ANSWER OUTLINE

FINAL EXAM: 100 POINTS TOTAL L. Tesfatsion/Econ 502/Fall 2014

Important Note: Please answer ALL FOUR questions in this packet.

In answering these questions, please do the following:

- (a) Use the provided answer packet for all answers (include all scratch work).
- (b) Read each question carefully before you begin your answer.
- (c) Define terms and concepts clearly.
- (d) Carefully label all graphs.
- (e) Watch your time. A useful rule of thumb would be to allocate 1 minute per point. This would leave you with 20 minutes at the end of the exam to check over your answers.
- (f) Justify your assertions as carefully as time permits.

QUESTION 1: [30 Points Total]

Consider an Economy E that is a basic pure-exchange two-period lived overlapping generations economy extending over the infinite past and infinite future (time periods $t = 0, \pm 1, \pm 2, \ldots$). Economy E has a single physical resource, Q, which is perishable (meaning it cannot be stored from one period to the next). Each agent in Economy E has a *positive* Q endowment in youth but *no* Q-endowment in old age.

More precisely, each young agent in each generation t has the same Q-endowment profile $w = (w^1, w^2)$ with $w^1 > 0$ and $w^2 = 0$. Each young agent in each generation t has the same strictly increasing and strictly concave lifetime utility function $U:R^2 \to R$, where $U(c_t^1, c_{t+1}^2)$ denotes the lifetime utility that would be obtained by the agent from consuming the consumption profile (c_t^1, c_{t+1}^2) during his youth and old age. Each young agent desires to maximize his lifetime utility. The number L_t of young agents in each generation t grows at a net rate g > 0, so that $L_{t+1} = (1+g)L_t$, with $L_0 > 0$. Finally, assume for each young agent that MRS(w) < (1+g), where MRS(w) denotes the agent's marginal rate of substitution evaluated at the endowment point w, i.e., $MRS(w) = U_1(w)/U_2(w)$.

- Q1.A:[8 Points] Present in analytical form with accompanying graphical illustration the budget-constrained lifetime utility maximization problem faced by a representative young agent in Economy E under each of the following two conditions:
 - 1. **Autarky:** Economy E has no institutional arrangements.
 - 2. Q-Banking System:
 - In each period t there exists a (gross) interest rate $(1+r_t) > 0$ that determines the total amount of Q (principal plus interest) that an agent will receive back from the Q-bank in period t+1 if he lends one unit of Q to the Q-bank in period t or must pay back to the Q-bank in period t+1 if he borrows one unit of Q from the Q-bank in period t.
 - Each agent in period t can borrow or lend as many units of Q as he wishes at the net interest rate $r_t > -1$ as long as he pays off all of his debts before he dies.
- Q1.B: [6 Points] Provide a careful verbal and analytical definition for each of the following concepts for Economy E: (i) feasible allocation; (ii) efficient allocation; and (iii) Pareto-efficient allocation
- Q1.C: [8 Points] Determine two distinct stationary net interest rates r > -1 for Economy E such that, for each r, the stationary allocation that results for Economy E is feasible. (Be sure to establish this feasibility.) Graphically depict in the $c^1 c^2$ plane the utility maximization problem that results for a representative young agent conditional on each of your two r values.
- Q1.D: [8 Points] Does each of the two net interest rates r you have determined in Q1.C necessarily result in an efficient allocation for economy E? a Pareto efficient allocation for economy E? Explain carefully why or why not.

Answer Outline for Q1.A

Compare Section C of Packet 21 ("The Basic Pure-Exchange Overlapping Generations Economy"), and in particular Figures 4 and 8, for answers to Q1.A-Q1.D for the case in which each young agent has a positive second-period Q-endowment. The only difference between the packet answers and the answers required here for Q1 is that each young agent's old-age endowment is zero.

