# DISCOVERING ARTIFICIAL ECONOMICS

## How Agents Learn and Economies Evolve

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# Sheep, Explorers and Phase Transitions

*"I have supposed a Human Being to be capable of various physical states,*
*and varying degrees of consciousness, as follows:*
*(a) the ordinary state, with no consciousness of the presence of Fairies;*
*(b) the eerie state, in which, while conscious of actual surroundings,*
*he is also conscious of the presence of Fairies;*
*(c) a form of trance, in which, while unconscious of actual surroundings, and apparently asleep,*
*he (i.e. his immaterial essence) migrates to other scenes in the actual world, or in Fairyland,*
*and is conscious of the presence of Fairies."*
LEWIS CARROLL

## {A}The Fallacy of Composition{/A}

In the previous chapter, we learnt two important things about how the economy works. First, human decisionmaking reflects the *different* beliefs and expectations of individuals. Second, the *interactions* beween these different individuals can produce unexpected collective outcomes. In turn, it's the evolution of these collective outcomes that shapes each agent's future behaviour. *What agents believe affects what happens to the economy and, in turn, what happens to the economy affects what agents believe*. The impacts of this positive feedback loop are what this book is all about. We call it coevolutionary learning.

For the time being, we'll regard the first problem as a *psychological* one and the second as a *systems* problem.[1] Having looked at aspects of psychology in the previous chapter, our aim in this chapter is to look more deeply into the systems problem. This problem lies at the core of the complexity issue. It's not difficult to understand why. All of us have the feeling that our economy is complex . Why is it so complex? One reason is that it involves a very large number of elements. In a major city economy, for example,

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[1] By the end of this chapter, however, we'll argue that *both* are systems problems; because a single human brain is just as much a complex adaptive system as a group of human brains.

several million human agents can link together in myriads of different ways. In a national economy, an even greater number of interactions can arise. Adopting this view of economic complexity is just one way of looking at the problem. It turns out to be a *combinatorial* view.

Most of us aren't expected to know very much about the whole economy and the way it works. What we often know (or can find out) are the prices of things we buy or sell, the interest rates at which we borrow or lend, and a little about the alternative ways we might earn our living and spend our money. Beyond these personal aspects, however, we tend to think of the rest of the economy as some giant accounting system, capable of balancing out all those transactions resulting from *specific* patterns of interaction between all the agents involved. Some of us believe we know more than this, others feel that they know even less. Most of us simply take our economy for granted, partly because we don't understand the way it really works.

What we tend to forget is that any specific pattern of transactions, whether in equilibrium or disequilibrium, is just one collective possibility. It's nothing more than one plausible set of agents' interactions from among many candidates. The magical thing is that, somehow, all of these chosen transactions seem to get coordinated. We have little idea of *how* this fantastically complex system selects this particular pattern of interactions, manages to balanced them out, and then somehow decides where to go next. Because inductive agents learn and adapt, forever changing the ways they choose to interact, a coevolving economy is actually open-ended. There's simply no way we can know how each and every agent is going to behave, least of all what the economy as a whole will do under different conditions. Future outcomes depend on the historical trajectory of choices made along the way.

Most economists acknowledge that what seems to be true for individuals isn't always true for society as a whole. Conversely, what seems to be true for all may be quite false for any one individual. In stressing that things aren't always what they seem at first, Paul Samuelson provides some paradoxical examples in the form of the following *true* statements:[2]

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1. – If all farmers work hard and nature cooperates to produce a bumper crop, total income may *fall*, and probably will;
2. – *One* man may solve his own unemployment problem by great ingenuity in finding a job or by a willingness to work for less, but *all* cannot necessarily solve their job problems in this way;
3. – Higher prices *for one industry* may benefit its firms; but if the prices of *everything* bought and sold increased in the same proportion, *no one* would be better off.
4. – It may pay the United States to *reduce* tariffs charged on goods imported, even if *other* countries refuse to lower their tariff barriers;
5. – *Attempts* of individuals to save more in depression *may lessen the total* of the community's savings;
6. – What's prudent behaviour for an *individual* may at times be folly for a *nation*.

Many of the above paradoxes hinge upon a single confusion or fallacy. Logicians have dubbed it the "fallacy of composition," In books on logic, you can find the following definition:

**FALLACY OF COMPOSITION**:
A fallacy in which what is true of a part is, on that account alone,
alleged to be also necessarily true.

The six statements mentioned above are typical of the many instances of the fallacy of composition that appear in economics texts. In the course of books like Samuelson's, these paradoxes are resolved. There are no magic formulas or hidden tricks. Each is explained in terms of standard economic principles. Some fallacies are due to comparative price or quantity changes at different levels of the economy, while others are due to structural properties of the whole economy. The interested reader might care to attempt an explanation of each.

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[2] See Samuelson (1976), page 14.

Our view is that paradoxical statements typically chosen to illustrate the fallacy of composition (like the six mentioned above) hardly scratch the surface compared to the full set of paradoxes that can arise. Furthermore, they're a select group which *can* be resolved with the conventional static equilibrium view of the economy. In other words, most are traps which exist inside the world of stasis only. Although they're treated as if they're exceptions to the rule, what's overlooked is the fact that the truly challenging economic paradoxes arise in that other world of economics: the world of *morphogenesis.* Many have nothing to do with equilibrium prices, but are due to the positive feedback loop associated with learning from others and from collective experiences. In other words, they're governed by the coevolutionary learning processes in the economy.

A central difficulty in the economy, then, is one of *dynamic interdependencies*. How do we find a way of handling the huge variety of behaviour – both individual and collective – that can arise when economic agents interact over space and time? Complexity arises, then, because of the ways in which economic agents choose to work together. Furthermore, unusual collective properties can arise if the agents choose to interact in particular ways. We caught a glimpse of some of these unexpected phenomena in Schelling's segregation model (Chapter 1) and Arthur's bar problem (Chapter 2). The point to remember is that we need to know what the agents are thinking and doing interactively over time, not simply what prices they're paying at specific points in time.

Before we probe the perplexing issue of dynamics more deeply, consider the following example of *how* the systems problem comes about. Instead of sandpiles, this simplifying illustration involves action on a billiard table. If you roll a billiard ball across such a table, any reasonable player can predict the path that it will follow. If you roll two balls across the table at the same time, it's still not difficult to calculate each ball's path individually. Once you add a few more balls, however, there's a strong chance that some balls will bump into others. The problem of keeping track of every ball's path and collision possibilities becomes more difficult. Now imagine what would happen if, with the help of friends, you rolled 100 balls across the table at the same time! They'd be bumping into each other all the time. Predicting their individual paths would be impossible. The system as a whole would become unpredictable at the individual level.

If you happen to be a Grand Master at Chess, you'd probably be interested in the pattern of behaviour over the entire table? Oddly enough, the problem begins to simplify again at this macrolevel. The more balls you roll, the less important it is to trace every ball exactly. Once there are lots of balls rolling across the table, a new behavioural paradigm takes over. Individual interactions start to average out. Suddenly you can make reasonable predictions about the average speed and the average time between collisions. Macroscopic patterns form. A new kind of order has arisen. These collective properties *emerge* unexpectedly out of the countless number of individual interactions.

The billiard ball model turns out to be the starting point for the kinetic theory of gases.[3] In a container full of many particles, all whizzing around and bumping into each other at all sorts of speeds, one property is always stable with respect to all the other elements (such as type of gas, shape of container, and so on). That property is temperature. A stable temperature is an emergent property because we would never have expected it by looking at the individual particles themselves, or the shape of the container, or anything else which is part of the whole system. By now, you've probably got the message; emergence is one of the hallmarks of complexity in both physical and socio-economic systems.

Certainly economic agents are more complicated than billiard balls. Nevertheless, the notions of interactive complexity and emergence are identical in both cases. Remember those music lovers at the El Farol bar. Nobody risked much chance of bumping into anyone else if the bar wasn't crowded. But as soon as the crowd grew to more than sixty, the chances of brushing up against one or two pushing-and-shoving bar louts were pretty high. From week to week, it was impossible to guess how many would turn up next Thursday evening. Despite all this uncertainty at the individual level, the mean attendance over time converged to a predictable value. Order emerged from seemingly random behaviour.

## {A}Irreducible Interactions{/A}

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[3] See Bossomaier and Green (1998).

There seems to be a consistent message here about simple and complex economic systems, at least in terms of two key properties. By and large, simple economic systems are *homogeneous* and *weakly-interactive*. Individual beliefs and expectations must be sufficiently uniform, and the level of interactions sufficiently weak or trivial, for us to be able to predict collective patterns of behaviour with any confidence. By way of comparison, complex economies are *heterogeneous* and *strongly-interactive*. They lie beyond the point where individual behaviour can be discerned with any degree of confidence. Somehow, the whole economic system gets transformed into a qualitatively different behavioural state.

What we find is a high degree of irregularities together with heterogeneity of individual behaviour. The surprising thing is that intrinsic unpredictability of individual behaviour does not imply any unpredictability of collective behaviour. Furthermore, any failure to predict does not imply a failure to understand or explain. Not knowing the full details, we may nevertheless build theories that seek to explain the *generic properties*. Collective order may depend little on the details of structure and function. This was the point of the billiard-ball example and the bar problem. Despite our ignorance of all the individual elements, we can still uncover interesting collective features like statistical averages.

[Fig. 3.1 near here]

Are such distinctions between simple and complex systems fundamentally important in economic life? I believe so. Plenty of examples may be found. Even when people appear to be doing very simple things - like driving a car, buying food or going to the movies - their pattern of interactions can quickly add up to more complexity than we can handle. One example, familiar to all of us, is the unwelcome traffic jam. If the flow pattern of traffic on a roadway happens to approach a critical flow density, a qualitatively different kind of collective behaviour appears. The traffic changes from a free-flowing state to one in which stop-start waves propagate back and disrupt the flow discontinuously. Which flow pattern you meet depends on what others are doing, not just on what you choose to do. Technically, this kind of unexpected change is known as a *phase transition*. But now we're getting ahead of ourselves again.

Suppose that you're planning to go to the movies. Which one do you see? Even if the reviews in the papers and magazines have kept you up to date with what's on - expanding your *know-what*, so to speak - you'll still want to get other opinions. So you talk to your family and friends, to find out what they thought of the ones they saw. Which movie you decide to see will depend on what you learn from other people, people who've already gone through similar choice processes to the one in which you're engaged. Afterwards, other people will ask you what film you saw and what you thought of it. Then you can boast a good degree of *know-whether*, i.e. whether any of the ones you've seen are worthy of their attention.

