Notes on Wilhite

(*Computational Economics,* 2001)

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https://www2.econ.iastate.edu/tesfatsi/

Main Reference:

Allen Wilhite, **"Bilateral Trade and `Small World' Networks,"** *Computational Economics* 18 (2001), 49-64

https://www2.econ.iastate.edu/tesfatsi/SmallWorldNetworksBilateralTrade.Wilhite.pdf

NOTE: **For details about the C++ Program implementation, contact the author Allen Wilhite at** wilhitea@email.uah.edu

Small-World Networks & Economics

A number of ACE researchers have begun to consider small-world networks in relation to economic processes.

For example, Wilhite (2001) uses an ACE model of a bilateral exchange economy to explore the consequences of restricting trade to four different types of networks, including a small-world trade network.

Small-World Network

As defined by Watts and Strogatz (1998), a ***Small-World Network*** is a network that can be represented as a simple connected graph G exhibiting two properties:

- ***Global Reach:*** The presence of "short cut" connections between vertices results in short characteristic path length L(G).

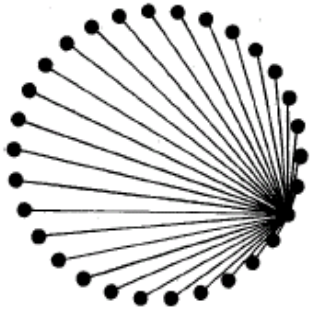
- ***Local Connectivity:*** Each vertex of G is linked to a relatively well-connected set of neighboring vertices, resulting in a large value for the clustering coefficient C(G).

Wilhite's Basic Approach

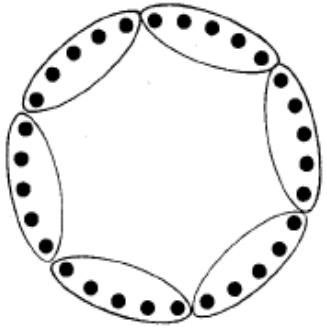
Examine the trade-off between market efficiency and transaction costs under four types of trade networks:

- ***Completely connected trade network*** (every trader can trade with every other trader);
- ***Locally disconnected trade network*** (disjoint trade groups);
- ***Locally connected trade network*** consisting of trade groups aligned around a ring with a 1-trader overlap at each meeting point;
- ***Small-world trade network*** constructed from the locally connected trade networks by permitting from 1 to 5 randomly specified short-cut trade links between members of non-adjacent trade groups.

Four Possible Trade Networks

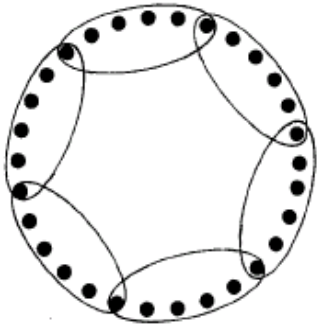


Panel a:
Global Network
Trade routes for one trader

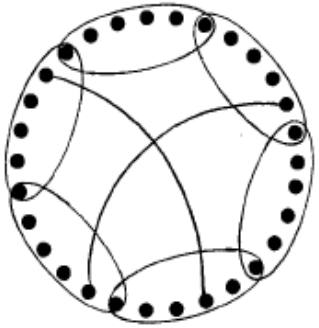


Panel b:
Local Disconnected Network
six groups, five agents per group

Note: **Depicted links are for a *typical* trader in the global network**



Panel c:
Local Connected Network
six groups, six agents per group



Panel d:
Small-world Network
two crossover agents

Figure 1. Sketches of four networks (population: 30 agents).

Basic Wilhite Model

- Exchange economy with 2 durable (non-perishable) goods: (1) a good g_1 that must be traded in whole units; and (2) an "infinitely divisible" good g_2.

- 500 traders initially endowed with ***random*** positive amounts of g_1 and g_2 (g_2 = numeraire good, i.e., the price of g_1 is expressed as units of g_2 per unit of g_1)

- Each trader i has the same form of utility function measuring preferences for the two goods:

