ANSWER OUTLINE

FINAL EXAM: 100 POINTS TOTAL **L. Tesfatsion/Econ 502/Fall 2009**

*****Important Note:***** Please answer **ALL FOUR** questions in this packet.

In answering these four questions, please do the following:

(a) Use the provided answer packet for all answers (include all scratch work).

(b) Read each question carefully before you begin your answer.

(c) Define terms and concepts clearly.

(d) Carefully label all graphs.

(e) Justify your assertions as carefully as time permits.

(f) **Watch the time. Each of the four questions is worth 25 points, so plan roughly to spend about one fourth (30 minutes) of your two-hour exam time on each question.**

QUESTION 1: SHORT QUESTIONS [25 Points Total]

Part Q1.A (4 Points): Briefly but carefully compare and contrast the standard economic meanings of "economic growth" and "economic development".

Answer Outline for Part Q1.A: See the required course materials related to Discussion Group 2.

Part Q1.B (12 Points):

Consider a basic pure-exchange two-period lived overlapping generations economy that extends over the infinite past and infinite future (time periods $t = 0, \pm 1, \pm 2, \ldots$).

Suppose the economy has a single physical resource, Q, which is perishable (non-storable). Suppose, also, that each young agent in each generation t has the same stationary Q-endowment profile (w^1, w^2) and the same strictly increasing and strictly concave utility-of-consumption function $U(c^1, c^2)$. Finally, suppose the number L_t of young agents in each generation t grows at a net rate $g > 0$, so that $L_{t+1} = [1 + g]L_t$.

Present in *analytical form*, with an accompanying *graphical illustration*, the budget-constrained lifetime utility maximization problem faced by a representative young agent in this economy under each of the following three conditions:

1. Autarky (i.e., the economy has no institutional arrangements).
2. Each young agent can save units of Q in a deposit account, where each unit saved results in a payment of [1+r] units of Q at the beginning of his old age ($r > 0$).
3. Each agent in each period t can buy (sell) as many units of Q as he wants to at a price $p_t > 0$, where p_t is the number of "units of account" paid (received) per unit of Q bought (sold) in period t, subject to the constraint that all debts must be paid off before death.

Answer Outline for Part Q1.B: See the required course materials for Section III.E, in particular Course Packet Reading 24.

Part Q1.C (9 Points):

- Provide a careful definition of *coordination failure* as given in the required course materials.
- Provide a simple analytical and/or graphical example that illustrates how coordination failure might arise in an economy. Be sure that your analytical/graphical example of coordination failure has an *economic* interpretation.

Answer Outline for Part Q1.C: See the required course materials for Sections III through VI related to coordination issues. In particular, see Course Packet Readings 23 (game theory concepts), 24 (failure of First Welfare Theorem for overlapping generations economies), 26 (credibility issues), and 28 (signalling issues and non-Walrasian equilibrium illustrations).

QUESTION 2: [25 Points Total]

Part Q2.A (9 Points): Briefly but carefully describe the modeling of consumer (household) behavior provided in each of the following three models covered in the required course materials:

Model 1: the basic Solow-Swan descriptive growth model

Model 2: the basic optimal growth model with an infinite final-time T (**NOTE**: Use the interpretation of this optimal growth model that does NOT involve a centralized social planner.)

Model 3: the Smets-Wouters Dynamic Stochastic General Equilibrium (DSGE) model

Answer Outline for Part Q2.A:

See the required course materials related to Sections III.C, III.D, and Discussion Group 3. In particular, see Course Packet Readings 17 and 20 for Models 1 and 2, and the Exercise 8 answer outline and DG3 required materials for Model 3. Here are the essential points in brief summarized form:

- Model 1: This model has *no utility maximization*. Rather, as in standard IS-LM models, a *one-period (myopic) aggregate consumer demand function* is directly specified that gives aggregate consumption as a proportion [1-s] of income net of depreciation expenditure, where [1-s] is the marginal propensity to consume (s is the savings rate). Savings is all directed to physical capital investment; there are *no financial assets* in the model. *Labor is assumed to be supplied inelastically* by consumers (no consideration of a leisure choice). The model is *deterministic*; there are no external sources of uncertainty (system shocks) or internal sources of uncertainty (behavioral uncertainty).
- Model 2: A *representative infinitely-lived consumer* undertakes the *maximization of lifetime utility* subject to a “budget constraint” dictating the income (real output) in each period is entirely divided between consumption and physical capital investment. There are *no financial assets in the model*. The representative consumer *supplies labor inelastically* (no leisure choice). The model is *deterministic*.
- Model 3: Infinitely lived consumers engage in *lifetime utility maximization*. These consumers display *some degree of heterogeneity with respect to labor skills*. Their

utility in each period is determined by consumption of both a produced good and leisure, so *labor is no longer supplied inelastically*. Consumers can save in each period either through physical capital investment or through bond holdings (*introduction of a financial asset*). System shocks are also introduced, so the model is *stochastic rather than deterministic*; but behavioral uncertainty plays no role.