If you also happen to follow the fortunes of the recent releases from the major studios, you may learn that one of the films you elected to see has taken off like a rocket. Later, however, it loses its momentum. It ends up making a nice profit. Another starts more slowly, but builds up an ever-increasing audience, doing well enough to earn a place on the all-time earners' list. Most of the others fizzle out, not even covering their production and distribution costs. You realize that you can't always pick the big winner beforehand. Because the movie selection business is strongly-interactive, individual market shares at the macrolevel are hard to predict. There are many subtle and surprising ways in which agent characteristics and connectivity structure affect the market shares of competing products.[4] We'll return to the issue of connectivity shortly.

As noted earlier, the defining characteristic of a coevolving economy is that some of its collective properties can't be predicted from our knowledge of the agents involved or their likely interactions. Even if we could recognize and understand the implications of all the two-way interactions between pairs of agents, we would still be ignorant about three-way, four-way, and larger groups of interaction. This suggests a second way of distinguishing between simple and complex economic systems. A complex economy is *never additive*. It behaves quite differently from what we'd expect by simply adding up

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[4] For an interesting information contagion model, designed to isolate the effects of informational feedback on the market-share allocation process, see Lane (1997).

these pairs, trios and quartets of interaction. Self-organizing economies are not additive; they're *emergent*. This is precisely why some of their collective behaviours can't be predicted in advance. There's simply no way of combining the parts into an aggregate when we're ignorant about the nature and extent of some of the interactions.

Aristotle may have been the first to recognize that the whole is something more than the sum of its parts. But it was the Scottish philosopher, David Hume, who impressed upon us the need to distinguish between the simple and the complex. Hume lived in the so-called Age of Enlightenment; along with such great French thinkers as Voltaire and Rousseau. His main work, *A Treatise of Human Nature*, was published when Hume was twenty-eight years old, though he claimed that the idea for the book came to him when he was only fifteen.

In Hume's day, there was a widespread belief in angels. Yes, we're talking about those human figures with wings. Many people of Hume's time also claimed that they had a very clear idea of heaven. According to Hume, however, "heaven" and "angels" are complex ideas. If we think about this for a moment, we quickly realize that our personal idea of "heaven" consists of a great many elements. It may include "pearly gates," "angels," "streets of gold", and so forth. Breaking heaven down into its various constituent parts doesn't solve the problem for us because each of the parts - pearly gates, angels and streets of gold - are still complex ideas in themselves.

The prize for the most amusing illustration of Aristotle's view should go to the systems scientist, John Casti. In recalling Mark Twain's tale about *Those Extraordinary Twins*, Siamese twins called Luigi and Angelo, Casti reminds us that the story was based on the lives of the first recorded Siamese twins in the real world.[5] These twins, Chang and Eng, were born in Siam in 1811 but ended up as American citizens. The truly fascinating thing was that both of them had an army of children: seven daughters and three sons for Chang, seven sons and five daughters for Eng. How they managed to be so productive is anyone's guess!

Hume and Casti were stressing the same point: if you want to study the behaviour of a system composed of several parts, breaking it up into its constituent parts and studying each of them separately won't always help you to understand the whole thing. It's pretty clear that Mark Twain, for one, would not have been the least bit interested if Chang and Eng had been separated at birth. Siamese twins are very special because they're linked together in an unusual manner. It's the connection itself which makes them interesting and unique. The result is a system far more complicated than that of a typical human being. In trying to understand the whole system "Chang-and-Eng", it's essential to take this connectivity into account.

Connectivity is not just a fundamental feature of living systems. As we showed in the example of movie selection, it's also an important feature of products which we buy and sell in our everyday economic activities. A car is an excellent example of connectivity. If you'd lived all your life in the remotest jungles of Africa, chances are that you might never have seen a car. Then one day you come to town and happen to see this strange object, the purpose of which is unknown to you. The functions of each of its components are carefully explained to you in your native language - what the carburettor does, where the fuel is injected, how the wheels turn on axles, and so on. But imagine if the aim of the whole exercise is never revealed. Chances are high that you might never guess that this strange object sitting before you is designed for one simple purpose - to transport human beings from one place to another. A car's detailed structure is complex, but its overall purpose is behaviourally simple.

As you may have guessed already, the ability of a car to move is an emergent simplicity, an ordered outcome that the car can carry out by virtue of its overall organization. So is the ability of a clock to "tell" the time or of a fan to cool the surrounding air.[6] Many products that we buy and sell possess regularities of behaviour that seem to transcend their own ingredients. We know what the emergent simplicities happen to be. But imagine if we didn't. Even after having all the intricate parts and

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[5] The tale of these twins can be found in a book addressing the possible creation of a "science of surprise." For the part on Twain's tale, see Casti (1994), pages 171-2.

[6] Time is rather special in other ways as well. Whether we think of it as flowing like sand, or turning on wheels within wheels, time escapes irretrievably. It simply keeps on keeping on. All that a clock really does is mark that progress for us. Timepieces don't really keep time, they just keep up with it!

mechanisms of a car or a clock or a fan explained to us in minute detail, we might never guess what the whole machine does.

Emergent simplicities can’t be deduced by a reductionist approach, i.e. reducing the apparent complicatedness of the problem by analyzing its constituent parts and then linking them together by relatively simple rules. Yet this is what much of economics attempts to do. Whole economies are subdivided into statistically convenient subclasses, like industries and households. These subclasses are often subdivided into even smaller parts, like jobs and persons. Then comes the problematical step. By making some simplifying assumptions about the behaviour of these smaller parts, economists draw conclusions about collective outcomes.

The reductionist strategy – take it apart, see what the pieces are, understand how they fit together – does help with simpler economic systems. But it runs into major headaches when the system of interest is truly complex. Regrettably, we cannot use the reductionist philosophy to explain more than the very simplest interactions between or within human populations. It simply won't work on more complicated problems.

Obviously, it’s difficult to predict the exact branchings of a complex coevolving system which involves living elements - like a human brain, a bustling city or an expanding economy. The sheer complexity of some human systems is mind-boggling. For example, a brain is a collection of immensely many parts. Estimates of just how many vary from 10 billion to 100 billion neurons. These parts are hooked up to each other in incredibly complex ways, most neurons being connected to several hundred others. Some are connected to thousands of others! Having so many connecting wires, it's impossible to even imagine what an accurate circuit diagram would look like. It would need a gigantic computer system just for its storage. Although we do understand some basic features of how this gigantic neural telephone system works, we know virtually nothing about the meaning of the messages flowing through it.

Speaking metaphorically, we’re left groping in the dark! Even an expert electronic engineer would have trouble understanding how a circuit worked if he didn't know what its components did or how they were linked together. This is precisely the situation in many parts of human society. Our cities are another example. Each of us knows very little about what other people and other organizations do and has limited knowledge of the circulatory patterns which the city generates. Yet, despite this uncertainty - or perhaps because of it - the city manages to survive! Most of the time it seems to be operating under the principle of "more of the same." Every now and then, however, it suffers abrupt and permanent change. The city's apparent equilibrium is suddenly punctuated by an avalanche of changes. To all intents and purposes, it undergoes a phase transition.

Phase transitions are quite commonplace in many physical systems. A lesser known fact is that they also occur in schools of fish, in human brains and on city highways. When the interactions between the components of the system are sufficiently dense, and when those interactions add up in such a way as to make for large-scale correlations, then a different kind of entity emerges. Remember that sandpile behaviour we met in Chapter One. Self-organized criticality takes hold. The fascinating thing is that this new entity is on a higher level of organization than its constituents. It obeys certain laws of its own. These higher-level laws are sometimes very simple.

The collective behaviour of crowds at a concert or voters at a meeting can undergo phase transitions. Before a public meeting starts, many of the individuals in the audience are unknown to each other. What can they possibly have in common? Perhaps only one thing, but one *important* thing. Once they begin to listen to the same performers, they begin to influence each other - by laughing, applauding or interrupting. These interactive modes of behaviour tend to get "locked-in" very quickly. Their contagious nature creates *self-reinforcing* loops of interaction between performers and audience. The performers are aware of the interaction between themselves and the audience. They can sense the mood of the audience *as a whole*. In fact, their very success or failure relies heavily on the audience's collective psyche.

Such self-reinforcing feedback loops are of paramount importance in collective modes of behaviour. They help to explain why a phase transition can arise. Once the density of interactions exceeds a critical threshold, then the collective character of the system can change unexpectedly. To further illustrate the pervasive nature of phase transitions, let's look at a toy experiment, one in which this kind of qualitative change is "catalysed" at critical thresholds of interaction.

{A}Getting Well-Connected{/A}

Simpler toy problems are useful in science because they allow us to gain insight into more complicated, real-world ones. The toy problem of interest here involves *random graphs*. Before we launch into a discussion of random graphs, however, a few introductory words about graph theory and modelling are in order.

Conventional economic modelling involves postulating causal relationships between known variables of the problem. These variables are often measured over aggregates of agents. Such models attempt to formalize relationships in the language of mathematics. As we learnt in the previous section, and will find throughout the course of this book, it may be impossible to model a truly complex system in this way.[7] In many ecological and economic systems, there are simply far too many variables and far too many interactions to measure. Some of our forebearers were well aware of this difficulty. For example, Joseph Schumpeter delivered a cautionary message about the use of economic models more than fifty years ago: "*The process of social life is a function of so many variables, many of which are not amenable to anything like measurement, that even mere diagnosis of a given state of things becomes a doubtful matter quite apart from the formidable sources of error that open up as soon as we attempt prognosis*."[8]

In many economic situations, the formulation and use of such conventional mathematical models is highly questionable. This is where graph theory can provide an alternative means of studying the relational processes involved. A graph is simply a set of vertices or *nodes* together with a set of edges or *links* connecting certain pairs of nodes. If each link also has a specific direction (i.e. a beginning and an end), then we call it a

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[7] Here we should distinguish conventional modelling from simulation. Many complex economic systems may be “modelled” with the aid of *agent-based simulation*, a new way of doing science and a topic which is discussed in later chapters.

[8] See Schumpeter (1934).

directed graph or *digraph*. Furthermore, if the strength of the causal relationship can be represented by a real number, the process results in a *weighted digraph*. This is simply a digraph in which every link has a real number associated with it.

