time series for all Model M endogenous

variable solutions). When  $\lambda = 1$ , all firms have strong-form rational expectations and the resulting time series for the Model M change-in-inventories process is then serially independent (as indeed are the resulting time series for all Model M endogenous variable solutions).

#### QUESTION 4: [25 Points Total, About 25 Minutes]

**PART Q4.A: (9 Points)** Consider an economy E in some period T that has one produced consumption good, C. The economy E has ONE utility-maximizing consumer with an exogenously given labor service endowment  $\bar{l}$  and a strictly increasing and strictly concave utility function  $U(c, Le)$  defined over consumption good amounts  $c$  and leisure amounts  $Le = \bar{l} - l$ , where  $l$  denotes supplied labor services. The economy E also has ONE profit-maximizing firm (owned by the consumer) that produces consumption good amounts via a strictly increasing and strictly concave production function  $c = F(l)$  that uses labor services  $l$  as its only input.

Carefully define, IN WORDS, what is meant by a *Walrasian equilibrium* for **the particular economy E**.

**Caution:** You are *not* being asked here to give the nine basic model assumptions for a Walrasian general equilibrium economy. You are asked to provide a careful verbal description of the specific defining conditions for a *Walrasian equilibrium* for the specific economy E outlined in Part Q4.A.

#### Answer Outline for Part Q4.A:

The general definition of a Walrasian equilibrium is provided in Section 3 of Course Packet Reading 3 (“Introduction to Walrasian General Equilibrium Modeling”). This general definition will now be specialized to the specific economy E, as asked for in Q4.A.

A specific vector  $e^*$  comprising consumer supplies and demands for services and consumption goods, firm demands and supplies for services and consumption goods, prices, expected prices, and expected dividends is a *Walrasian equilibrium* for the specific economy E if the following three conditions hold:

- **(a) Individual Optimality:** At  $e^*$ , the consumer is maximizing his utility subject to physical feasibility and budget constraints, conditional on expected prices and expected dividends, and the firm is maximizing its profits subject to physical feasibility and technology constraints, conditional on expected prices.
- **(b) Fulfilled Expectations:** At  $e^*$ , expected prices coincide with actual prices and expected dividends coincide with actual dividends calculated as the consumer’s shares of the firm’s profits.
- **(c) Market Clearing:** At  $e^*$ , excess supply is greater than or equal to zero in each market, which for economy E includes a market for consumption good and a market for labor services.

**Note:** Since the consumer is strictly non-satiated, Walras Law (total value of excess supply = 0 at  $e^*$ ) automatically holds, given conditions (a), (b), and (c), and so does not need to be included.

**REMARK:** The precise analytical formulations of the three defining conditions (a), (b), and (c) for a Walrasian equilibrium for the economy E at hand (i.e., an economy consisting of one consumer and one firm) can be found in Section 5 of Course Packet Reading 28 (“Non-Walrasian Equilibrium: Illustrative Examples”).

**PART Q4.B: (6 Points)** Assuming all equilibrium prices are positive, provide a *graphical depiction* of a Walrasian equilibrium for the specific economy E as follows:

- (i) Depict in graphical terms the demand and supply curves for the labor services market and the consumption good market for economy E.
- (ii) Identify which points on these graphs represent a Walrasian equilibrium.
- (iii) Explain carefully why the points you identified in (ii) satisfy your definition of Walrasian equilibrium as given in Part Q4.A.

**Answer Outline for Part Q4.B:**

See Course Packet Reading 28 (“NonWalrasian Equilibrium: Illustrative Examples”), Figure 3.

**Technical Note:** In Figure 3, observe that the demand and supply curves for the consumption good are expressed in the  $c - w$  plane rather than in the more usual  $c - p$  plane, where  $W$  denotes the nominal wage,  $P$  denotes the nominal price of the consumption good,  $w = W/P$  denotes the real wage (measured in units of the consumption good), and  $p = W/P$  denotes the real price of consumption good (measured in labor units).

Since consumption demand is normally a *decreasing* function of  $p = P/W$ , it follows that consumption demand appears as an *increasing* function of  $w = W/P$  in the  $c - w$  plane, and conversely for consumption supply. Consequently, the slopes of these consumption demand and supply curves in the  $c - w$  plane are the reverse of what they would be in their more usual  $c - p$  plane representation.

A graphical depiction that plots the demand and supply curves for the consumption good market in the  $c - p$  plane is perfectly acceptable *as long as the connections between the market clearing points in the consumption good market and the labor services market are clearly made*.

