

# Investigation of one-dimensional photonic bandgap structures containing lossy double-negative metamaterials through the Bloch impedance

Fenghua Shi,<sup>1</sup> Yihang Chen,<sup>1,2,\*</sup> Peng Han,<sup>1,4</sup> and Costas M. Soukoulis<sup>2,3</sup>

<sup>1</sup>Laboratory of Quantum Information Technology, School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou 510006, China

<sup>2</sup>Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA

<sup>3</sup>Institute of Electronic Structure and Lasers (IESL), FORTH, Heraklion, Crete 71110, Greece

<sup>4</sup>e-mail: hanp@scnu.edu.cn

\*Corresponding author: chenyh@ameslab.gov

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The Bloch impedance is studied and used to understand the properties of the absorption loss in one-dimensional photonic crystals (PCs) composed of air and metal-based double-negative metamaterials. We find that as the frequency increases across the zero- $\bar{n}$  gap of the considered structures, the modulus of the Bloch impedance always decreases from a maximum to a minimum value. On the other hand, the frequency dependence of the phase angle of the Bloch impedance is greatly influenced by the ratio of the electric to the magnetic damping coefficient  $\gamma_e/\gamma_m$  of the metamaterials. When the phase angle of the Bloch impedance reaches maximum inside the zero- $\bar{n}$  gap, the impedance mismatch between the incident medium and the considered PC becomes greatest, the reflection will be strongest and a minimum absorption will be observed. As  $\gamma_e/\gamma_m$  increases, the frequency corresponding to the minimum absorption shifts from the lower to the upper gap edge. We also show that the main characteristics of both the Bloch impedance and the absorption loss are insensitive to the geometrical parameters. Our study offers a valuable reference in the designs of zero- $\bar{n}$  gap with optimized properties. © 2013 Optical Society of America

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## 1. INTRODUCTION

Photonic crystals (PCs) [1–3], a type of artificial composites, have attracted much interest in the last two decades because of their potential ability to control the motion of photons in a similar way as the semiconductor crystals affect the motion of electrons. Conventional PCs are composed of periodic dielectric or metallo-dielectric nanostructures in which photonic bandgaps (PBGs) exist due to the interference of multiple Bragg scattering. Potential applications of the PCs have been proposed in scientific and technical areas, such as filters [4], optical switches [5], and suppression of spontaneous emission [6]. However, such applications are substantially limited since the electromagnetic (EM) properties of the Bragg gaps strongly depend on the lattice constant, incident angle, and disorder.

A possible way to overcome such limitations is to introduce metamaterials with negative refractive indices into the PC structure. The so-called negative-index or double-negative (DNG) metamaterials, possessing simultaneously negative permittivity  $\epsilon$  and negative permeability  $\mu$  [7], have attracted a great deal of attention since their first experimental realization in 2001 [8,9]. It has been demonstrated that stacking alternating layers of ordinary double-positive (DPS) materials (with positive  $\epsilon$  and positive  $\mu$ ) and DNG metamaterials can

lead to a special type of PBG when the volume averaged effective refractive index ( $\bar{n}$ ) equals zero [10]. Such a PBG, named the zero- $\bar{n}$  gap, originates from the Bragg scattering, possesses properties different from the traditional Bragg PBGs. The zero- $\bar{n}$  gap has weak dependence on the lattice constant and disorder [10,11], can be designed to be insensitive to the incident angle [12,13].

For DNG metamaterials, negative permeability is the result of a strong resonant response to an external magnetic field while negative permittivity appears by either a plasmonic or a resonant response to an external electric field. Several types of designs were proposed to realize both electric and magnetic responses simultaneously, for example, split ring resonators combined with thin wires [8,9], cut wire pairs, and fishnet structures [14–16]. These artificial materials are designed based on the metallic structures, in which absorption loss is inevitable. Such losses have great effect on the properties of the DNG metamaterials, which may eventually influence the EM properties of the zero- $\bar{n}$  gap. Therefore, it is necessary to investigate the properties of the zero- $\bar{n}$  gap when absorption loss is considered. It should be noticed that there may exist a great difference of the absorption between the electric and the magnetic field components of the wave propagating through the DNG metamaterials. On the other hand, as the frequency varies, the EM field distribution inside the

PC changes significantly, which may lead to large changes of the absorption loss. Considering the above factors, the Bloch impedance, determined by the electric and the magnetic field distributions inside the periodic structure, is used to understand and predict the characteristics of the absorption loss.