Specifically, as in Packet C, it can be argued for Economy E that no trade will take place under autarky. Thus, for Economy E, each young agent's autarkic budget set in the $c^2 - c^1$ plane reduces to the line segment from 0 to w^1 along the w^1 axis (compare Figure 4 in Packet 21). Consequently, since each agent's utility function U is strictly increasing, each young agent under autarky will choose the same lifetime consumption profile $(c^{1,w}, c^{2,w}) = (w^1, 0)$.

Alternatively, under the Q-banking system with a gross interest rate $(1 + r_t) > 0$ in each period t, a young agent in generation t faces the following lifetime utility-maximization problem:

$$\max U(c_t^1, c_{t+1}^2) \tag{1}$$

with respect to choice of (c_t^1, c_{t+1}^2, s_t) subject to

$$c_t^1 + s_t \le w^1; (2)$$

$$c_{t+1}^2 \le (1+r_t)s_t; \tag{3}$$

$$c_t^1 \ge 0; c_{t+1}^2 \ge 0 \tag{4}$$

The Q-banking system permits s_t to be either lending or borrowing. However, note that the restriction $c_{t+1}^2 \ge 0$ in (4) and $(1+r_t) > 0$ forces the young agent to choose $s_t \ge 0$; i.e., the young agent cannot borrow when young.

For later purposes, it will be useful to express this utility maximization problem in the following equivalent manner with savings substituted out:

$$\max U(c_t^1, c_{t+1}^2) \tag{5}$$

with respect to choice of (c_t^1, c_{t+1}^2) subject to

$$c_t^1 \le w^1; \tag{6}$$

$$c_t^1 + \frac{c_{t+1}^2}{1+r_t} \le w^1;$$

$$c_t^1 \ge 0; c_{t+1}^2 \ge 0$$
(8)

$$c_t^1 \ge 0; c_{t+1}^2 \ge 0 \tag{8}$$

Answer Outline for Q1.B

Compare Section B.1 of Packet 21. An allocation for Economy E is an assignment of a nonnegative consumption level to each young agent and each old agent in each period t, where all of the young agents in any particular generation t are assigned the same lifetime consumption profile. Analytically, an allocation for Economy E can be expressed in the following form:

$$\mathbf{c} = \{ (c_t^1, c_t^2) \ge 0 \mid t = 0, \pm 1, \pm 2, \dots \}$$
(9)

An allocation \mathbf{c} for Economy E is *feasible* if and only if, for each period t, the aggregate consumption of Q designated under this allocation for period t is less than or equal to the aggregate amount of Q available in period t. Using the fact that Q is assumed to be perishable for Economy E, this condition can be expressed in the following analytical form: For each t,

$$L_t c_t^1 + L_{t-1} c_t^2 \le L_t w^1 \tag{10}$$

or equivalently, using $L_{t+1} = [1+g]L_t$ for each t,

$$c_t^1 + \frac{c_t^2}{1+q} \le w^1 \tag{11}$$

An allocation \mathbf{c} for Economy E is *efficient* if and only if, for each period t, the aggregate consumption of Q designated under this allocation for period t equals the aggregate amount of Q available in period t. This condition can be expressed in the following analytical form: For each t,

$$L_t c_t^1 + L_{t-1} c_t^2 = L_t w^1 (12)$$

or equivalently, using $L_{t+1} = [1+g]L_t$ for each t,

$$c_t^1 + \frac{c_t^2}{1+q} = w^1 (13)$$

Finally, an allocation \mathbf{c}^* for Economy E is *Pareto efficient (PE)* if and only if it is a feasible allocation and there exists no other feasible allocation that yields at least as much lifetime utility for each young agent in each generation and strictly higher lifetime utility for at least one young agent in some generation. These PE conditions for \mathbf{c}^* can be expressed in the following analytical form: Allocation \mathbf{c}^* satisfies (11) for each t, and there does *not* exist any other allocation \mathbf{c}' satisfying (11) for each t such that

$$U(c_t^{1\prime}, c_{t+1}^{2\prime}) \ge U(c_t^{1*}, c_{t+1}^{2*}) \tag{14}$$

for all t, with strict inequality holding in (14) for at least one t.