Part Q2.B (16 Points):

In what ways (if any) does the modeling of consumer (household) behavior provided in each successively listed model in Part Q2.A represent a theoretical and/or empirical improvement over the earlier listed models? More precisely, with respect to consumer (household) modeling, in what theoretical/empirical ways (if any) does:

- Model 2 improve upon Model 1?
- Model 3 improve upon Model 2 and/or Model 1?

Justify your assertions with care.

Answer Outline for Part Q2.B:

The essential differences among the models are outlined in Part A. What is sought here is a reasonable discussion regarding the extent to which these differences represent theoretical and/or empirical improvements. This discussion should ideally reflect both personal reflection and relevant discussions in required course materials.

QUESTION 3: [25 Points] Consider the following one-period **Model M**:

$$
\begin{aligned}
C^d &= a + b[1-t]\cdot \mathrm{E}_h Y^s + \epsilon \ ; && (1)\\
Y^d &= C^d + G \ ; && (2)\\
Y^s &= 20N^d \ ; && (3)\\
Y^s &= C^s + G && (4)\\
N^s &= 10[1-t]w \ ; && (5)\\
Y^s &= Y^d \ ; && (6)\\
N^s &= N^d \ . && (7)
\end{aligned}
$$

Endogenous Variables: Y^d=Real aggregate output demand; Y^s=Real aggregate output supply; N^d=Labor demand; N^s=Labor supply; w=Real wage; C^d=Real aggregate consumption demand; C^s = Real aggregate consumption supply.

Admissible Exogenous Variables: $0 < a$; $0 < b < 1$ (marginal propensity to consume); $0 < t < 1$ (income tax rate); $0 < G$ (real government expenditure); InfoSet = the information set of the representative household for Model M; $\mathrm{E}_h\mathrm{Y}^s$ = the value of Y^s expected by the representative household in Model M conditional on InfoSet; and ϵ = mean-0 stochastic shock term that is independent of all other variables for Model M.

Part Q3.A (6 Points): According to the required packet reading on rational expectations, what does it mean to say that an agent in a modeled economy has a *strong-form rational expectation* at the beginning of some time period t regarding the value that a variable v will take on in some period $t+k$ with $k \geq 0$?

CAUTION: Here you are being asked for a *general definition*, not a specific example.

Answer Outline for Part Q3.A:

From Packet Reading 25: An agent i in a model of an economy has a *weak-form rational expectation (RE)* at the beginning of period t regarding the value that a variable v will take on in some period $t+k$ with $k \geq 0$ if agent i's subjective expectation $E_{t-1,i}[v_{t+k}]$ for v_{t+k} at the beginning of period t, conditional on his information set $I_{t-1,i}$ at the beginning of period t, coincides (up to an unsystematic error term) with $E[v_{t+k} \mid I_{t-1,i}]$, the *objectively true* expectation of v_{t+k} conditional on $I_{t-1,i}$; that is, if

$$E_{t-1,i}v_{t+k} \ = \ E[v_{t+k}|I_{t-1,i}] \ + \ \mu_{t,i}, \tag{8}$$

where $\mu_{t,i}$ is a forecasting error satisfying $E[\mu_{t,i}|I_{t-1,i}] = 0$. Moreover, agent i's expectation $E_{t-1,i}[v_{t+k}]$ is a *strong-form RE* if in addition to being a weak-form RE the information set I_{t-1} for agent i at the beginning of period t contains the following information:

(a) The true structural equations and classification of variables for the model, including the actual decision rules used by any other agent (private or public) in the model to generate their actions and/or expectations;

(b) The true values for all deterministic exogenous variables for the model as expressed in the model classification of variables.

(c) The properties of the true probability distributions governing all stochastic exogenous variables as expressed in the model classification of variables.