We'll use a special kind of weighted digraph in this chapter. It's called a *signed digraph*, and is simply a weighted digraph in which the weightings are either +1 or -1. It usually corresponds to a situation where we can identify possible interactions or relationships between pairs of socio-economic variables, but only to the extent of deciding whether they're positive or negative for the pairs involved. We simply don't know enough about the strength of the various interactions to assign real numbers to them. Even if we can measure the strength of an interaction at one point in time, we can't be sure that the same relationship will prevail at another point in time.

Take a look at Figure 3.2. It's a signed digraph representing the causal linkages in a key subsystem of an urban economy: a waste disposal system. The arrows indicate the direction of influences. A + sign indicates that the changes occur in the same direction, but not necessarily positively. For example, the + between P and G indicates that an *increase* in the population of the city causes an *increase* in the amount of garbage per unit area. At the same time, it also indicates that a *decrease* in the population of the city causes a *decrease* in the amount of garbage per unit area. The - between S and D indicates that an *increase* in sanitation facilities causes a *decrease* in the number of diseases. At the same time, it also indicates that a *decrease* in sanitation facilities causes an *increase* in the number of diseases.

[Fig. 3.2 near here]

Note how some of the arrows form loops. For example, there's a loop of arrows from P to M, M to C, and then from C back to P. A loop indicates a mutual causal relationship. By this we mean that an initial influence on an element comes back to amplify itself by way of other elements. For example, in the loop P-M-C-P, an increase in the city's population causes an increase in modernization, which in turn increases migration into the city, which in turn increases the population in the city. In short, an increase in population causes a further increase in population through modernization and migration. On the other hand, a decrease in population causes a further decrease in population through decreased modernization and decreased migration.

In such a loop, therefore, each element has a positive influence on all other elements, either directly or indirectly, and therefore each element influences itself positively through other elements. The thing which distinguishes this approach from typical closed-form modelling is that there's no hierarchical causal priority in any of the elements. That's why the loop depicts a *mutual* causal relationship. None of the elements are assumed to be the principal causal element affecting the others. The resulting mutual causal relationship in this loop is *deviation-amplifying*. For simplicity, we'll refer to it as a *positive* feedback loop.

Now look at the loop P-G-B-D-P. This loop contains a negative influence from D to P. An *increase* in population causes an *increase* in the amount of garbage per unit area, which in turn causes an *increase* in the number of bacteria per unit area, which in turn causes an *increase* in the number of diseases, which in turn causes a *decrease* in population. In short, an increase in population causes a decrease in population through garbage, bacteria and diseases. On the the other hand, a decrease in population causes a decrease in garbage, bacteria and diseases, and thus causes an increase in population. In this loop, therefore, any change in population is counteracted by itself. The mutual causal relationship in this loop is *deviation-counteracting*.[9] Such a deviation-counteracting process may result in stabilization or oscillation, depending on the time lag involved. We'll refer to it as a *negative* feedback loop.

The loop P-M-S-D-P has *two* negative influences. An increase in population causes causes an increase in modernization, which in turn causes an increase in sanitation facilities, which in turn causes a decrease in the number of bacteria per unit area, which in turn causes a decrease in the number of diseases, which in turn causes an increase in

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[9] According to Magoroh Maruyama, the field of cybernetics has been more-or-less identified as a science of self-regulating and equilibrating systems. By focusing mostly on the *deviation-counteracting* aspects of mutual causal relationships, cyberneticians have paid less attention to systems in which the mutual causal relationships are *deviation-amplifying*. Yet such systems are ubiquitous in nature and in human society. For a more complete discussion of the distinctions, see Maruyama (1963).

population. This is therefore a deviation-amplifying or positive feedback loop. The two negative influences cancel each other out and become positive overall. In general, any feedback loop with an *even* number of negative influences is *positive*, and any feedback loop with an *odd* number of negative influences is *negative*. An economy contains many positive as well as negative feedback loops. What matters most is that all the pertinent loops affecting the system of interest be identified and their influences considered.

Now that we're familiar with the idea, let's see how digraphs can be used to gain deeper insights into some economic complexities. Our next example focuses on a controversial issue in the global arena: the terms of trade between nations. Appropriate tariff and quota levels have been debated vigorously for decades, especially for agricultural products. During the nineties, for example, American newspapers were full of criticism of Japan's restrictive policy on beef imports from the United States. There's little doubt that the Japanese do play by different rules when it comes to agricultural goods. In the case of beef, they try to protect their cattle farmers and promote the "home-grown" product. In today's "borderless" world, however, it's not that easy to figure out what's really in a nation's best interest anyway.[10] Who stands to gain most from increased beef exports from the United States to Japan? Let's try to work through the arguments, visualizing the relationships between the key factors raised in the form of a digraph.

[Fig. 3.3 near here]

Conventional economic wisdom – like the Hecksher-Ohlin theory – attributes international trade to underlying differences between countries. The key idea is that each country produces and exports goods that reflect that country's comparative advantage over other countries. In determining the volume of beef exports from the U.S. to Japan, for example, four variables seem to be important: consumption levels of comparable beef in Japan (B), the size of Japan's cattle herds (J), exports of American beef to Japan (A),

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[10] For a penetrating insight into misleading trade statistics in the global economy, including an assertion that the United States has no "foreign" trade as long as it's buying in dollars, see Ohmae (1994, especially Chapter 9).

and the dollar/yen exchange rate (X). We've depicted these causal linkages in Figure 3.3. Having witnessed rising family incomes in Japan, the Americans reasoned that growing demand for beef in Japan could be satisfied by expanding American beef shipments to Japan. This is indicated by the positive sign on the arrow from B to A.

While this reasoning seems sound, it really only scratches the surface. In their desperation to get American beef onto Japan's dinner tables, the U.S. Government failed to address some related issues of fundamental importance. Firstly, is beef that's grown in America "American"? For that matter, are the cattle raised in Japan really "Japanese"? The answers may look obvious. But they're not obvious. Why? One complicating factor is that Japanese cattle are raised almost entirely on American grain. If more Japanese beef is eaten in Japan, then more American grain will be consumed. Although an increase in the size of Japan's herds may have a negative impact on imports of beef from America, almost certainly it will have a positive impact on imports of "American" grain. Furthermore, grain agriculture has much higher levels of productivity in the United States than does cattle raising. Thus the Americans are more likely to be able to win the grain race. Competition from the Australians and the Argentinians makes the beef race much tougher.

Another complication is the issue of ownership. Who owns the ranches, feedlots and packing plants that make up the lion's share of the beef industry in the United States? Japanese importers like Zenchiku Ltd. own many of them, mainly because it's easier for the Japanese to invest in the U.S. industry than for Americans to invest in Japan. They and others will soon be shipping in more beef from the United States than Americans do today. Japanese owners can control all stages of cattle raising and handling to meet their own market's requirements. They can also use Japanese know-how and repatriate profits back to Japan.

As Paul Krugman suggests, whenever a Japanese firm buys an American firm, we need to know whether that firm will be run differently, and if so, whether the U.S. economy will be hurt or helped by the difference.[11] It's not at all clear in what sense the

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[11] See Krugman (1994b), Chapter 11.

beef shipped by Zenchiku from the United States to Japan represents a net increase in American exports. Not only does growing Japanese ownership in the United States make it difficult to answer the two earlier questions. More importantly, it points to the growing interdependencies between the Americans and the rest of the world. Rising levels of trade and foreign ownership mean that *the U.S. economy is becoming increasingly multinational and interdependent*. In other words, the US-led global economy is rapidly becoming a *strongly-interactive* economy.

[Fig. 3.4 near here]

Now let's return to our main question: Will an increase in beef exports from the United States to Japan be of net benefit to the U.S. economy? The answer is far from obvious. Less obvious linkages and relationships need to be factored in when an issue like this is debated. What we're seeing in Figure 3.4 is just a small sample of the full range of economic interdependencies affecting beef exports between the two countries. When a change occurs in one part of this interlinked economy, it can lead to myriads of chain reactions elsewhere. A handful of these responses are large and direct, but many others are small and indirect. Their "global" repercussions may be unpredictable. Shades of Schelling's segregation model and Bak's sandpile avalanches spring to mind.

Now let's move on to the topic of random graphs. A random graph is similar to the graphs discussed earlier, except that the nodes are connected *at random* by a set of links. Although we know how the network graph looks at one point in time - i.e. which pairs of nodes are already connected – we've no way of knowing which pair of unconnected nodes may be connected at the next point in time. This means that we're less concerned with the direction and strength of pairs of interactions (like those shown in Figure 3.4), and more concerned with the overall pattern of connectivity that develops.[12]

This is the starting point for the toy problem we wish to discuss. To give it a physical interpretation, think of the nodes as "buttons" and the links as "threads."

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[12] For a fuller discussion of the connectivity properties of random graphs, see Erdos and Renyi (1960) and Kauffman (1993, 1995).

Imagine that alot of buttons lie scattered on a wooden floor. Choose any two buttons at random, pick them up, and connect them with a thread. After putting this pair down, randomly choose two more buttons and do the same. As you continue to do this, at first you'll mostly pick up buttons that you haven't threaded earlier. Sooner or later, however, you're likely to pick up a pair of buttons and find that you've already threaded one of them. When you thread that button again, you'll have linked together three buttons. As you go on choosing pairs of buttons randomly to link together with threads, you'll find that some of the buttons soon become interconnected into larger clusters. We've used a random number generator to produce the pattern of linkages shown in Figure 3.5.

[Fig. 3.5 near here]

The interesting thing to note is that random graphs exhibit very regular statistical behaviour as one tunes the ratio of threads to buttons. Let's denote this ratio by R. Once R passes the 0.5 mark, something seemingly magical occurs. All of a sudden most of the clusters become cross-connected into one giant structure![13] When this giant web forms, the majority of buttons are directly or indirectly connected to each other. As R approaches one, virtually all remaining isolated buttons and small clusters become cross-connected into the giant web. Note how closely the situation resembles the sudden transformation of a weakly-connected transportation network into a strongly-connected one, by the addition of a "critical link" connecting key subnetworks; or the sudden transformation of a weakly-connected social group into a strongly-connected one once there's unanimous acnowledgment of a common accord. It's the sandpile effect again!