Answer Outline for Q1.C

Claim: Under the assumptions of Q1, the selection of a net interest rate r > -1 results in a feasible allocation for Economy E if and only if $r \leq g$.

Proof:

The selection of any stationary net interest rate r > -1 satisfying $r \leq g$ results in a feasible stationary allocation for Economy E. This is because the budget set (7) resulting for each young agent is then nested within the collection in the $c^1 - c^2$ plane of all (macro) feasible stationary consumption profiles (c^1, c^2) , characterized by

$$c^1 + \frac{c^2}{1+g} \le w^1 \tag{15}$$

Conversely, given r > g, each young agent will perceive a budget set whose budget-constraint boundary line lies outside this collection of feasible stationary consumption profiles apart from the endowment point w. Consequently, unless $(1+r) \leq \text{MRS}(w)$, (implying each utility-maximizing young agent chooses to consume w), each young agent will choose a stationary consumption profile leading, in the aggregate, to a non-feasible stationary allocation. However, under the assumptions of Q1, r > g implies (1+r) > (1+g) > MRS(w); thus each young agent will choose an optimal consumption profile strictly to the left of his endowment point w along his perceived budget constraint, which will result in an infeasible allocation.

In summary, under the assumptions of Q1, the selection of a net interest rate r > -1 results in a feasible allocation for Economy E if and only if $r \leq g$. QED

As in Packet 21 (Section C, especially Fig. 8), two special stationary gross interest rates that result in a feasible allocation are as follows:

- SPECIAL CASE 1: Set $r = r^w$ where $(1 + r^w) = MRS(w)$, i.e., set the net interest rate so that the gross interest rate is equal to the marginal rate of substitution of each young agent at his endowment point w. Given r^w , each young agent will choose to consume his endowment profile w, which results in a feasible allocation.
- SPECIAL CASE 2: Set r = g, i.e., set the net interest rate equal to the "golden rule" net growth rate g. Given r = g, when each young maximizes his lifetime utility subject to the budget constraint (7), each young agent will select the same optimal consumption profile $c^o = (c^{1,o}, c^{2,o})$; and these choices in aggregate will result in an efficient (hence feasible) stationary allocation for Economy E because c^o satisfies the efficiency condition (13).

The statement of Q1 includes the assumption MRS(w) < (1+g), hence the two net interest rates r^w and r^g are distinct.

Whatever two stationary net interest rates r > -1 are selected for Q1.C, the student should prove that each selection results in a feasible allocation, as defined in Part Q1.B.

Also, the student should graphically depict in the $c^1 - c^2$ plane the utility maximization problem that results for a representative young agent conditional on each of these net interest rates, along the lines of Fig. 8 in Packet 21. The graphical depictions should be consistent with the fact that, if $(1+r) \leq \text{MRS}(w)$, each agent's optimal consumption profile selection will coincide with the endowment point w. Alternatively, if (1+r) > MRS(w), each young agent will choose an optimal consumption profile that lies strictly to the left of his endowment point w along his perceived budget line.

Answer Outline for Q1.D

Let $r^1 > -1$ and $r^2 > -1$ denote the two distinct stationary net interest rates selected for Q1.C, each resulting in a feasible stationary allocation. Assume $r^1 < r^2$. It then follows from the answer outline for Q1.C that these net interest rates must satisfy $-1 < r^1 < r^2 \le g$.

Recall, also, that Q1 includes the assumption MRS(w) < (1+g). There are then five possible situations to consider.

SITUATION 1: $(1+r^2) \leq \text{MRS}(w) < (1+g)$. In this case, given either r value, each utility-maximizing young agent will simply choose to consume his endowment w and the resulting autarkic stationary allocation will be efficient. However, this autarkic stationary allocation will be Pareto-dominated by the golden-rule allocation that would result from setting r = g.