(d) All past realized values for variables as observed by the modeler through the beginning of period t (equivalently, through the end of period $t-1$).

Part Q3.B (4 Points): Using Part Q3.A, explain carefully what specific information must be included in InfoSet in order for the representative household in Model M to have a strong-form rational expectation for Y^s.

Answer Outline for Part Q3.B:

From Part Q3.A, the information that must be included in InfoSet is as follows.

(a) The true structural equations (1) through (7) for Model (M), along with the classification of variables for Model M, as set out in the statement of Question 3.

(b) The true values for all of the deterministic exogenous variables for Model M: namely, a, b, t, and G.

(c) The true probability distribution governing the stochastic exogenous variable ϵ as set out in the statement of Model M's classification of variables: namely, that ϵ is a mean-0 stochastic shock term independent of all other variables for Model M.

(d) There are no past realized values for endogenous and stochastic exogenous variables for the one-period Model M, so the part (d) requirement of the definition of strong-form RE is trivially satisfied.

Part Q3.C (10 Points): Suppose the representative household's conditional expectation for Y^s in equation (1) is a strong-form rational expectation. Carefully derive an *explicit* analytical form for this strong-form rational expectation expressed entirely in terms of exogenous variables. **Be sure to show your derivation step by step, and be sure to justify carefully each of these derivation steps.**

Answer Outline for Part Q3.C:

Let $E[Y^s|\text{InfoSet}]$ denote the desired strong-form rational expectation. As explained in Section 3 of Packet #25, this strong-form rational expectation can be deduced for the linear model Model M using four steps.

The first step is to determine the explicit type of information to be included in the conditioning information set InfoSet – this was done in Part Q3.B.

The second step is to replace any subjective conditional expectations by true conditional expectations. Thus, let $E[Y^s|\text{InfoSet}]$ replace the expression for the representative household's subjective conditional expectation in equation (1).

The third step would be to take the InfoSet-conditional expectation of each side of each model equation, *explaining carefully* how the specific information in InfoSet has been used to simplify the form of these expectations. However, this becomes much simpler if Model M is first reduced down by variable substitution.

In particular, let equation (5) be used to solve for $w = N^s/(10[1-t]$, and let equations (6) and (7) be used to substitute out for $Y^d = Y^s$ and $N^d = N^s$. Also, let equations (2), (4), and (6) be used to substitute out for $C^d = C^s$, and let (1) be used to substitute out for C^d. This leaves the following reduced Model M^R, a two-equation model in the two unknowns Y^s and N^d:

$$
\begin{aligned}
Y^s &= a + b[1-t]E[Y^s|\text{InfoSet}] + \epsilon + G \;\; ; && (9) \\
Y^s &= 20N^d \;\; . && (10)
\end{aligned}
$$

Now take InfoSet-conditional expectations of each side of each equation () and (**??**), using that InfoSet contains the values of the exogenous variables a, b, t, and G (so these variables can be taken outside of the expectation) as well as the fact that ϵ is a mean-0 stochastic variable independent of all of these exogenous variables. One then obtains:

$$
\begin{aligned}
E[Y^s|\text{InfoSet}] &= a + b[1-t]E[Y^s|\text{InfoSet}] + G \;\; ; && (11) \\
E[Y^s|\text{InfoSet}] &= 20E[N^d|\text{InfoSet}] \;\; . && (12)
\end{aligned}
$$

The fourth step is then to use the resulting equations, in particular equation (11), to solve explicitly for the required strong-form rational expectation $E[Y|InfoSet]$. Noting that dependence on G will be critical for answering Part Q3.D, below, this solution is expressed below in a form that makes explicit exactly how G enters this solution. The positive signs of A and B follow from the admissibility conditions for Model M.

$$
\begin{aligned}
E[Y|\text{InfoSet}] &= A \; + \; B \cdot G \;\; ; && (13) \\
A &= \frac{a}{(1-b[1-t])} \; > \; 0 \; ; && (14) \\
B &= \frac{1}{(1-b[1-t])} \; > \; 0 \; . && (15)
\end{aligned}
$$

Part Q3.D (5 Points): Suppose the representative household's conditional expectation for Y^s in equation (1) is a strong-form rational expectation, and suppose government's objective is to keep labor demand N^d as close as possible to some targeted value $N^* > 0$. Using your results from Part Q3.C, determine the extent to which the government can use its expenditure level G to achieve this objective. Discuss the economic implications of your findings.