This sudden and unexpected change in the size of the largest cluster, as R passes 0.5, is the signature of a nonlinear process we met in Chapter 1. It resembles a phase transition. The size of the largest cluster of buttons increases slowly at first, then rapidly, then slows again as the value of R increases further. Had we used an infinite number of buttons, then the size of the largest web would jump discontinuously from tiny to enormous as R passed 0.5. The steep part of the curve would become more vertical than it is in the figure. This is typical of a phase transition, just like when separate water molecules freeze to form a block of ice.

[Fig. 3.6 near here]

The point of this toy example is to highlight the nonlinear nature of transitions from a weakly-interactive to a strongly-interactive system. Technically speaking, an isotropic random graph crosses a threshold when the system passes from "nearly unconnected" to "nearly connected." Such threshold transitions are rampant in graph theory. Suddenly, many small clusters are cross-linked to form one large cluster. But it's also reminiscent of Bak's sandpile experiment and Schelling's segregation model. A state of self-organized criticality is reached once local interactions between individual elements are replaced by global communication throughout the whole system. Could this be some kind of universal law when it comes to the dynamics of complex systems? Perhaps it may apply to socio-economic systems?

The argument for universality is strengthened by insights into random behaviour stemming from another field, known as *percolation theory*. In a random medium like a porous stone, large scale penetration of water depends on the proportion of passages which are broad enough to allow water to pass along them. We'll call this proportion R. By simulating the stone's porous structure as a sequence of open and closed edges (or connected and unconnected links) on a square grid, large-scale water penetration is seen to be related to the existence of strongly (i.e. infinitely) connected clusters of open edges. When R = 0.25, the connected clusters of open edges are isolated and rather small. As R increases, however, the sizes of clusters increases. There's a critical value of R at which a "super cluster" forms, pervading the entire grid. As we throw in more and more open edges, suddenly we reach a threshold when large-scale connections emerge.

Just like the onset of self-organized criticality, the occurrence of a critical phenomenon is central to the process of percolation. One may surmise that the wetting of a stone is only a "local effect" when R is small, but becomes a "global effect" when R

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[13] Erdos and Renyi (1960) were the first to demonstrate this rapid transition, in which a single gigantic

reaches a critical value. Drinkers of Pernod acknowledge such phase transitions. Transparent Pernod is undisturbed by the addition of a small amount of water, but as more water is added drop by drop, an instant suddenly arrives when the mixture becomes cloudy. Recent simulation studies of bond percolation have shown that the random value of R at which such global paths appear is very close to 0.5.[14] It's the same ratio as the one that emerged in our toy example of random graphs. Is this a coincidence? We think not.

The physical theory of phase transitions and critical phenomena is well developed, together with multidimensional scaling and power laws. But what about phase transitions in socio-economic life. Do they apply to human behaviour? Consider what happens when a bunch of urban residents meet frequently to discuss an issue of common interest. As the intensity of their interaction increases, clusters of "like-minded" residents begin to emerge spontaneously. Like-minded residents don't know in advance that they're like-minded. They don't even know who their closest allies may be. These kinships emerge spontaneously during the meetings. Such like-minded clusters may do more than simply interact among themselves. To pursue their common interests more widely, eventually they may link up with other like-minded clusters; creating even larger clusters. Sounds familiar, doesn't it? People behaving like buttons and threads or grains of sand! It's precisely how the weak may grow strong.

In fact, the socialization processes by which performers enchant an audience, politicians sway voters, or common interest groups gain support are analogous to the toy problem of buttons and threads. People form clusters (e.g. political parties, unions, clubs) in order to pursue their joint interests. These weakly-interactive clusters can gather strength unexpectedly, especially with the help of a key catalyst or critical link. The latter can play a powerful role in the shaping of society as a whole. Electoral outcomes can be swayed spontaneously by charismatic or forceful agruments from one of the protagonists. The collective outcomes can be quite different from those intended or expected at the

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connected component emerges – linking most of the nodes!

[14] For a comprehensive discussion of the mathematical aspects of percolation theory, see Stauffer (1985) or Grimmett (1998).

outset by some individuals. Such unpredictable outcomes are further examples of emergent behaviour.

As we'll learn in Chapter 6, traffic jams on city highways are another form of emergent behaviour, springing from the collective interactions of a bunch of drivers on a road network. Once the critical flow density has been exceeded, smooth laminar flow changes abruptly to stop-start waves. Emergent phenomena can arise in many other socio-economic situations. Their common bond is that the population of interacting individuals "spontaneously" develop collective properties that were neither intended nor expected by individuals *a priori*. Order through fluctations again!

{A}Sheep and Explorers[/A}

Granted that strongly-interactive systems display self-organized criticality, an obvious question arises. Who triggers these phase transitions? Who are the architects of strongly-interactive socio-economic systems? The answer's pretty obvious. We all are. We, the interactors. But "we' aren't identical. Quite the opposite in fact. As we learnt in the previous chapter, our expectations, decisions and experiences vary greatly, even under identical circumstances. Because we're forced to reason inductively in situations where information is limited, and such reasoning is open-ended, our chosen behaviour usually differs.

Inductive reasoning places different demands on us as thinking individuals than the deductive metaphor. We're all constrained by our personal stock of know-ware and the cognitive abilities within us. Inductive reasoning involves pattern formation and pattern recognition, often by intuition and creativity. Clearly some people are more intuitive or creative than others. They're better at seeking and discovering novel solutions to problems. They're willing to experiment, adapt and instigate change. Others merely follow existing patterns, often resisting change under almost any circumstances. It's pretty clear that we don't possess the same catalytic potential" to create novel solutions or adapt to changing circumstances. Like the spectrum of light, cognitive equipment consists of a mixture of cognitive skills of varying intensities. Some of us are strongly creative, others only weakly creative; some of us are strongly adaptive, others only weakly adaptive.

For convenience, we'll classify all economic agents in terms of two extreme forms of behaviour. We'll call those who actively search for new possibilities *explorers* and those who prefer to remain with the status quo *sheep*.[15] The abovementioned spectrum of cognitive skills implies that we all possess sheep and explorer qualities, albeit in different doses. Pure explorers tend to be imaginative, creative, highly strung individuals who constantly search for better solutions to the problems they face. They're more inclined to reason inductively, to learn quickly and to adapt willingly to changing circumstances. Sheep are more placid, patient and resigned than explorers. Preferring to reason deductively, they're prone to choosing a well established pattern. They mostly cling to particular beliefs because they've worked well in the past. Sheep are slow learners who must accumulate a record of failure before discarding their favourite beliefs.

Success in the economic world, as in life itself, requires both these facets of behaviour. Yet the two traits are almost contradictory. First, an ability to organize one's behaviour so as to exploit the available information to the fullest extent possible. In short, sheep like to think deductively and act predictably. Second, an ability to ignore the available information to some extent and to "explore" beyond the boundaries of current knowledge. Explorers tend to think inductively and their decisions are unpredictable. Both kinds of behaviour can be found in all walks of economic life. For example, sheep and explorer strategies have been observed among fishing fleets searching for profitable fishing zones. The only difference is one of nomenclature. Nonequilibrium systems scientists like Peter Allen, who've studied the behaviour of fishing fleets, call the sheep "Cartesians" and the explorers "Stochasts". As Allen notes, the first group makes good use of information, but the second generates it! At the root of all creative activity lies an explorer, punctuating the restful equilibrium of the sheep with unexpected change.

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[15] The terms "sheep" and "explorers" were suggested by the traffic planner, Anthony Downs (1961), to explain how peak-hour traffic congestion causes some drivers to search actively for faster alternatives while others stubbornly tolerate the long delays caused by congestion. We'll explore this issue more deeply in Chapter 7.

[Fig 3.7 near here]

Because they’re driven by a constant struggle between creative (explorers) and conservative (sheep) forces, economies evolve in an unpredictable way. Over short time horizons, the deductive agent may outperform the inductive one. Deductively optimal behaviour can rule supreme in a world of stasis. Sheep can strut about with confidence in a frozen world which isn't doing anything or going anywhere. But not in that dynamic world of morphogenesis. Invariably, sheep will be found lacking in the long run. Unfortunately for them, the best performance doesn’t follow optimization principles. Instead it amounts to an adaptive compromise. Remember our findings from the Trader's Dilemma game in the last chapter. If the game’s played once only, the best strategy for each trader is to defect (i.e. leave empty bags). But a Nash equilibrium is only imperturbable because it compels all traders to think and act rationally. In the iterated version of the game, however, the best strategy is to cooperate. Emergent cooperation evolves out of noncooperation in the long run. Order for free again!

And the long run is where our main interest lies. To the extent that the real “laws” of economics exist at all, the key point to remember is that they can’t be fully understood by studying economic change within a time-frame which is short compared with the economy's overall evolution. It’s remarkable that the fallacy of a simple supply-demand equilibrium has persisted for so long. In the medium to long run, supply and demand functions cannot be specified in isolation of one another. They’re not independent functions of price. Each depends crucially on chance events in history - like the way in which fads start, rumours spread, and choices reinforce one another. Supply and demand affect each each other, as well as being subject to common factors like the media.

If we look at the key economic agents who control technology in the world of morphogenesis, such fallacies become more transparent. Once again, sheep and explorers can be found under different names. This time they're called *imitators* and *innovators*. Innovation is the domain of creative explorers. Whenever their exploratory search uncovers an area where positive feedbacks outweigh negative ones, then new growth-inducing development can occur. It’s only when we wish to rationalize about what’s happening that we insist there must have been some "pent-up" demand which justified the supply. We allow ourselves to slide back into the world of stasis. *There's really no static hill to climb*. The economic landscape's too complex for that. It's heaving and deforming incessantly because it's formed by the interacting agents themselves. All the agents are coevolving continuously, learning-by-interacting. Some are innovative explorers, others are imitative sheep. Once we admit even to just the presence of innovative or imitative mechanisms, then the potential demand for something becomes a *dynamic* variable which itself depends on the unfolding of events.

A small, growing band of economists have begun to treat technological change as an evolutionary process. Work carried out by Richard Nelson, Sidney Winter, and others has emphasized the role of innovations and analysed conditions under which firms should invest in innovations or imitate others. Some firms invest huge sums in innovation, thereby climbing up the the learning curve of a technological trajectory. Others simply copy the innovator. IBM invested in innovation; Compaq cloned, selling IBM imitations. Sheep follow explorers, just as long periods of relative stasis follow short periods of morphogenesis.