SITUATION 2: $(1+r^1) \leq \text{MRS}(w) < (1+r^2) < (1+g)$. In this case the autarkic allocation resulting under r^1 will be efficient; but the allocation resulting under r^2 will be inefficient because it will lie strictly in the interior of the set of all feasible stationary allocations. In effect, a certain portion of the Q units set aside as savings by young agents are simply thrown away under r^2 . However, the allocations resulting under either r^1 or r^2 are Pareto-dominated by the golden-rule allocation resulting under r^2 .

SITUATION 3: $(1+r^1) \leq MRS(w) < (1+r^2) = (1+g)$. In this case, the allocations resulting under r^1 and r^2 are both efficient. However, the allocation resulting under r^1 is Pareto-dominated by the allocation resulting under r^2 , which coincides with the Pareto-efficient golden rule allocation.

SITUATION 4: $MRS(w) < (1 + r^1) < (1 + r^2) < (1 + g)$. In this case, the allocations resulting under r^1 or r^2 are both inefficient. They are both Pareto-dominated by the golden-rule allocation resulting under r = g.

SITUATION 5: $MRS(w) < (1 + r^1) < (1 + r^2) = (1 + g)$. In this case, the allocation resulting under r^1 is inefficient and Pareto-dominated by the allocation resulting under r^2 , which coincides with the Pareto-efficient golden-rule allocation.

QUESTION 2: [18 Points Total] Consider the following Model M of an economic process over periods $t \geq 1$, where Model M is assumed to incorporate the expectations of a single representative consumer:

$$y_t = a \cdot y_{t-1} + b \cdot E[y_t | I_{t-1}] + \epsilon_t \tag{16}$$

where:

- y_t denotes the natural log of real GDP in period t. (*Note:* It is assumed the modeler is able to observe all past realized values for y_t as time proceeds.)
- I_{t-1} denotes a period-t predetermined information set that is available to the representative Model M consumer at the beginning of period t (equivalently, at the end of period t-1).
- $E[y_t|I_{t-1}]$ denotes the objectively true I_{t-1} -conditional expectation for y_t , assumed to be the expectation of the representative Model M consumer.
- ϵ_t denotes a random shock term.

Classification of Variables:

- y_t is period-t endogenous for $t \ge 1$.
- y_{t-1} and $E[y_t|I_{t-1}]$ are period-t predetermined variables for t > 1.
- a, b, y_0 , and $E[y_1|I_0]$ are deterministic exogenous variables satisfying the admissibility condition $b \neq 1$, and $\{\epsilon_t \mid t = 1, 2, ...\}$ are stochastically determined exogenous variables that satisfy the following admissibility conditions: $E[\epsilon_t|I_{t-1}] = 0$ for all $t \geq 1$.
- **Q2.A** (6 Points): As presented in Packet 23 ("Introduction to Rational Expectations") what does it mean to say for any economic model that an agent i in a model of an economy has a *strong-form rational expectation* (RE) at the beginning of period t regarding the value that a variable v will take on in some period t + k with $k \ge 0$?
- **Q2.B** (2 Points): Focusing specifically on Model M, above, explain carefully and precisely what information must be included in the representative consumer's information set I_{t-1} at the beginning of any period $t \geq 1$ in order for this information set to be in accordance with the definition of strong-form RE for Model M.
- Q2.C (6 Points): Using your results from Part Q2.B, determine an *explicit* analytical expression for the strong-form RE for y_t at any time $t \ge 1$, i.e., an analytical expression that represents $E[y_t|I_{t-1}]$ in Model M equation (1) as a function solely of the information in I_{t-1} . Be sure to show your derivations step by step, and be sure to justify carefully each of these steps.
- **Q2.D** (4 Points): In what sense, if any, does the form of the strong-form expectation you derived in Q2.C differ from a traditional adaptive expectation?

