

REVISED VERSION OF EXERCISE 3: DISTRIBUTED 9 SEPTEMBER 2017

EXERCISE 3: INDIVIDUAL (20 Points Total)

DUE: Friday, September 15, 2017, 12:10 PM

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Econ 502/Fall 2017

****CAUTION: Late assignments will not be accepted – no exceptions.**

Note 1: Please make an **EXTRA** copy of your exercise to bring to class on the due date for use in class discussion after you turn in your exercise.

Note 2: Students are permitted to work together in study groups on this exercise, but each student is asked to *separately* prepare and turn in their exercise answer. The techniques covered in this exercise are essential tools for subsequent exercises and exams, so “free riding” on other people’s work should definitely be avoided!

Effects of an Increase in the Income Tax Rate in a Dynamic Sticky-Price IS-LM Model

Background Materials:

- Packet Materials on the Dynamic Sticky-Price IS-LM Model (PACKETS 6-8);
- “Dynamic Sticky-Price IS-LM Model: Illustrative Exercise” (PACKET 9)
- Answer Key for Exercise 2 (handed out in class on 9/8/2017)

Packet 9 provides an illustrative exercise for a partially reduced-form version of the dynamic sticky-price IS-LM model developed in Packets 6-8. More precisely, a “four-diagram graphical analysis” is used in Packet 9 to determine (as far as possible) the effects of an *increase in the government budget deficit* D on the solution values for various endogenous variables in time periods 1 and 2, assuming that the increase in D at the beginning of period 1 is maintained for all time periods $T \geq 1$.

As a continuation of Exercise 2, this Exercise 3 asks you to use a carefully explained four-diagram graphical analysis (as in Packet 9) to analyze the effects of an *increase in the income tax rate* t on the period-1 and period-2 solution values for a partially reduced-form version of a dynamic sticky-price IS-LM model that *differs* in several respects from the model developed in Packets 6-8 and in Packet 9. The Exercise 3 model, referred to as Model M*, is given below. Exercise 3 asks you to use Model M* to determine (as far as possible) the effects of an increase in the income tax rate t on the solution values for all endogenous variables in time periods 1 and 2, assuming that the increase in t at the beginning of period 1 is maintained for all periods $T \geq 1$.

Model M* Equations: For each time period $T \geq 1$:

$$\begin{aligned}
(M1) \quad Y(T) &= C(T) + I(T) + G(T) + NE(T) \\
(M2) \quad C(T) &= a + b[1 - t^o]Y(T) \\
(M3) \quad I(T) &= e - dR(T) \\
(M4) \quad NE(T) &= g - mY(T) - nR(T) \\
(M5) \quad G(T) - t^oY(T) &= D \\
(M6) \quad M/P(T) &= kY(T) - hR(T) \\
(M7) \quad N^*(T) &= H([1 - t^o]w^*(T)) \\
(M8) \quad Y^*(T) &= F(\beta(T)N^*(T), K(T)) \\
(M9) \quad w^*(T) &= F_N(\beta(T)N^*(T), K(T)) \\
(M10) \quad \beta(T+1) &= [1 + \theta G(T)] \\
(M11) \quad \pi(T, T+1) &= [P(T+1) - P(T)]/P(T) \\
(M12) \quad \pi(T, T+1) &= \lambda\pi(T-1, T) + f \cdot [Y(T) - Y^*(T)]/Y^*(T) \\
(M13) \quad K(T+1) &= K(T) + I(T)
\end{aligned}$$

Model M* Classification of Variables & Admissibility Conditions:

Thirteen Period-T Endogenous Variables ($T \geq 1$):

$Y(T), C(T), I(T), G(T), NE(T), R(T), N^*(T), w^*(T), Y^*(T),$
 $\beta(T+1), \pi(T, T+1), P(T+1), K(T+1)$

Four Period-T Predetermined (State) Variables ($T > 1$):

$\beta(T), \pi(T-1, T), P(T), K(T)$

Admissible Exogenous Variables and Functional Forms:

Initial Conditions for State Variables ($T = 1$):

$$\beta(1) = 1, \quad \pi^o(0, 1) = 0, \quad P^o(1) > 0, \quad K^o(1) > 0$$

Coefficients: $0 < a, 0 < b < 1, 0 < e, 0 < d, 0 < g, 0 < m < 1, 0 < n, 0 < k,$
 $0 < h, 0 < \theta, 0 < \lambda, 0 < f$

Government Policy Variables: $0 < t^o < 1; 0 < D; 0 < M$

Functional Forms:

$H(z)$ with $H(0) = 0$ and $dH(z)/dz > 0$ for all $z = [1 - t]w \geq 0$

Production function $F(L, K)$ with inputs *effective labor* $L = \beta N$ and capital K , where:

- (i) β measures the effectiveness (skill level) of each unit of raw labor N ;
- (ii) for all $L, K > 0$, $F(L, K)$ satisfies $F(0, 0) = F(0, K) = F(L, 0) = 0$,
 $F_L > 0, F_K > 0, F_{LL} < 0, F_{KK} < 0, F_{LK} > 0$

