

EXERCISE 7: INDIVIDUAL (28 Points Total)
DUE: Friday, November 3, 2017, 12:10pm

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Econ 502/Fall 2017

****CAUTION: Late assignments will not be accepted – no exceptions.**

Note 1: Please make an **EXTRA** copy of your exercise to bring to class on the due date for use in class discussion after you turn in your exercise.

Note 2: Students are permitted to work together in study groups on this exercise, but each student is asked to *separately* prepare and turn in their exercise answer. The techniques covered in this exercise are essential tools for subsequent exercises and exams, so “free riding” on other people’s work should definitely be avoided!

Background Materials:

- “The Basic Pure-Exchange Overlapping Generations Economy” (Packet 16)

Exploring the Implications of a Social Security System in a Simple Overlapping Generations Economy

Consider a pure-exchange overlapping generations model with two-period lived agents distinguishable only by their date of birth. The population grows at the rate g , where g is a **positive** constant. Thus, if L_t denotes the number of young agents born in period t , the number of young agents born in period $t + 1$ is $L_{t+1} = (1 + g)L_t$.

There is one **durable** resource, Q , that can be stored from one period to the next at the rate of return x , where x is a constant satisfying $x > -1$. That is, one unit of Q stored in period t results in $(1 + x)$ units of Q in period $t + 1$. Each agent is endowed with $w^1 > 0$ units of Q in the first period of life and with $w^2 = 0$ units of Q in the second period of life.

Finally, each young agent desires to maximize his lifetime utility of consumption $U(c^1, c^2)$, where $(c^1, c^2) \geq 0$ denotes the young agent’s lifetime consumption profile. Each young agent has the same lifetime utility of consumption function $U: R_+^2 \rightarrow R$, which is strictly increasing and strictly concave over $R_+^2 = \{(c^1, c^2) \in R^2 \mid c^1 \geq 0, c^2 \geq 0\}$.

SEE THE FOLLOWING PAGE FOR QUESTION DETAILS

Part A (4 Points) Provide an analytical characterization of the lifetime utility maximization problem faced by a representative young agent in this economy in the absence of any institutional or legal structure.

Part B (4 Points) Illustrate *graphically* the lifetime utility maximization problem for a typical young agent that you analytically represented in Part A.

Part C (20 Points Total) Suppose the government introduces a pay-as-you-go social security system that imposes a positive lump-sum tax T on each young agent in each period t satisfying $0 < T < w^1$, and that pays out all tax receipts in the form of equal benefits to each old agent in each period t . Under this social security (SS) system, the amount of benefits paid out to each old agent in each period t is $B = (1 + g)T = [L_t/L_{t-1}] \cdot T$. Consequently, the government in effect earns a net return rate of g on each dollar of the public savings (tax receipts) T it collects from private agents through the SS system.

- (i) **(4 Points)** Provide an analytical characterization of the lifetime utility maximization problem faced by a representative young agent in this economy under this SS system.
- (ii) **(6 Points)** Suppose $x > g$. Using a carefully labelled graph for illustration, explain why the introduction of the SS system necessarily *lowers* the lifetime utility of each young agent relative to the lifetime utility this agent would achieve in the absence of the SS system (i.e., in the system analyzed in Parts A and B). Discuss the *economic* implications of this finding.
- (iii) **(8 Points)** Suppose, instead, that $x < g$. Using carefully labeled graphs for illustration, explain why the introduction of the SS system in this case will “typically” increase the lifetime utility of each young agent relative to the lifetime utility this agent would achieve in the absence of the SS system (i.e., in the system analyzed in Parts A and B). However, also explain why – in some special cases – the young agent would be worse off under the SS system even though $x < g$. Discuss the *economic* implications of these findings.
- (iv) **(2 Points)** Suppose, instead, that the storage net return rate x is a random variable with an *expected* value \bar{x} satisfying $\bar{x} > g$ and with a large positive variance σ_x . Discuss *briefly* how this might affect the answer you provided in Part C(ii).