Research into technological evolution sometimes ignores the fact that evolution is actually *coevolution*.[16] As we've said already, agents change their minds when they interact with other agents. Explorers are constantly learning-by-interacting. So are technologies. They live in the niches afforded by other technologies. For example, the arrival of the automobile technology put the smithy's hammer out of business. But it also spawned new markets for traffic lights, gas stations, motels and drive-in food chains. We're all hustling our wares, creating and destroying niches for one another. Many of the goods and services in our economy are *intermediate* goods and services; they're used in the creation of other goods and services that are finally consumed by households.

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[16] Richard Nelson, Sidney Winter, Stuart Metcalfe, Giovanni Dosi, Gerry Silverberg and others have developed an interesting class of evolutionary models in which technology and the structure of industry are said to *coevolve*; see, for example, Nelson and Winter (1982). This process leads to productivity growth which is a statistical property of the system as a whole. The *technological* notion of coevolution described in their work complements the *behavioural* one discussed in this book.

Searching for new possibilities among this vast web of goods and services is essentially a stochastic activity. Multiple possibilities abound, but few choices or solutions are uniquely superior. New ideas, products, or methods are mostly recombinations of old ones. Schumpeter recognized that recombination is a valid way of defining economic development. He argued that the the carrying out of new combinations meant the different employment of the economic system's existing supplies of productive means, which may provide another definition of economic development.[17] In the chapters to follow, we'll look at several examples of how and when the recombination of old ideas takes place, what's recombined, and what's actually created.

How do we, as economic agents, trade off economic necessity against the elements of chance? Should we behave like low-risk sheep or dare to enter the domain of the high-rolling explorers? Putting it in more explicit economic terms, how do we decide between the certain prospect of earning modest profits now as against the uncertain prospect of earning far higher profits in an unknown future? Once again, this parallels the Traders' Dilemma game played once against the *iterated* version played many times. Perhaps the outcome of the iterated game can provide a clue to the answer. Over the longer term, not only do we search for novel solutions, but also for longlasting cooperative strategies with those agents whose custom we learn to value most. To soften the impacts of an uncertain trading environment, for example, merchants seek to develop bilateral trading agreements with their principal trading partners. These long-term agreements are designed to hedge against the volatility of a series of once-off transactions with different partners. Wherever the continuity of shipment and quality are important, supplier loyalty is likely to prevail.

There's another important reason for seeking bilateral trading agreements. Our choice of preferred partners is based on personal experience - what we've learnt about our potential trading partners. Once we've spent time and money identifying suitable partners, bilateral agreements enable us to "lock-in" the benefits of what we've learnt. Mutual advantages accrue to the partners. Our cooperative network grows. Learning-by-interacting proves to be an adaptive process out of which a desire for cooperation

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[17] See Schumpeter (1934).

emerges. We can capitalize jointly on what we've learnt. And capitalizing on what we've learnt is the key attribute of an explorer. In the next section, we'll look at an example of how we do capitalize on what we've learnt. Whether consciously or unconsciously, each of us behave like sheep or explorers in our everyday activities.

{A}Are You an Inductive Graph Theorist?{/A}

Most people use graph theory in their daily lives without ever realizing it. I don't mean that they're unsuspecting designers of signed digraphs, like the waste disposal system and the beef story we discussed earlier.[18] Instead, people use a much simpler kind of graph theory as they move about conducting their daily business. We all travel on networks of various kinds. For example, the London Underground Map (see Figure 3.8 below) is a network graph used by millions of commuters each year.[19] Although it only shows the rail connections between stations, that information assists Londoners to make informed decisions about the journeys they can make. For example, if the stations of embarkation and disembarkation are on the same Line, then the journey can be made without changing trains. Alternative routes can also be compared in terms of the number of intermediate stops between points of embarkation and disembarkation.

The London Underground System has several hundred stations. There are literally millions of possible ways of travelling between them all. It looks like a very big combinatorial headache for most commuters. Despite the network's apparent complexity, the above map and a few simple rules are sufficient to allow most commuters to select reasonable routes at a glance. Let's see how it works.

[Fig. 3.8 near here]

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[18] Perhaps one should not rule out this idea too quickly. Signed digraphs may be a natural way of attempting to simplify a complex set of relationships into manageable components. Such links and arrows may play a basic part in the "mental modelling" of human beings. For a discussion of the role of links, arrows and networks in human thought, see Johnson (1995).

[19] I am grateful to Jeff Johnson, a systems scientist at the Open University in Milton Keynes, for pointing this fact out to me; see Johnson (1995), pages 26-28.

Suppose that you, the reader, want to use the Loop for the very first time. You wish to catch a train from Victoria (near the bottom left of the map) to Notting Hill Gate (at the left centre). First you must search the map for routes or sequences of stations which will allow you to do this. You find two that seem feasible. One sequence is Victoria - Sloane Square - South Kensington - Gloucester Road - High Street Kensington - Notting Hill Gate. The other is Victoria - Green Park - Bond Street - Marble Arch - Lancaster Gate - Queensway - Notting Hill Gate.

Choosing between these two possibilities turns out to be a relatively simple task. You opt for the Sloane Square route for one of three reasons. First, you can make the whole journey on one line without changing trains. You'll simply get on a westbound Circle Line train at Victoria and duly arrive at Notting Hill Gate. If you took the second route, you'd need to change trains twice - at Green Park and again at Bond Street. Second, there are only four intervening stations on the first route compared to five on the second route. Third, you've been told that trains on the Circle Line are invariably more frequent than those on the Victoria and Jubilee Lines.

Because the problem is well-defined, you're able to deduce the optimal solution. Moreover, you feel confident about your choice. You have all the information you need to make an objectively rational decision. And you're absolutely correct! Most London commuters do take the Circle Line train to reach Notting Hill Gate from Victoria. Commuters have no need to resort to intuition or fancy guesswork. The problem is simple enough to be solved by deduction.

Now suppose that you've been making the journey from Victoria to Notting Hill Gate on a daily basis for several months. In fact, you've been using the Loop for some other journeys as well, most of them on a single line. From these commuting experiences, gradually you've formed a picture of the Loop system in your mind. Let's call it a "mental model." It's your own mental impression of how the Loop operates in time terms – trip times, delay times, line frequencies, etc. Of course, this model's only a crude, partial approximation to reality. It may even be flawed. But it's helped you to select routes for several months now and you've been satisfied with the resulting travel times. You begin to wonder if the commuting life of a London Loop traveller is always so simple!

I'm willing to bet that your mental model turns out to be rule-based.[20] By this I mean that your mental representation of the Loop system is formed and altered by the application of condition-action rules, which take the general form, IF (condition 1, condition 2, ...... condition n), THEN (action). In choosing between alternative route possibilities, for example, experience has taught you that an important rule to observe is:

> IF you can make your whole journey on one line
> THEN choose that line.

Another rule, which you may apply, is:

> IF trains run more frequently on one line
> THEN choose that line.

A third rule, which may form part of your mental model, is:

> IF one route has fewer intervening stations
> THEN choose that route.

Undoubtedly, your mental model will consist of more than just a set of IF/THEN rules. For example, experience may have taught you to apply the above rules in a different order on weekdays to weekends. The key point is that you're reasonably happy with the net result. Travel times experienced have fallen within your expectations. You

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[20] Rule-based mental models are central to the dynamic analysis of problem solving and induction described by Holland, Holyoak, Nisbett and Thagard (1986). The reader is directed to this source for a comprehensive discussion of the kind of inductive behaviour described in this book. This is not to imply that a rule-based approach to artificial intelligence is likely to succeed. To build a device that even approaches real intelligence would require a rule-based program far larger than anything that could be managed in a human's lifetime.

feel confident that you understand how the Loop system operates and that you can estimate travel times, albeit roughly.

Then, one day, you find the need to commute between a pair of stations that you haven't tried before. You must get from Baker Street to St. James's Park. Searching the map for feasible routes, you find a direct link (westbound) between these two stations on the Circle Line, with eleven intervening stations. There's a second route that goes via Victoria, Green Park and Bond Street, with only three intervening stations on this route. But you'll have to change trains at two of them (Victoria and Green Park). There's a third route via Victoria, Green Park, Oxford Circus and Regent's Park. This time there are four intervening stations, but again you'll have to change trains at two of them (Victoria and Oxford Circus).

If you apply your usual mental model, the direct route via the Circle Line is the obvious choice. But will it be the quickest? Suddenly you realize that you're facing a more complicated decision problem. Diabolically, you lack some key information. You've no way of knowing what the likely delays will be if you choose either of the indirect routes. How long will you have to wait when you change trains? The two indirect routes certainly *look* much shorter on the map, because there's only a few intervening stations. Should you relinquish familiar determinism (the direct approach) and test the elements of chance (unknown waiting times)? In other words, should you behave as a sheep or as an explorer?

The truth of the matter is that you simply don't have sufficient information to make a rational choice between these alternatives? Having never travelled on the Loop between St. James's Park and Baker Street before, you're forced to rely on intuition and an ounce of luck. But once you've made this journey several times, you can begin to form a more accurate picture of the relative merits of each alternative. If you're an explorer at heart, you'll test all three routes. Only by trying out the two indirect routes can you hope to estimate the relative frequency of trains on different lines and the average delays incurred by changing trains. Sheep tend to resist this kind of experimentation. They favour the certainty of familiarity, i.e. the direct option. By way of contrast, explorers tend to learn incessantly from their own experiments. Gradually they begin to form a more accurate impression of the typical behavioural patterns of the Loop System. Then they adapt and change routes accordingly.

In summary, explorers are adept at *learning-by-circulating*. They believe in testing and updating their own mental models of the Loop System on a regular basis. This means judging how well their favoured rules work when applied to the reality of their day-to-day experiences. They also compare these experiences with their prior expectations. If this experimentation suggests that their mental model may be inaccurate, then they discard some of their old rules and add some new ones in order to improve it.. Then they repeat the experiments. Testing, adapting, testing, adapting. This is the way of an inductive explorer. And this is how to travel on the road to know-ware.