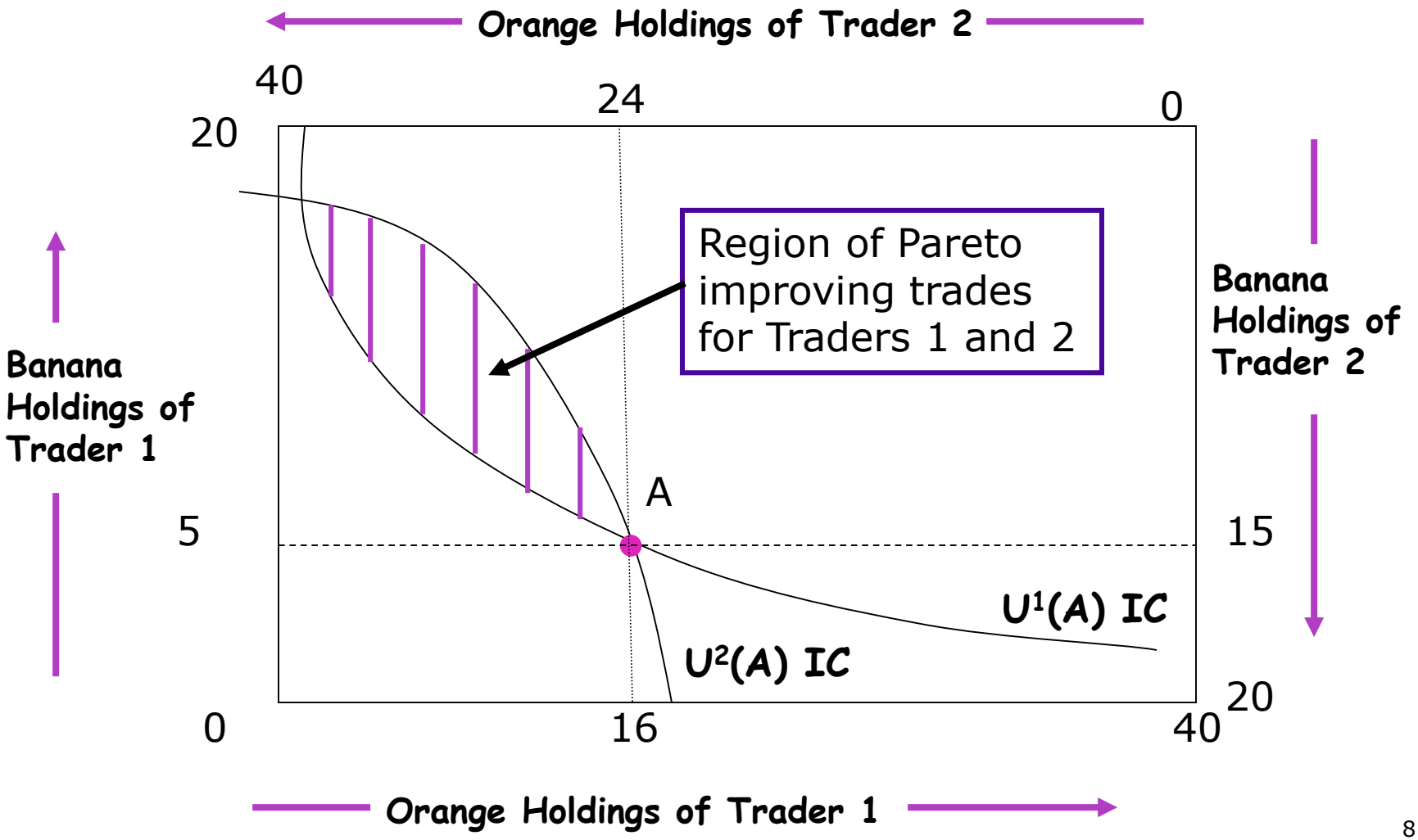
$$U^i = g^i_1 g^i_2 \text{ , } i = 1,\ldots,500$$

- Traders are rational, non-strategic, and myopic agents who try to improve their utility in each period by voluntary feasible trades of g_1 and g_2.

Basic Model…Continued

- Opportunity for mutually beneficial trade exists for two traders if they have different reservation prices ("marginal rates of substitution" MRS) giving the rates at which they are ***just willing*** to exchange g_2 for g_1.
- To see the intuition for this, consider an "Edgeworth Box" pure exchange economy with two goods - oranges g_1 and bananas g_2 in fixed supplies 40 and 20 – that are currently allocated between Traders 1 and 2 at point A.
  - $MRS^i(A) = [\Delta U^i(A)/\Delta g^i_1]/\Delta U^i(A)/\Delta g^i_2] = \Delta g^i_2/\Delta g^i_1 = g^i_2/g^i_1$
  - $U^i(A)$ IC = Indifference Curve of Trader i passing through A
  - $MRS^i(A)$ = -1 times the slope of Trader i's indifference curve passing through A

Edgeworth Box Illustration



Basic Model…Continued

- Each trader is limited to trades within its own particular neighborhood as determined by the trade network.
- By random selection (without replacement), each trader in each neighborhood searches for trade partners within its neighborhood and selects a trade partner offering a "best" mutually beneficial price (if such a trade partner exists).
- Reservation prices are assumed to be truthfully revealed.