For any $T \geq 1$, let the solution values for the 13 period- T endogenous variables for Model M^* with $t = t^o$ be referred to as the *benchmark solution for period T* , denoted by

$$\begin{aligned} \mathbf{BenchSol}(T) = & \\ & Y^o(T), C^o(T), I^o(T), G^o(T), NE^o(T), R^o(T), N^{*o}(T), w^{*o}(T), Y^{*o}(T), \\ & \beta^o(T+1), \pi^o(T, T+1), P^o(T+1), K^o(T+1) \end{aligned}$$

Suppose at the beginning of period 1 the government *increases* the income tax rate t^o to a higher admissible value t' (i.e., $0 < t^o < t' < 1$), and the government maintains the income tax rate at the higher value t' for all periods $T \geq 1$. For any $T \geq 1$, let the solution values for the 13 period- T endogenous variables for Model M^* with $t = t'$ be referred to as the *new solution for period T* , denoted by

$$\begin{aligned} \mathbf{NewSol}(T) = & \\ & Y'(T), C'(T), I'(T), G'(T), NE'(T), R'(T), N^{*'}(T), w^{*'}(T), Y^{*'}(T), \\ & \beta'(T+1), \pi'(T, T+1), P'(T+1), K'(T+1) \end{aligned}$$

*****IMPORTANT NOTES ON EXERCISE 3***:**

- Assume in your answers for all parts of Exercise 3 that, for the *benchmark* solution $\mathbf{BenchSol}(1)$ for period 1, actual real GDP in period 1 is equal to potential real GDP in period 1, and investment is strictly positive; that is, assume $Y^o(1) = Y^{*o}(1)$ and $0 < I^o(1)$.
- As in Packet 9, to simplify the analysis it is assumed in equation (M6) for Model M^* that there is no “Keynes ex ante effect,” that is, it is assumed that real money demand in each period T depends on the *real* interest rate $R(T)$ rather than on the *nominal* interest rate $R(T) + \pi(T, T+1)$.
- In contrast to Packet 9, there is a multiplicative labor effectiveness factor $\beta(T)$ appearing within the period- T aggregate production function F in equation (M8), classified as period- T predetermined. The variable $\beta(T+1)$ is classified as period- T endogenous for Model M^* because its solution value is determined by equation (M10) and other period- T equations for Model M^* . Also, in contrast to Packet 9, in Model M^* the TFP coefficient A for the aggregate production function is set equal to 1 for all periods $T \geq 1$ and the growth rate u for raw labor N is set equal to zero.
- Observe that the first six equations (M1)-(M6) for Model M^* in Exercise 3 are identical to the six equations (E1)-(E6) for model MOD in Exercise 2, apart from the appearance in (M6) of the real interest rate R rather than the nominal interest rate R^N . You should make use of this fact in Exercise 3 to save yourself some calculations.

QUESTION 1 [10 POINTS TOTAL]:

Part Q1.A [6.5 points total; 1/2 pt for each period-1 endogenous variable]: Using a careful 4-diagram graphical analysis as illustrated in Packet 9, determine for each of the 13 period-1 endogenous variables for Model M^* whether its NewSol(1) solution value resulting under the higher income tax rate t' is larger, the same, smaller, or indeterminate in size relative to its corresponding BenchSol(1) solution value resulting under the original income tax rate t^o .

For example, for the particular endogenous variable Y (real GDP), can you sign the difference $[Y'(1) - Y^o(1)]$? Why or why not? Be sure all of your graphs are carefully labeled and explained, and that all of your assertions are carefully justified *in terms of Model M^* assumptions*.

Part Q1.B [3.5 points]: Provide a *careful economic interpretation* for your findings in Part Q1.A. That is, carefully discuss what you believe to be the *economic* meaning of these findings. Don't just restate the mathematical findings.

QUESTION 2 [10 POINTS TOTAL]:

Part Q2.A [6.5 points total; 1/2 point for each period-2 endogenous variable]: Using a careful 4-diagram graphical analysis as illustrated in Packet 9, determine for each of the 13 period-2 endogenous variables for Model M^* whether its NewSol(2) solution value resulting under the higher income tax rate t' is larger, the same, smaller, or indeterminate in size relative to its corresponding BenchSol(2) solution value resulting under the original income tax rate t^o .

For example, for the particular endogenous variable Y (real GDP), can you sign the difference $[Y'(2) - Y^o(2)]$? Why or why not? Be sure all of your graphs are carefully labeled and explained, and that all of your assertions are carefully justified *in terms of Model M^* assumptions*.

Part Q2.B [3.5 points]: Provide a *careful economic interpretation* for your findings in Part Q2.A. That is, carefully discuss what you believe to be the *economic* meaning of these findings. Don't just restate the mathematical findings.