Answer Outline for Part Q2.A:

From Packet 23: An agent i in a model of an economy has a weak-form rational expectation (RE) at the beginning of period t regarding the value that a variable v will take on in some period t+k with $k \geq 0$ if agent i's subjective expectation $E_{t-1,i}[v_{t+k}]$ for v_{t+k} at the beginning of period t, conditional on his information set $I_{t-1,i}$ at the beginning of period t, coincides (up to an unsystematic error term) with $E[v_{t+k} \mid I_{t-1,i}]$, the objectively true expectation of v_{t+k} conditional on $I_{t-1,i}$; that is, if

$$E_{t-1,i}v_{t+k} = E[v_{t+k}|I_{t-1,i}] + \mu_{t,i}, \tag{17}$$

where $\mu_{t,i}$ is a forecasting error satisfying $E[\mu_{t,i}|I_{t-1,i}] = 0$.

Moreover, agent i's expectation $E_{t-1,i}[v_{t+k}]$ is a strong-form RE if in addition to being a weak-form RE the information set I_{t-1} for agent i at the beginning of period t contains all of the information available to the modeler, as follows:

- (i) The true structural equations and classification of variables for the model, including the actual decision rules used by any other agent (private or public) in the model to generate their actions and/or expectations;
- (ii) The true values for all deterministic exogenous variables for the model.
- (iii) The true probability distributions governing all stochastic exogenous variables.
- (iv) Realized values for all endogenous variables as observed by the modeler through the beginning of period t (equivalently, through the end of period t-1).

Answer Outline for Part Q2.B:

From Part Q2.A, the information that must be included in the information set I_{t-1} of the representative Model M consumer at the beginning of any period $t \geq 1$ in order for these information sets to be in accordance with the definition of strong-form RE for Model M is as follows.

- (a) The true structural equation (16) for Model (M), along with the classification of variables for Model M, as set out in the statement of Question 2. [Note: Model M does not include any agents using decision rules to generate their actions and/or expectations apart from the one representative consumer whose information sets are being described.]
- (b) The true values for all of the deterministic exogenous variables for Model M: namely, a, b, y_0 , and $E[y_1|I_0]$.

- (c) The true probability distribution governing the stochastic exogenous variables $\{\epsilon_t \mid t = 1, 2, ...\}$. [NOTE: As will be seen in Part Q2.C, given the linearity of Model M, it actually suffices to include in I_{t-1} only the following specific distributional property in order to solve for strong-form RE solutions: $E[\epsilon_t | I_{t-1}] = 0$ for any $t \ge 1$.]
- (d) The realized values $\{y_s \mid s=t-1,t-2,\ldots\}$ for the Model M endogenous variables observed by the modeler through the end of period t-1 (equivalently, the beginning of period t).

Answer Outline for Part Q2.C:

As explained in Packet 23, linear models such as Model M can be solved for strong-form RE expressions using four steps. The first step is to determine the explicit type of information to be included in the information sets – this was done in Q2.B. The second step is to replace any subjective expectations by symbols denoting true conditional expectations, but this has already been done in equation (16) of Model M.

The third step is to take I_{t-1} -conditional expectations of each side of equation (16) for Model M for any given $t \geq 1$, using the specific information in the information set I_{t-1} to simplify the form of these expectations. In particular, note that the values of a, b, and y_{t-1} are in I_{t-1} for each $t \geq 1$, so these terms can be pulled out of any expectation conditional on I_{t-1} . In addition, I_{t-1} contains the distributional information that $E[\epsilon_t | I_{t-1}] = 0$ for all $t \geq 1$.