## TABLE 1.1:
## Two Economic Worlds - The Simple and the Complex

| NECESSITY | CHANCE |
|---|---|
| Stasis | Morphogenesis |
| Resource-Based | Knowledge-Based |
| Unique Outcome | Multiple Outcomes |
| Equilibrium | Path-Dependent |
| Mechanistic | Organic |
| Predictable | Unpredictable |
| Diminishing Returns | Increasing Returns |
| Convex | Nonconvex |
| Easy to Model | Difficult to Model |
| A SIMPLE WORLD | A COMPLEX WORLD |

## TABLE 2.1:
## Information and Knowledge

| *Characteristic* | Information | Knowledge |
|---|---|---|
| *Source* | External | Internal |
| *Nature* | Weakly-interactive | Strongly-interactive |
| *Primary exchange mode* | Interface | Face-to-face |
| *Learning rate* | Fast | Slow |
| *Usefulness* | Temporary | Longlasting |
| *Exchange process* | Simple | Complex |
| *Unit of measurement* | Quantitative<br>(e.g. bits) | Qualitative<br>(e.g. deep) |

### TABLE 4.1
### Population Growth in Europe

| Date | European Population | Margin of Error (%) |
|---|---|---|
| 200 | 48 | 35 |
| 500 | 36 | 30 |
| 800 | 32 | 30 |
| 1000 | 39 | 20 |
| 1300 | 75 | 20 |
| 1500 | 76 | 10 |
| 1700 | 102 | 8 |

## TABLE 4.2:
## The Ten Largest Cities in Europe by Population, 1000-1400

| 1000 | 1100 | 1200 | 1300 | 1400 |
|---|---|---|---|---|
| Cordova | Constantinople | Constantinople | Paris | Paris |
| Constantinople | Fez | Palermo | Granada | Bruges |
| Seville | Seville | Seville | Constantinople | Milan |
| Palermo | Palermo | Paris | Venice | Venice |
| Kiev | Cordova | Venice | Genoa | Genoa |
| Venice | Granada | Cordova | Milan | Granada |
| Thessalonika | Venice | Granada | Sarai | Prague |
| Ratisbon | Kiev | Milan | Seville | Constantinople |
| Amalfi | Salerno | Cologne | Florence | Rouen |
| Rome | Milan | London | Cologne | Seville |

Table 5.1: Changes in Rank of Selected American Cities, 1810-1910

| City | ----------------Rank in-----------------<br>1810 | <br>1860 | <br>1910 |
|---|---|---|---|
| New York | 1 | 1 | 1 |
| Philadelphia | 2 | 2 | 3 |
| Baltimore | 3 | 3 | 7 |
| Boston | 4 | 4 | 5 |
| New Orleans | 6 | 5 | 14 |
| Cincinatti | 42 | 6 | 13 |
| St. Louis | - | 7 | 4 |
| Chicago | - | 8 | 2 |
| Buffalo | - | 9 | 10 |
| Louisville | - | 10 | 22 |
| Albany | 17 | 11 | 44 |
| Washington | 12 | 12 | 16 |
| San Francisco | - | 13 | 11 |
| Providence | 8 | 14 | 21 |
| Pittsburgh | 28 | 15 | 8 |
| Rochester | - | 16 | 23 |
| Detroit | - | 17 | 9 |
| Milwaukee | - | 18 | 12 |
| Cleveland | - | 19 | 6 |
| Charleston | 4 | 20 | 77 |

Table 5.2: Similarities Between CAs and Socio-Economic Dynamics

| | Cellular Automata | Socio-Economic Dynamics |
|---|---|---|
| Basic elements | Cells are the basic units or "atoms" of a CA | Individual agents are the basic units of an economy |
| Possible states | Cells assume one of a set of alternative states | Agents form mental models which enable them to make choices from alternatives |
| Interdependence | The state of a cell affects the state of its closest neighbors | The choices made by agents affect the choices made by other agents |
| Applications and tasks | Modeling the emergence of order, macro outcomes explained by micro rules, and the path dependence of dynamic processes | Important tasks include: understanding the emergence of order, macro to micro relationships, and economic dynamics |

# Bibliography

Albin, P. (1975) The Analysis of Complex Socio-Economic Systems. Lexington, MA: Lexington Books.

Allen, P. and M. Sanglier (1979) A Dynamic Model of Growth in a Central Place System, Geographical Analysis, vol. 11, pp. 256-272.

______ and ______ (1981) Urban Evolution, Self-Organization, and Decisionmaking, Environment and Planning A, vol. 13, pp. 167-183.

Andersson, Å.E. (1986) "Presidential Address: The Four Logistical Revolutions," Papers of the Regional Science Association, vol. 59, pp. 1-12.

______ (1995) "Economic Network Synergetics", in Batten, D.F., Casti, J.L. and R. Thord (eds.), Networks in Action, Berlin: Springer Verlag, pp. 309-318.

Andersson, Å.E. and O. Persson (1993) “Networking Scientists,” The Annals of Regional Science, vol. 27, pp. 11-21.

Arnott, R., A. de Palma and R. Lindsay (1993) “A Structural Model of Peak-Period Congestion: a Traffic Bottleneck with Elastic Demand,” American Economic Review, vol. 83, pp. 161-179.

Arrow, K. (1962) "The Economic Implications of Learning by Doing," Review of Economic Studies, vol. 29, pp. 155-173.

Arthur, W.B. (1994a) "Inductive Behaviour and Bounded Rationality", American Economic Review, vol. 84, pp. 406-411.

______ (1994b) Increasing Returns and Path Dependence in the Economy. Ann Arbor: University of Michigan Press.

______ (1995) “Complexity in Economic and Financial Markets,” Complexity, vol. 1, pp. 20-25.

Arthur, W.B., Holland, J.H., LeBaron, B., Palmer, R. and P. Tayler (1997) “Asset Pricing under Endogenous Expectations in an Artificial Stock Market,” in Arthur, W.B., Durlauf, S.N. and D.A. Lane, eds. The Economy as an Evolving Complex System II, Reading, Ma: Addison-Wesley, pp. 15-44.

Auerbach. F. (1913) “Das Gesetz der Bevölkerungskonzentration,” Petermanns Geographische Mitteilungen, no. 59, pp. 74-76.

Axelrod, R. (1984) The Evolution of Cooperation. New York: Basic Books.

______ (1997) The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration. Princeton: Princeton University Press.

Bachelier, L (1900) Théorie de la Spéculation. Doctoral Dissertation in Mathematical Sciences, Faculté des Sciences de Paris, defended 29 March.

Bairoch, P. (1988) Cities and Economic Development: From the Dawn of History to the Present. (Translated by Christopher Braider) Chicago: The University of Chicago Press.

Bak, P. (1996) How Nature Works: The Science of Self-Organized Criticality. New York: Springer-Verlag.

Bak, P., Chen, K., Scheinkman, J.A. and M. Woodford (1993) "Aggregate Fluctuations from Independent Shocks: Self-Organized Criticality in a Model of Production and Inventory Dynamics," Ricerche Economiche, vol. 47, pp. 3-24.

Barrett, C.L., Thord, R. and C. Reidys (1998) "Simulations in Decision Making for Socio-Technical Systems," in M.J. Beckmann, B. Johansson, F. Snickars and R. Thord, eds. Knowledge and Networks in a Dynamic Economy. Berlin: Springer-Verlag, pp. 59-82.

Batten, D.F. (1982) "On the Dynamics of Industrial Evolution," Regional Science and Urban Economics, vol. 12, pp. 449-462.

______ (1995) "Network Cities: Creative Urban Agglomerations for the Twenty-First Century," Urban Studies, vol. 32, pp. 313-327.

______ (1998) "Coevolutionary Learning on Networks," in M. Beckmann, B. Johansson, F. Snickars and R. Thord, eds., Knowledge and Networks in a Dynamical Economy. Berlin: Springer-Verlag, pp. 311-332.

Batten, D.F. and B. Johansson (1989) "The Dynamics of Metropolitan Change," Geographical Analysis, vol. 19, pp. 189-199.

Batten, D.F., Kobayashi, K. and Å.E. Andersson (1989) "Knowledge, Nodes and Networks: An Analytical Perspective", in Å.E. Andersson, D.F. Batten and C. Karlsson, eds. Knowledge and Industrial Organization, Berlin: Springer-Verlag, pp. 31-46.

Batty, M., Couclelis, H. and M. Eichen (1997) "Urban Systems as Cellular Automata," Environment and Planning B, vol. 24, pp. 159-164.

Beckmann, M.J. (1958) “City Hierarchies and the Distribution of City Size,” Economic Development and Cultural Change, vol. 6, pp. 243-248.

______ (1994) “On Knowledge Networks in Science: Collaboration among Equals,” The Annals of Regional Science, vol. 28, pp. 233-242.

Bell, D. (1981) “Models and Reality in Economic Discourse,” in Bell, D. and I. Kristol (eds.) The Crisis in Economic Theory, New York: Basic Books.

Ben-Akiva, M., A. de Palma and I. Kaysi (1991) "Dynamic Network Models and Driver Information Systems," Transportation Research A, vol. 25, pp. 251-266.

Bendor, J. and P. Swistak (1998) “The Evolutionary Advantage of Conditional Cooperation,” Complexity, vol. 4, pp. 15-18.

Benhabib, J., ed. (1992) Cycles and Chaos in Economic Equilibrium. Princeton: Princeton University Press.

Berry, B.J.L. (1961) “City-Size Distributions and Economic Development, Economic Development and Cultural Change, vol. 9, pp. 573-588.

Biham, O., Middleton, A. and D. Levine (1992) "Self-Organization and a Dynamical Transition in Traffic-Flow Models", Physical Review A, vol. 46, pp. R6124-7.

Bloch, M. (1962). Feudal Society. (Translated by L.A. Manyon), London: Routledge & Kegan Paul.

Bossomaier, T. and D. Green (1998) Patterns in the Sand: Computers, Complexity and Life, Sydney: Allen and Unwin.

Bower, G.H. and E.R. Hilgard (1981) Theories of Learning, Englewood Cliffs: Prentice Hall.

Boyd, R. and J.P. Lorberbaum (1987) “No Pure Strategy is Evolutionarily Stable in the Iterated Prisoner’s Dilemma Game,” Nature, vol. 327, pp. 58-59.

Braess, D. (1968) “Über ein Paradoxon aus der Verkehrsplanung,” Unternehmensforschung, vol. 12, pp. 258-268.

Braudel, F. (1982) The Wheels of Commerce. London: William Collins.

Brock, W., Hsieh, D. and B. LeBaron (1991) Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence. Cambridge, MA: MIT Press.

Bunge, W. (1966) Theoretical Geography. Lund Studies in Geography, The Royal University of Lund, Series C, No.1.

Casti, J.L. (1989) Paradigms Lost, New York: Avon.

