EXERCISE 6: INDIVIDUAL (16 Points Total) **L. Tesfatsion**
DUE: Friday, October 20, 12:10pm **Econ 502/Fall 2017**

****CAUTION: Late assignments will not be accepted – no exceptions.**

Introductory Exercise on Intertemporal Optimization

Background Materials:

- **Optimization Subject to Constraints (Econ 500/501)**
- **The Pure-Exchange Overlapping Generations Economy (Course Packet 16)**

Consider a consumer, Robin, who lives for just two periods: youth (period 1) and old age (period 2). The economy inhabited by Robin has only one consumable resource, coconuts, assumed to be completely divisible and also completely perishable (i.e., coconuts cannot be stored from one period to the next).

Suppose Robin has a *positive* coconut endowment w^y when young and a *positive* coconut endowment w^o when old. Let c^y and c^o denote the coconut consumption levels of Robin when young and old, respectively. Robin's preferences over consumption profiles $c = (c^y, c^o)$ are measured by a twice continuously differentiable utility function $U{:}R^2_+ \to R$ that is strictly increasing and strictly concave over R^2_+.

To ensure that at least some amount of coconut consumption is desired by Robin in each period of his life, it will also be assumed that $U(c^y, 0) = U(0, c^o) = U(0, 0)$. The latter condition, together with the assumption $U(c^y, c^o)$ is strictly increasing in c^y and in c^o, rules out any indifference curve for Robin that passes through a positive consumption point $\hat{c} = (\hat{c}^y, \hat{c}^o) > 0$ from intersecting either the c^y or c^o axis.

Finally, suppose the only option Robin has for securing *additional* future coconuts in his old age (i.e., more than his old-age coconut endowment w^o) is to engage in coconut production. Specifically, when young, Robin can invest a portion k of his endowment w^y in a production process described by a production function $f(k)$ satisfying: (i) $f'(k) > 0$ and $f''(k) < 0$ for all $k > 0$; (ii) f(0) = 0; (iii) $f'(k) \to 0$ as $k \to +\infty$; and (iv) $f'(k) \to +\infty$ as $k \to 0$. If Robin allocates a portion k of w^y to coconut production in his youth, he will have $f(k)$ additional coconuts to consume in his old age.

Part A: [5 Points Total]
Suppose Robin at the beginning of his youth (period 1) wants to maximize his lifetime utility subject to technological and non-negativity constraints.

A.1 (2.5 points) Provide a careful *analytical* representation for Robin's constrained utility-maximization problem, hereafter referred to as **Robin's UMax problem**.

A.2 (2.5 points) Provide a careful *economic* interpretation (in words) for your analytical representation in Part A.1.

PART B: [6 Points]
Carefully derive the first-order *necessary* conditions for Robin's UMax Problem to have an optimal solution $(c^{y*}, c^{o*}, k^*) \geq 0$, showing all steps of your derivation. Provide a careful *economic* interpretation for these first-order necessary conditions.

Hint: First prove that the properties assumed for Robin's utility function U and endowments (w^y, w^o) guarantee that k^* must be strictly less than w^y. Then separately consider two possibilities: (i) $k^* = 0$; and (ii) $k^* > 0$.

PART C: [5 Points]
Using a carefully labeled and carefully explained graph, illustrate how Robin's constraints and utility indifference map might appear in relation to his optimal consumption profile (c^{y*}, c^{o*}) when plotted in the $c^y - c^o$ plane, assuming Robin's optimal investment choice k^* satisfies $0 < k^* < w^y$. Be sure your graph is carefully and completely labeled with respect to axis variables, relations being graphed, and so forth.

Hint: This graphical depiction is a bit tricky because the production process $f(k)$ induces a non-linearity in Robin's constraints. Proceed carefully.