It follows from these observations, together with the linearity of the expectation operator, that the third step results in the following equation:

$$[1-b] \cdot E[y_t \mid I_{t-1}] = a \cdot y_{t-1} ; (18)$$

By admissibility, $b \neq 1$. Consequently, it follows from (18) that the strong-form rational expectation for y_t is given by

$$E[y_t \mid I_{t-1}] = \frac{a \cdot y_{t-1}}{1 - b} . {19}$$

Answer Outline for Part Q2.D:

The strong-form expection (19) in Q2.C superficially has an adaptive expectation form, being a weighted average over past y realizations (with all weight placed on the most recent observation). However, the weight in (19) is structurally derived from the form of Model M equation (16) rather than being a weight determined statistically on the basis of historically observed correlations between y-variables in successive time periods. Consequently, if any shock should disrupt the structural form of Model M equation (16), this could immediately and substantially change the strong-form expectation (19) for y_t . In contrast, an adaptive expectation for y_t would only slowly adjust over time as the changed model structure results in changes in historically observed y-correlations.

QUESTION 3: [30 Points Total]

- Q3.A: [18 Points] Using verbal descriptions and one or more carefully labeled figures, briefly but carefully explain the basic structure of a standard Dynamic Stochastic General Equilibrium (DSGE) model, as reviewed by Sbordone et al. (2009).
- Q3.B: [6 Points] Advocates of DSGE modeling believe that DSGE modeling provides a better framework for assessing macroeconomic policy than traditional dynamic IS-LM modeling. Briefly but carefully describe two advantages of DSGE relative to traditional dynamic IS-LM modeling that have been stressed by advocates of DSGE.
- Q3.C: [6 Points] Briefly but carefully discuss two distinct key criticisms (practical and/or conceptual) that have been raised regarding the use of DSGE models for macroeconomic policy assessments following the 2007-2009 financial/economic crisis.

Answer Outline for Q3.A

Below is a summary discussion of the basic characteristics of a standard DSGE model as surveyed by Sbordone et al. (2009) in terms of main actors (decision-makers), key building blocks of equations, and exogenous shock terms. A completely satisfactory answer to Q3.A should contain at least some discussion of each of these aspects.

As discussed by Sbordone et al. (2009) in Sections 2 and 3 of their paper, the standard DSGE model is built around three interrelated blocks of equations for the determination of three key endogenous variables: namely, output; the inflation rate; and the nominal interest rate. The three interrelated blocks of equations are a demand block, a supply block, and a monetary policy equation.

The equations that define these three block are derived from microfoundations: namely, explicit assumptions about the behavior of the main actors in the economy. These main actors consist of:

- (1) a very large representative household (representing the entire population of the economy) which, at the starting point of the economy, chooses a complete state-contingent plan for consumption, asset (bond) holdings, and differentiated labor supplies for each current and future period t to maximize its expected discounted lifetime utility subject to a sequence of budget constraints;
- (2) a single f-firm producing a final (consumption) good in a competitive market for sale to the representative household, using differentiated intermediate goods for input;
- (3) a collection of infinitely many monopolistically-competitive *i*-firms producing differentiated intermediate goods for sale to to the *f*-firm, using differentiated labor from households as input;
- (4) a government that sets interest rates in accordance with a monetary policy rule.

Brief characterizations of each of the three interrelated blocks of equations are as follows:

- The demand block (derived from the household's behavior) is similar to a traditional "IS" relationship except that it is dynamic and forward looking (i.e., it involves future expected variables as well as current variables).
- The supply block (derived from firm behavior) is similar to an expectations-augmented Phillips curve except that it, too, is forward looking (i.e., it involves future expected variables as well as current variables).
- The monetary policy equation (representing government behavior) is a monetary policy rule that has a general Taylor-Rule form.

The economy is stochastic because it is subject to multiple exogenous shocks in each time period (productivity, discount rate, mark-up, Calvo-fairy, monetary policy shock, etc.). However, the shock realizations are observable to the consumers and firms, meaning they can condition their plans for each period t on the shock realizations they have observed up through time t. That is, the state of the economy in each period t includes all shock realizations through period t.