Basic Model…Continued

- Whenever a suitable pair of trade partners i and j is determined, the two traders trade at the following "split the difference" price as long as the resulting trades are feasible and mutually beneficial:

$$p_{i,j} = \frac{g_2^i + g_2^j}{g_1^i + g_1^j} \quad i, j \in \{1, \ldots, 500\}.$$

Note: $p_{i,j} = MRS^i \bullet [G^i_1] + MRS^j \bullet [1 - G^i_1]$,

where $G^i_1 = g^i_1/[g^i_1+g^j_1]$

Experimental Design: Four Distinct Trade Networks

Global network:

(i)	number of groups:	1
(ii)	agents in each group:	500
(iii)	end-agents overlap?	no
(iv)	number of crossovers:	0

Local connected network:

(i)	number of groups:	50
(ii)	agents in each group:	11
(iii)	end-agents overlap?	yes
(iv)	number of crossovers:	0

Local disconnected network

(i)	number of groups:	50
(ii)	agents in each group:	10
(iii)	end-agents overlap?	no
(iv)	number of crossovers:	0

Small-world network

(i)	number of groups:	50
(ii)	agents in each group:	11
(iii)	end-agents overlap?	yes
(iv)	number of crossovers:	5

50 runs with the same 500-trader populations were conducted for each of the four tested trade networks

Illustrative depiction of the four tested trade networks for a 30-tradereconomy:

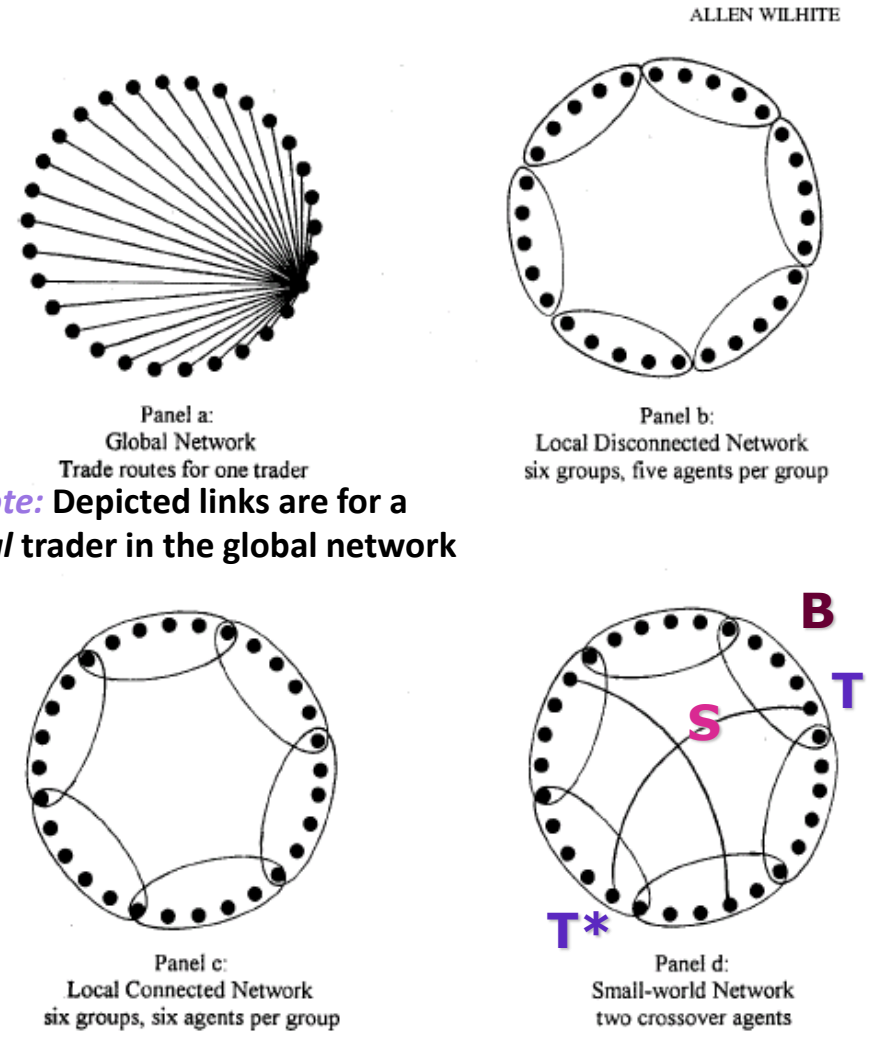


Figure 1. Sketches of four networks (population: 30 agents).