## PART Q4.C: (9 Points)

- (i) Briefly but carefully explain in what sense, if any, the Walrasian equilibrium you have defined in Part Q4.A and graphed in Part Q4.B is a “benchmark of coordination success”.
- (ii) Briefly but carefully explain Clower’s criticism of the Walrasian conceptualization of market clearing used in the definition of a Walrasian equilibrium for which demands are “notional” rather than “effective.” Be sure to include in your answer a verbal explanation of Clower’s conception of an *effective demand*.
- (iii) Discuss, briefly, how Clower’s critique – in particular, his insistence that all demands be effective – has contributed to the debate regarding the possible existence of “unemployment equilibrium.” Be sure to include in your answer a verbal explanation of the concept of an *unemployment equilibrium*.

### Answer Outline for Part Q4.C:

**KEY POINT FOR (i):** A Walrasian equilibrium represents a precisely formulated set of conditions under which a feasible allocation of goods and services can be supported in a decentralized manner by a price system in a market economy consisting of individually optimizing agents in a way that ensures *all agents’ plans are fulfilled*, *all agents’ expectations are realized*, and – given consumer nonsatiation – *the price-supported allocation is Pareto efficient*, a consequence of the famous First Welfare Theorem. In view of these three highlighted properties, Walrasian equilibrium is generally considered to be a benchmark model of coordination success for market economies.

**ADDITIONAL REMARKS ON POINT (i):** The properties that characterize a Walrasian equilibrium can be more specifically cast in game theoretic terms as follows:

- (1) **Coordination:** Individual optimization conditional on expected prices and dividends, fulfilled expectations, and market clearing.
- (2) **Pareto efficiency:** By the First Welfare Theorem, given nonsatiation, every Walrasian equilibrium allocation is Pareto efficient.

The three defining conditions summarized in (1) for a Walrasian equilibrium imply that each micro agent is both willing and able to carry out his planned activities, given his current price and dividend expectations. Moreover, these expectations are correct. Thus, although strategic interaction is absent, the economy satisfies the “no individual incentive to deviate” property of a Nash equilibrium.

Recall from the examples in Course Packet Reading 28, however, that economies can be in a situation in which all agents’ plans are fulfilled (in an effective planning sense) and

all agents' expectations are fulfilled, implying that no agent with the power to deviate has any incentive to do so, and yet the situation is not Pareto efficient. Situations like these – i.e., Nash equilibria that are Pareto dominated – are generally referred to as situations of *coordination failure* in game theory. Thus, property (2), the Pareto efficiency of Walrasian equilibrium allocations, is critical for the characterization of Walrasian equilibria as examples of coordination *success*.

**KEY POINT FOR (ii):** Clower's key criticism of Walrasian equilibrium is the inattention paid to the importance of credible signalling (believable communication).

More precisely, Clower argues that, in real world markets, only *effective demands* – i.e., *demands backed by actual purchasing power* – can be credibly signalled to other market participants. Clower criticizes the concept of a Walrasian equilibrium because the demands arising in a Walrasian equilibrium are not fully backed by actual purchasing power at the time of their transmittal to other market participants. Rather, they are conditioned on anticipated income from anticipated sales of labor and capital services to firms that have not been consummated at the time the demands are transmitted. Clower argues that firms will not pay attention to such “notional” demands; they will only pay attention to demands for their goods and services that are made “effective” by being credibly backed by purchasing power (whether in cash or credit terms) at the time of their communication.

**KEY POINT FOR (iii):** Clower (and many other new Keynesians) argue that, since only demands backed by purchasing power can be credibly signalled to other market participants, an economy can persist in an “effective equilibrium” in which every market clears in terms of effective (credibly signalled) demands and supplies even though markets do not clear in terms of notional demands and supplies.

The reason why is that firms that have the *power* to hire more workers simply do not receive any credible signals that they could sell more of their goods and services than they are currently selling, so they have *no incentive* to hire more workers. Conversely, even though workers have an *incentive* to work more in order to earn more income to use to purchase more goods and services from firms, they cannot credibly signal these buying intentions to firms and hence have *no power* to change things.

In particular, then, an effective equilibrium could persist in which the effective supply of labor (the labor actually supplied by consumers who are constrained to only make demands backed by actual purchasing power) is less than the notional supply of labor (the amount of labor that consumers would actually like to supply at prevailing wages). The difference between the notional and effective labor supplies can be interpreted as “involuntary unemployment” in the sense of Keynes. Some economists refer to such a situation as an (*involuntary*) *unemployment equilibrium*.