The key figure providing an overall flow-diagram depiction of the three key DSGE building blocks appears on page 25 of Sbordone et al. (2009).

Answer Outline for Q3.B

Some of the perceived advantages of DSGE relative to traditional dynamic IS-LM modeling that have been stressed by DSGE advocates include: (a) microfoundations that postulate agents are rational decision makers who optimize objective functions subject to budget and/or technology constraints; (b) microfoundations that provide a logically consistent treatment of stock and flow relationships; (b) microfoundations that postulate that households and other decision makers have rational expectations rather than ad hoc adaptive expectations; (d) a government modeled as a decision-maker within the economy that sets nominal interest rates to achieve interest rate, output, and inflation rate targets rather than as an unmodeled exogenous source of M, G, and t policy variables; (e) a government that uses a monetary policy rule to generate its policies instead of engaging in discretionary (ad hod or unexplained) policy setting.

Answer Outline for Q3.C

The brief but careful presentation of any two reasonable criticisms will be sufficient here for full credit.

QUESTION 4: [22 Points Total] This question focuses on key issues raised in DG1 and DG2 about Thomas Piketty's provocative 2014 book Capital in the Twenty-First Century.

Piketty's analysis of capitalist economies rests on the following three building blocks:

$$\alpha = r \times \beta \quad ; \tag{20}$$

$$\beta = s/g ; \qquad (21)$$

$$r > g , \qquad (22)$$

$$r > g$$
, (22)

where α denotes the share of capital in national income, r denotes the return rate on capital; β denotes the capital/income ratio, s denotes the savings rate (net of capital depreciation), and q denotes the growth rate of national income.

- Q4.A: [6 Points] For each of the three relationships (20) through (22), briefly state whether Piketty considers this relationship to be (i) an accounting identity, true by the definition of the terms appearing in the relationship; (ii) an empirical regularity ("stylized facts") derived from empirical data; or (iii) a theoretical hypothesis (assumption) about the way a capitalist economy works.
- Q4.B: [16 Points] As many critics of Piketty have noted, his book does not provide a fully-articulated theoretical model of a dynamic capitalist economy for which the three relationships (20) through (22) simultaneously hold (in the limit if not in finite time). Explain briefly but clearly how you might go about setting up an agent-based model of a dynamic capitalist economy to investigate under what conditions (if any) all three relationships (20) through (22) would hold for the economy in the long run.

Answer Outline for Q4.A

Relationship (20) is identified by Piketty as an accounting identify. Relationship (22) is interpreted and used by Piketty as an empirical regularity observed for dynamic capitalistic economies during some historical time periods but not all.

Piketty's explanation of relationship (21) is less clear. He motivates this relationship as an empirical regularity that one expects to see for dynamic capitalistic economies in the long run, but he then uses it to interpret data from advanced capitalist economies as if it holds for them now. Moreover, he never explains, in a cause and effect way, why one should expect relationship (22) to hold for a capitalist economy in the long run. In particular, he does not provide a structural model of a dynamic capitalist economy in which this relationship is seen to hold in the long run, starting from initial conditions where it does not hold.

Answer Outline for Q4.B

Any clearly and reasonably argued answer for Q4.B will be sufficient for full credit, as long as the proposed modeling for a dynamic capitalistic economy is indeed an agent-based modeling of a dynamic capitalist economy.

- For agent-based modeling: Starting from initially given conditions (set by the modeler), all subsequent events in the modeled economy should arise from the successive actions and interactions of the decision-making agents that reside in the modeled economy, as constrained by physical processes (e.g., random weather events), biological processes (e.g., birth and death, natural human behavioral dispositions), and institutional arrangements.
- For dynamic modeling: There should be changes over time in the state of the modeled economy.
- For a capitalist economy: The means of production (capital, labor, ...) should be largely owned by private citizens rather than by a government. Moreover, distribution, production, and distribution processes should be largely carried out through a system of decentralized markets.