Note: **Depicted links are for a *typical* trader in the global network**

Note:
The short-cut **S** adds one more trader **T*** to group **B**, but only trader **T** in **B** can directly trade with **T*.**

Key Questions Examined

H1: Price convergence: Is there a significant difference in the dispersion of prices across each trade network?

H2: Speed of convergence: Do the different trade networks require a significantly different number of rounds of trading to reach their steady state?

H3: Number of trades: Is there a difference in the number of trades it takes for each network to reach its steady state?

H4: Search: Is there a difference in the amount of search and negotiation in each trade network?

Key Experimental Findings

Note: *Round* = One pass through all traders as initiators of trades

***Equilibrium* = No more mutually beneficial trade opportunities**

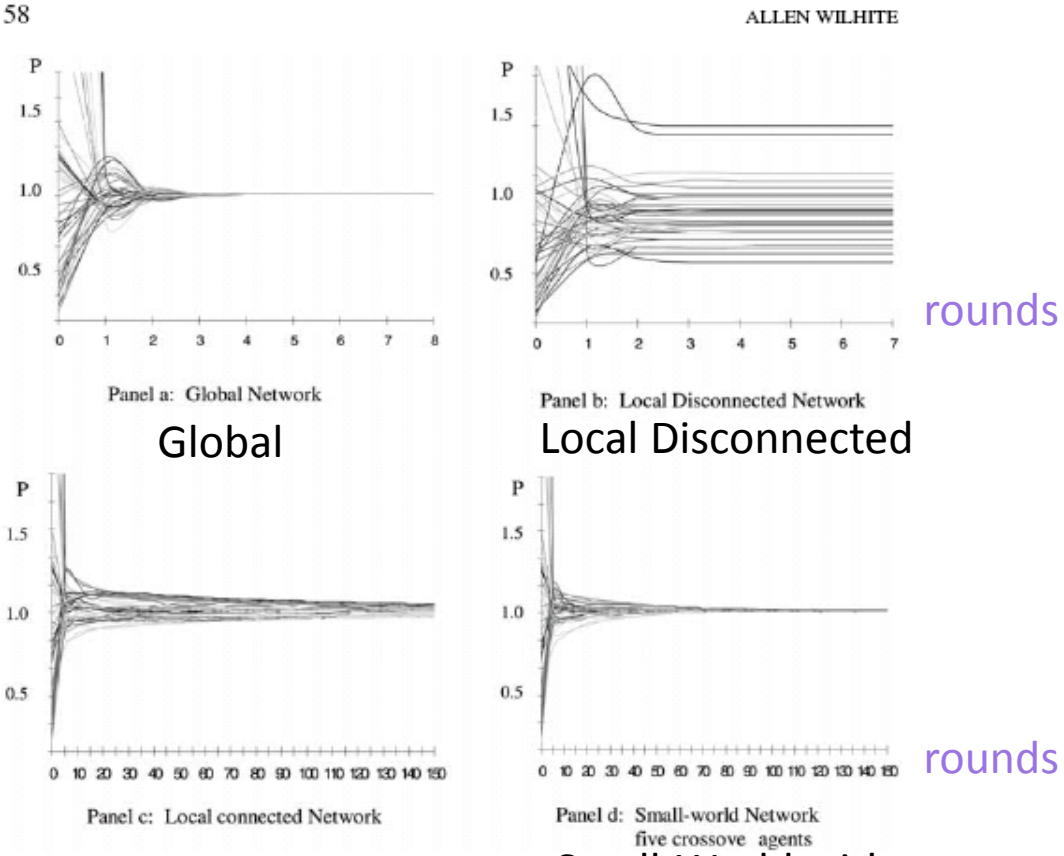
Table I. Average equilibrium characteristics. [a]

	Prices (standard deviation)	Rounds	Total trades	Total searches
Global network	1.0046 (0.00168)	8.08	1953.38	2,015,960 *
Local disconnected network	1.0396 (0.2771)	7.02	1727.7	31,590
Local connected network	1.0048 (0.0146)	497.14	93,975.72	2,734,270
Small-world network	1.0045 (0.00724)	242.54	45,944.56	1,236,954

[a] Averages calculated from 50 simulations of each network configuration.