______ (1994) Complexification, London: Abacus.

______ (1997) Would-Be Worlds, New York: Wiley.

Chandler, A.D. Jr. (1965) The Railroads: The Nation's First Big Business. New York: Harcourt Brace Jovanovich.

Christaller, W. (1933) Central Places in Southern Germany. (Translated by Carlisle W. Baskin, Englewood Cliffs, N.J.: Prentice Hall, 1966).

Cohen, J. and I. Stewart (1994) The Collapse of Chaos, New York: Penguin.

Conquest, L., Spyridakis, J., Haselkorn, M. and W. Barfield (1993) “The Effect of Motorist Information on Commuter Behaviour: Classification of Drivers into Commuter Groups,” Transportation Research C,” vol.1, pp.183-201.

Cronon, W. (1991) Nature’s Metropolis: Chicago and the Great West. New York: W.W. Norton.

Dafermos, S. and A. Nagurney (1984) “On Some Traffic Equilibrium Theory Paradoxes,” Transportation Research B, vol. 18, pp. 101-110.

Daganzo, C. and Y. Sheffi (1977) "On Stochastic Models of Traffic Assignment," Transportation Science, vol. 11, pp. 253-274.

Darley, V. (1995) “Emergent Phenomena and Complexity,” in Brooks, R.A. and P. Maes (eds.), Artificial Life IV: Proceedings of the Fourth International Workshop on the Synthesis and Simulation of Living Systems. Cambridge, MA.: The MIT Press, pp. 411-416.

Day, R.H. and J.L. Walter (1995) “Economic Growth in the Very Long Run: on the Multiple-Phase Interaction of Population, Technology, and Social Infrastructure,” in Barnett, W.A., Geweke, J. and K. Shell (eds.), Economic Complexity: Chaos, Sunspots, Bubbles, and Nonlinearity. Cambridge: Cambridge University Press, pp. 253-288.

De Bondt, W. and R. Thaler (1985) “Does the Stock Market Overreact?” Journal of Finance, vol. 60, pp. 793-805.

Delong, J.B., Schleifer, A., Summers, L.H. and J. Waldmann (1990) “Positive Feedback and Destabilizing Rational Speculation,” Journal of Finance, vol. 45, pp. 379-395.

Dillard, D. (1967) Economic Development of the North Atlantic Community. New Jersey: Prentice-Hall.

Downs, A. (1962) "The Law of Peak-Hour Expressway Congestion," Traffic Quarterly, vol. 16, pp. 393-409.

Dunlap, R.A. (1997) The Golden Ratio and Fibonacci Numbers. Singapore: World Scientific.

Dyckman, T.R. and D. Morse (1986) Efficient Capital Markets and Accounting: A Critical Analysis. Englewood Cliffs, N.J.: Prentice-Hall.

Eiser, J.R. (1994) Attitudes, Chaos and the Connectionist Mind. Oxford: Blackwell.

Eldredge, N. and S. Gould (1972) “Punctuated Equilibria: an Alternative to Phyletic Gradualism,” in Schopf, T.J.M. (ed.) Models in Paleobiology, San Francisco: Freeman Cooper, pp. 82-115.

Elliott, R.N. (1946) Nature’s Law: the Secret of the Universe (Reprinted in R.N. Elliott, R.N. Elliott’s Masterworks: The Definitive Collection, Gainesville, GA: New Classics Library, 1994).

Epstein, J.M. and R. Axtell (1996) Growing Artificial Societies. Cambridge, MA.: The MIT Press.

Erdos, P. and A. Renyi (1960) “On the Evolution of Random Graphs,” Institute of Mathematics, Hungarian Academy of Sciences, Publication No.5.

Fama, E. (1970) “Efficient Capital Markets: a Review of Theory and Empirical Work,” Journal of Finance, vol. 25, pp. 383-417.

Farmer, R. A. (1993) The Macroeconomics of Self-Fulfilling Prophecies. Cambridge, MA.: MIT Press.

Fogel, R.W. (1964) Railroads and American Economic Growth. Baltimore, Maryland: Johns Hopkins Press.

Forrester, J.W. (1987) "Nonlinearity in High-order Models of Social Systems," European Journal of Operational Research, vol. 30, pp. 104-109.

French, R.M. and A. Messinger (1995) “Genes, Phenes and the Baldwin Effect: Learning and Evolution in a Simulated Population,” in Brooks, R.A. and P.

Maes (eds.), Artificial Life IV: Proceedings of the Fourth International Workshop on the Synthesis and Simulation of Living Systems. Cambridge, MA.: The MIT Press. pp. 277-282.

Frost, A.J. and R.P. Prechter (1985) Elliott Wave Principle: Key to Stock Market Profits. Gainesville, GA: New Classics Library.

Fujita, M. (1996) "On the Self-Organization and Evolution of Economic Geography," The Japanese Economic Review, vol. 47, pp. 34-61.

Gell-Mann, M. (1995) "What is Complexity?" Complexity, vol. 1, pp. 16-19.

Goodin, S.H. (1851) "Cincinatti – It's Destiny," in C. Cist, ed. Sketches and Statistics of Cincinatti in 1851. Cincinatti: William Moore.

Gorman, M.E. (1992) Simulating Science: Heuristics, Mental Models, and Techno-Scientific Thinking. Bloomington: Indiana University Press.

Greising, D. and L. Morse (1991) Brokers, Bagmen, and Moles: Fraud and Corruption in the Chicago Futures Markets. New York: Wiley.

Griffin, K. (1998) "Friction in Economics," in Åkerman, N. The Necessity of Friction. Boulder: Westview Press, pp. 119-131.

Grimmett, G. (1998) Percolation. Berlin: Springer.

Haag, G. (1994) "The Rank-Size Distribution of Settlements as a Dynamic Multifractal Phenomena," Chaos, Solitons, and Fractals, vol. 4, pp. 519-534.

Haag, G and H. Max (1993) "Rank-Size Distribution of Settlement Systems: A Stable Attractor in Urban Growth," Papers in Regional Science, vol. 74, pp. 243-258.

Haken, H. (1977) Synergetics: An Introduction, Berlin: Springer-Verlag.

______ (1998) "Decision Making and Optimization in Regional Planning," in M.J. Beckmann, B. Johansson, F. Snickars and R. Thord, eds. Knowledge and Networks in a Dynamic Economy. Berlin: Springer-Verlag, pp. 25-40.

Hardin, R. (1982) Collective Action, Baltimore: Resources for the Future.

______ (1995) One for All: The Logic of Group Conflict, Princeton, New Jersey: Princeton University Press.

Harker, P. (1988) "Multiple Equilibrium Behaviours on Networks", Transportation Science, vol. 22, pp. 39-46.

Hegselmann, R. (1996) “Understanding Social Dynamics: The Cellular Automata Approach,” in K.G. Troitzsch, U. Mueller, G.N. Gilbert and J.E. Doran, eds. Social Science Microsimulation. Berlin: Springer, pp. 282-306.

Hinton, G.E. and S.J. Nowlan (1987) “How Learning Can Guide Evolution,” Complex Systems, vol. 1, pp. 495-502

Holland, J.H (1988) “The Global Economy as an Adaptive Process,” in Anderson, P.W., Arrow, K.J. and D. Pines (eds.) The Economy as an Evolving Complex System. Reading, MA: Addison-Wesley, pp. 117-124.

Holland, J.H., Holyoak, K.J., Nisbett, R.E. and P.R. Thagard (1986) Induction: Processes of Inference, Learning, and Discovery, Cambridge, MA: The MIT Press.

Horowitz, I.A. (1964) Chess Openings: Theory and Practice. London: Faber and Faber.

Hume, D. (1957) An Inquiry Concerning Human Understanding. New York: Library of Liberal Arts.

Jacobs, J. (1969) The Economy of Cities. New York: Random House.

Johansson, B. and L-G. Mattsson (1995) "Principles of road pricing", in Johansson, B. and L-G. Mattsson (eds.) Road Pricing: Theory, Empirical Assessment and Policy, Boston: Kluwer.

Johnson, J. (1995) "The multidimensional networks of complex systems", in Batten, D.F., Casti, J.L. and R. Thord (eds.) Networks in Action, Berlin: Springer Verlag, pp. 49-79.

Johnson-Laird, P.N. and R.M.J. Byrne (1991) Deduction, Hove: Lawrence Erlbaum Associates.

Katz, J.S. (1993) “Geographical Proximity and Scientific Collaboration.” Scientometrics, vol. 31 (1): pp. 31-43.

Kauffman, S. (1993) The Origins of Order: Self-Organization and Selection in Evolution, New York: Oxford University Press.

______ (1995) At Home in the Universe: The Search for Laws of Complexity, London: Penguin.

Kobayashi, K. (1993) "Incomplete Information and Logistical Network Equilibria, in Andersson, Å.E., Batten, D.F., Kobayashi, K. and K. Yoshikawa (eds.) The Cosmo-Creative Society, Berlin: Springer, pp. 95-119.

Kobayashi, K., Kunihisa, S. and K. Fukuyama (2000) "The Knowledge Intensive Nature of Japan's Urban Development, in Batten, D.F., Bertuglia, C.S., Martellato, D. and S. Occelli (eds.) Learning, Innovation and Urban Evolution, Boston: Kluwer, in press.

Krugman, P. (1993) "On the Number and Location of Cities," European Economic Review, vol. 37, pp. 293-298.

______ (1994a) "Complex Landscapes in Economic Geography," American Economic Association, Papers and Proceedings, vol. 84, pp. 412-416.

______ (1994b) The Age of Diminished Expectations. Cambridge, Ma.: The MIT Press.

______ (1994c) Peddling Prosperity: Economic Sense and Nonsense in the Age of Diminished Expectations. New York: W.W. Norton.

______ (1996) The Self-Organizing Economy. New York: Blackwell.

Kuhn, T. (1962) The Structure of Scientific Revolutions. Chicago: University of Chicago Press.

Lane, D. (1993) "Artificial Worlds and Economics, Part I," Journal of Evolutionary Economics, vol. 3, pp. 89-107.

______ (1997) "Is What is Good for Each Best for All? Learning from Others in the Information Contagion Model," in Arthur, W.B., Durlauf, S.N. and D.A. Lane (eds.) The Economy As An Evolving Complex System II, Reading, Ma: Addison-Wesley, pp. 105-127.