Answer Outline for Part Q3.D:

From (9), (10), and (13),

$$
\begin{aligned}
N^d &= Y^s/20 \\
&= [a+G]/20 \; + \; b[1-t][A+B\cdot G]/20 \; + \; \epsilon/20 \\
&= (a+b[1-t]A)/20 \; + \; (1+b[1-t]B)\cdot G/20 \; + \; \epsilon/20 \\
&= [D \; + \; H\cdot G] \; + \; \epsilon/20 \;\; , && (16)
\end{aligned}
$$

where $D = A/20$ and $H = B/20$ are strictly positive constants and $\epsilon/20$ is a mean-zero random variable. It then follows from (16) that

$$E[N^d] \;=\; D + H \cdot G \;\; . \tag{17}$$

Consequently, the government can set $E[N^d]$ to any level N^* in the half-open interval [D,∞) through choice of a non-negative value for government expenditures G, as follows:

$$G \;=\; \frac{[N^* - D]}{H} \;\; . \tag{18}$$

Thus, for $N^* \geq D$, the government can guarantee that N^d in (16) equals N^* up to the mean-zero random error μ. However, if the target employment demand level N^* is less than D, the closest government can come to achieving this target through its setting of G alone is to ensure that $E[N^d] = D$.

A key implication is that an assumption of strong-form rational expectations does not, in and of itself, imply a lessening of government's ability to control an economy through the setting of its policy variables, nor does it imply complete controllability. The degree to which the formation of expectations matters for the effectiveness of government policy depends on the structure of the economy and the exact availability of government policy instruments.

QUESTION 4: [25 Points Total]

PART Q4.A: (10 Points)

Consider a one-period economy E consisting of one corporate firm F and two consumers $C1$ and $C2$. The firm F uses labor services L to produce beans b by means of a strictly increasing and strictly concave production function $b = f(L)$.

The two consumers have identical endowments and tastes. More precisely, each consumer C1 and C2 has a money endowment $M^* > 0$, a labor endowment $L^* > 0$, 50% of the stock shares for firm F, a bean consumption subsistence need level $B^* > 0$, and a strictly increasing and strictly concave utility-of-consumption function $U(b - B^*, Le)$, where b denotes bean consumption, L denotes labor services, and $Le = [L^* - L]$ denotes leisure.

- Using a carefully labeled graph for illustration, explain how economy E can be modeled as a Walrasian General Equilibrium (WGE) model. Call your resulting WGE model the *WGE Bean Economy Model*.
- Carefully express what is meant by a *Walrasian equilibrium* for your WGE Bean Economy Model.

Answer Outline for Part Q4.A: See the required course materials related to Section I.C and Section VI.A, in particular Course Packet Readings 4 and 27.

PART Q4.B: (15 Points)

Now consider, instead, an open-ended multi-period version E^* of economy E that starts in time period 1 and proceeds over time periods $T \geq 1$ with no external imposition of coordination conditions (e.g., no assumed market clearing).

In particular, suppose economy E^* consists of a corporate firm F^* and two consumers $C1^*$ and $C2^*$. In the initial time period $T = 1$, each consumer $C1^*$ and $C2^*$ has an identical money endowment $M^* > 0$. In each time period $T \geq 1$ the two consumers $C1^*$ and $C2^*$ have identical labor endowments $L^* > 0$, identical ownership shares (50%) of firm F^*'s stock, identical subsistence needs $B^* > 0$, and identical utility functions $U(b - B^*, Le)$. Suppose, also, that a consumer dies at the end of time period T if his level of bean consumption for time period T is at or below B^*.

Finally, for each time period $T \geq 1$, suppose firm F^* becomes insolvent and must exit the economy if its net worth at the end of the period is negative.

- Explain briefly but carefully what minimal kinds of firm and consumer processes you would reasonably need to include in a modeling of economy E^* in order for firm F^* and consumers $C1^*$ and $C2^*$ to have a chance of surviving over time. Call the resulting process model the *Dynamic Bean Economy Model*.
- Explain briefly but carefully how an "equilibrium" might reasonably be defined for your Dynamic Bean Economy Model.

Answer Outline for Part Q4.B: See the required course materials for Sections VI.B and VI.C of the course, in particular Course Packet Reading 28 and the required in-class ppt presentation based on Course Packet Reading 29.