* 2,015,960 = [500 x 499 x 8.08] = 500 traders seeking best price from 499 other traders for 8.08 rounds

Price Convergence in the Four Networks



Price Convergence for Different Numbers of Crossover Traders

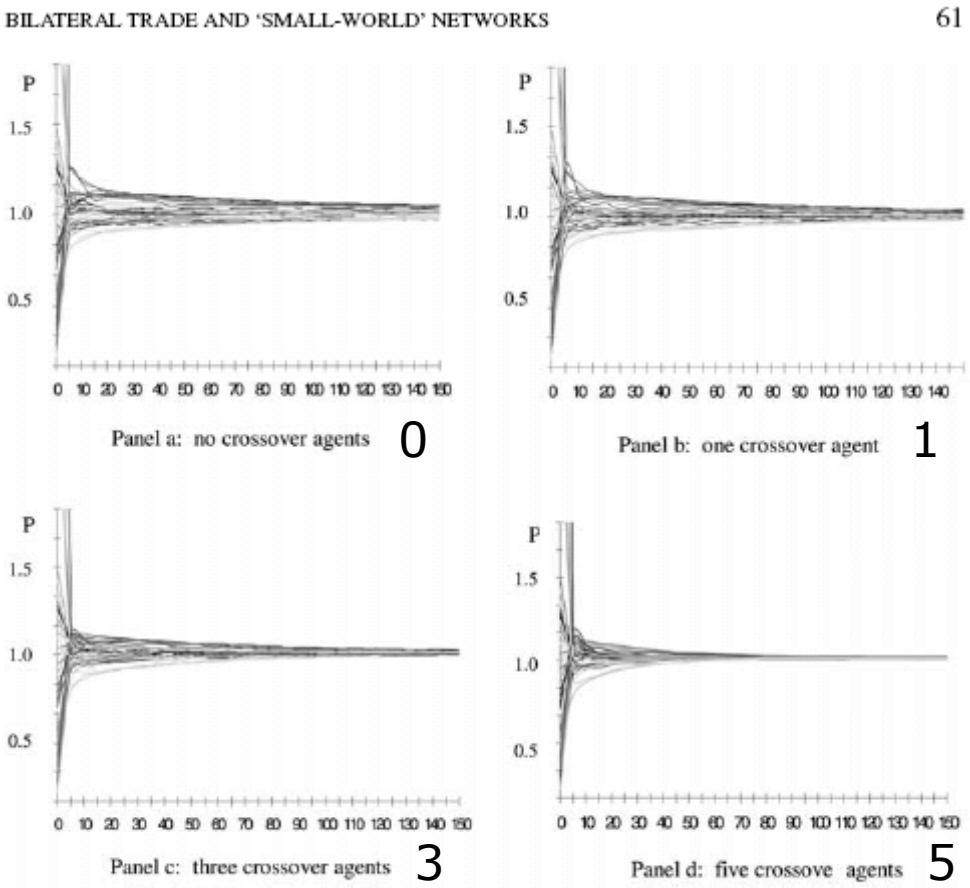


Figure 3. Dynamic price adjustment: adding crossover agents.

Relative Wealth of Crossover Traders

Table III. Wealth of crossover agents.[a]

	Agent *A*	Agent *B*	Agent *C*	Agent *D*	Agent *E*
0 crossovers	1929.61	887.73	1267.45	670.246	2059.95
1 crossover	**1997.17**	874.167	1269.23	675.557	2059.83
2 crossovers	1990.07	**1385.97**	1267.61	677.555	2058
3 crossovers	1984.94	1388.68	**1318.54**	677.591	2062.74
4 crossovers	1991.28	1389.3	1308.88	**985.451**	2066.52
5 crossovers	1981.82	1376.47	1307.04	981.252	**2105.27**

[a] The boldfaced number is the point at which that particular agent established a bridge to another trade group. Wealth from population #23, seed number 18847.

Summary of Key Findings

- The small-world trade network with 5 crossover traders provides most of the market-efficiency advantages of the completely connected trade network while retaining most of the transaction cost economies of the locally connected trade network.
- Wilhite's findings also suggest that there ***might*** exist private micro-level incentives for the ***formation*** of small-world trade networks.
- Specifically, the traders in the locally connected network who become crossover traders in the small-world trade network tend to amass greater wealth.

Extensions?

- For economic-social networks it is not satisfactory to focus solely on the implications of a fixed or parametrically varied network structure.
- Feedback mechanisms at work in economic-social networks can result in endogenous changes in the network structure over time.
- **EXAMPLE:** Network structure (whom you trade with) can affect how you behave in trading relationships, which in turn can affect whom you trade with in the future (***e.g. choice and refusal of trade partners***).