Langton, C.G. (1996) "Artificial Life," in Boden, M.A. (ed.) The Philosophy of Artificial Life, Oxford: Oxford University Press, pp. 39-94.

LeRoy, S.F. and R.D. Porter (1981) "Stock Price Volatility: Tests based on Implied Variance Bounds," Econometrica, vol. 49, pp. 97-113.

Lindgren, K. (1992) "Evolutionary Phenomena in Simple Dynamics," in Langton, C.G., Farmer, J.D., Rasmussen, S. and C. Taylor (eds.) Artificial Life II, Redwood City, CA: Addison-Wesley, pp. 295-312.

Lindsay, C.L. (1991) Trident: A Trading Strategy. Brightwaters, N.Y.: Windsor Books.

Lösch, A. (1944) The Economics of Location (Translated from the 2nd Revised Edition by William H. Woglom and published by Yale University Press, New Haven, 1954).

Machlup, F. (1962) The Production and Distribution of Knowledge in the United States, Princeton, N.J.: Princeton University Press.

Mandelbrot, B. (1963) “The Variation of Certain Speculative Prices,” Journal of Business, vol. 36, pp. 394-419.

______ (1997) Fractals and Scaling in Finance. New York: Springer-Verlag.

______ (1999) “A Multifractal Walk Down Wall Street,” Scientific American, February, pp. 50-53.

Marshall, A. (1920) Principles of Economics. London: Macmillan.

Maruyama, M. (1963) "The Second Cybernetics: Deviation-Amplifying Mutual Causal Processes," American Scientist, vol. 51, pp. 164-179.

Marx, K. (1973) Grundrisse: Foundations of the Critique of Political Economy. (Translated by Martin Nicolaus). Harmondsworth, UK: Penguin.

de la Maza, M. and D. Yuret (1995) “A Futures Market Simulation with Non-Rational Participants,” in Brooks, R.A. and P. Maes, Artificial Life IV: Proceedings of the Fourth International Workshop on the Synthesis and Simulation of Living Systems. Cambridge, MA.: The MIT Press. pp. 325-330.

Mees, A. (1975) "The Revival of Cities in Medieval Europe", Regional Science and Urban Economics, vol. 5, pp. 403-425.

Monod, J. (1971) Chance and Necessity. London: Penguin.

Nagel, K. and S. Rasmussen (1995) "Traffic at the Edge of Chaos," in Brooks, R.A. and P. Maes, Artificial Life IV: Proceedings of the Fourth International Workshop on the Synthesis and Simulation of Living Systems. Cambridge, MA.: The MIT Press. pp. 222-235.

Nagel, K. and M. Schreckenberg (1992) "A Cellular Automaton Model for Freeway Traffic," Journal de Physique I, vol. 2, p. 2221.

Nelson, R.R. and S.G. Winter (1982) An Evolutionary Theory of Economic Change, Cambridge, MA.: Harvard University Press.

Neumann, J. von (1966) Theory of Self-Reproducing Automata (edited and completed by Arthur Burks), Urbana: University of Illinois Press.

Nicolis, G. and I. Prigogine (1977) Self-Organization in Nonequilibrium Systems, New York: Wiley.

______ and ______ (1989) Exploring Complexity: An Introduction, New York: W.H. Freeman.

North, D.C. and R.P. Thomas (1973) The Rise of the Western World: A New Economic History, Cambridge: Cambridge University Press.

Nowak, M. and K. Sigmund (1993) "A Strategy of Win-Shift, Lose-Stay that Outperforms Tit-for-Tat in the Prisoner's Dilemma Game," Nature, vol. 364, pp. 56-58.

Ohmae, K. (1991) The Borderless World: Power and Strategy in the Interlinked Economy. New York: HarperPerennial.

Olson, M. (1965) The Logic of Collective Action, Cambridge, Ma.: Harvard University Press.

Pareto, V. (1896) Oeuvres Completes. Geneva: Droz.

Pigou, A.C. (1927) Industrial Fluctuations, London: Macmillan.

Pirenne, H. (1925) Medieval Cities: their Origins and the Revival of Trade. (Translation by F.D. Halsey, Princeton: Princeton University Press, 1952).

______ (1936) Economic and Social History of Medieval Europe. (Translation by I.E. Clegg, London: Routledge & Kegan Paul, 1965).

Portugali, J., Benenson, I. and I. Omer (1994) "Sociospatial Residential Dynamics: Stability and Instability within a Self-Organizing City," Geographical Analysis, vol. 26, pp. 321-340.

______, ______ and ______ (1997) "Spatial Cognitive Dissonance and Sociospatial Emergence in a Self-Organizing City," Environment and Planning B, vol. 24, pp. 263-285.

Pred, A. (1966) The Spatial Dynamics of U.S. Urban-Industrial Growth, 1800-1914. Cambridge, Ma.: Harvard University Press.

Prigogine, I. and R. Herman (1971) Kinetic Theory of Vehicular Traffic, New York: Elsevier.

Puu. T. (1997) Mathematical Location and Land Use Theory, Berlin: Springer.

Rapoport, A. and A.M. Chammah (1965) Prisoner's Dilemma: A Study in Conflict and Cooperation, Ann Arbor: University of Michigan Press.

Rasmussen, S. and C.L. Barrett (1995) “Elements of a Theory of Simulation,” in ECAL 95, Lecture Notes in Computer Science. New York: Springer.

Ray, T.S. (1992) “An Approach to the Synthesis of Life,” in Langton, C., Taylor, C., Farmer, J.D. and S. Rasmussen, eds. Artficial Life II. Redwood City, Ca: Addison-Wesley, pp. 371-408.

Rhea, R. (1932) The Dow Theory. New York: Barron’s.

Riising, A. (1952) “The Fate of Henri Pirenne’s Thesis on the Consequences of Islamic Expansion,” Classica et Medievalia, vol. 13.

Rostow, W.W. (1960) The Stages of Economic Growth. New York: Cambridge University Press.

Rothstein, M. (1982) “Frank Norris and Popular Perceptions of the Market,” Agricultural History, vol. 56, pp. 50-66.

Rouse, W.B. and Morris, N.M. (1986) “On Looking into the Black Box: Prospects and Limits in the Search for Mental Models,” Psychological Bulletin, vol. 100, pp. 349-363.

Rubinstein, M. (1975) “Securities Market Efficiency in an Arrow-Debreu Economy,” American Economic Review, vol. 65, pp. 812-814.

Ryle, G. (1949) The Concept of Mind. London: Hutchinson.

Saari, D. (1995) “Mathematical Complexity of Simple Economics,” Notices of the American Mathematical Society, vol. 42, pp. 222-230.

Sakoda, J.M. (1971) “The Checkerboard Model of Social Interaction,” Journal of Mathematical Sociology, vol. 1, pp. 119-132.

Samuelson, P. (1976) Economics. New York: McGraw Hill.

Sanders, L., Pumain, D., Mathian, H., Guérin-Pace, F. and S. Bura (1997) “SIMPOP: A Multiagent System for the Study of Urbanism,” Environment and Planning A, vol. 24, pp. 287-305.

Sargent, T.J. (1993) Bounded Rationality in Macroeconomics, New York: Oxford University Press.

Scheinkman, J.A. and M. Woodford (1994) “Self-Organized Criticality and Economic Fluctuations, American Journal of Economics, vol. 84, pp. 417.

Schelling, T.S. (1969) “Models of Segregation,” American Economic Review, Papers and Proceedings, vol. 59 (2), pp. 488-493.

______ (1978) Micromotives and Macrobehaviour. New York: W.W. Norton and Company.

Schroeder, M. (1991) Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise. New York, W.H. Freeman.

Schrödinger E. (1956) Mind and Matter (Reprinted together with What is Life, Cambridge: Cambridge University Press, 1967).

Schumpeter, J. (1934) The Theory of Economic Development. Cambridge, MA.: Harvard University Press.

Scott, J.W.(1876) A Presentation of Causes Tending to Fix the Position of the Future Great City of the World in the Central Plain of North America: Showing that the Centre of the World's Commerce, Now Represented by the City of London, Is Moving Westward to the City of New York, and Thence, within One Hundred Years, to the Best position on the Great Lakes. Toledo.

Sendut, H. (1966) "City-Size Distributions of South-East Asia," Asian Studies, vol. 4, pp. 165-172.

Sheffi, Y. (1985) Urban Transportation Networks, New Jersey: Prentice Hall.

Shiller, R.J. (1981) "The Use of Volatility Measures in Assessing Market Efficiency," Journal of Finance, vol. 36, pp. 291-304.

______ (1989) Market Volatility, Cambridge, MA: The MIT Press.

Simon, H. (1987) "Giving the Soft Sciences a Hard Sell," Boston Globe, 3 May.

Singer, H.W. (1936) "The 'Courbes des Populations': A Parallel to Pareto's Law," Economic Journal, vol. 46, no. 182, pp. 254-263.

Soros, G. (1994) The Alchemy of Finance: Reading the Mind of the Market, New York: Wiley.

Spence, A.M. (1981) "The Learning Curve and Competition," The Bell Journal of Economics, vol. 12, pp. 49-70.

Stauffer, D. (1985) Introduction to Percolation Theory. London: Taylor and Francis.

Taylor, G.R. (1951) The Transportation Revolution: 1815-1860. New York: Rinehart.

Turner, F.J. (1920) The Frontier in American History. New York: Holt, Rinehart and Winston.

Vickrey, W.S. (1969) "Congestion Theory and Transport Investment", American Economic Review, vol. 59, pp. 251-260.

Thünen, J.H. von (1826) Der Isolierte Staat in Beziehung auf Landtschaft und Nationalökonomie. Hamburg. (English translation by C.M. Wartenburg: von Thünen's Isolated State, Oxford: Pergamon Press, 1966).

Vries, J. de (1984) European Urbanization: 1500-1800. London: Methuen.

Ward, M. (1999) Virtual Organisms. London: Macmillan.

Yang, H. and M.G.H. Bell (1998) "A Capacity Paradox in Network Design and How to Avoid It," Transportation Research, vol. 32 (7), pp. 539-545.

Zhang, W-B. (1993) Synergetic Economics. Berlin: Springer-Verlag.

Zipf, G.K. (1941) National Unity and Disunity. Bloomington, Indiana: Principia Press.

______ (1949) Human Behaviour and the Principle of Least Effort. New York: Hafner.

Åkerman, N. (1998) The Necessity of Friction. Boulder: Westview Press.