In this paper, we investigate the main features of the Bloch impedance inside the multilayered structure combining DPS and DNG materials when losses are taken into account. We discuss the relationship between the properties of the Bloch impedance and those of the absorption loss. The discussions are organized as follows. In Section 2, we introduce the theoretical methods. In Section 3, we discuss the Bloch impedance and its relevance to the absorption loss of the considered system. In Section 4, we investigate the influence of the structural parameters on the Bloch impedance and the absorption loss inside the zero- $\bar{n}$  gap. Finally, our conclusions are presented in Section 5.

## 2. MODEL AND NUMERICAL METHODS

Consider a one-dimensional PC consisting of alternating layers of DPS and DNG materials, as shown in Fig. 1. Layers 1 and 2 represent the DPS and DNG material layers with the thicknesses of  $d_1$  and  $d_2$ , respectively.  $d = d_1 + d_2$  is the lattice constant. In this paper, we choose air as the DPS material with  $\epsilon_1 = 1, \mu_1 = 1$ . The DNG materials are considered to be realized from metal-based metamaterial. The dispersion of such metal-based metamaterials can be described by the Drude model [11,17,18], that is,

$$\epsilon_2(\omega) = \epsilon_0 - \frac{\omega_{ep}^2}{\omega(\omega + i\gamma_e)}, \quad (1)$$

$$\mu_2(\omega) = \mu_0 - \frac{\omega_{mp}^2}{\omega(\omega + i\gamma_m)}, \quad (2)$$

where  $\omega_{ep}$  and  $\omega_{mp}$  correspond to the electronic plasma frequency and magnetic plasma frequency.  $\gamma_e$  and  $\gamma_m$  denote the electric and magnetic damping coefficients that contribute to the absorption loss.  $\omega$  is the angular frequency measured in gigahertz. In the following calculations, we choose  $\omega_{ep} = \omega_{mp} = 10$  GHz,  $\epsilon_0 = 1.21, \mu_0 = 1$ .

We assume that the layers of the considered PC are oriented in the  $yo$ z plane, as shown in Fig. 1. Let a plane EM wave propagating in the  $+z$  direction be incident normally on the boundary  $z = 0$  from free space into the periodic structure.

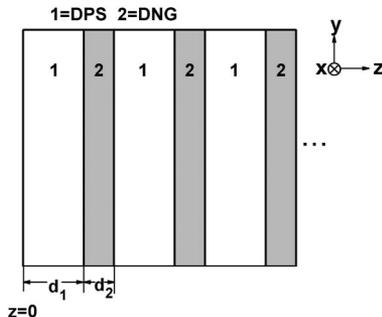


Fig. 1. Schematic representation of the DPS/DNG multilayered structure.

The electric and magnetic fields of adjacent layers can be related via a transfer matrix [19–21]

$$M_j = \begin{bmatrix} \cos(k_j d_j) & i/q_j \sin(k_j d_j) \\ iq_j \sin(k_j d_j) & \cos(k_j d_j) \end{bmatrix}, \quad (3)$$

where  $k_j = \omega \sqrt{\epsilon_j} \sqrt{\mu_j}$  and  $q_j = \sqrt{\mu_j} / \sqrt{\epsilon_j}$  ( $j = 1, 2$ ).

The transfer matrix of a unit cell of the PC can be obtained as

$$M = M_1 M_2 = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}, \quad (4)$$

where  $u_{i,j}$  ( $i, j = 1, 2$ ) is the matrix element. The relationship between the EM fields at  $z = nd$  ( $n = 0, 1, 2, 3, \dots$ ) and those at  $z = (n + 1)d$  can be written as

$$\begin{bmatrix} E_n \\ H_n \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} E_{n+1} \\ H_{n+1} \end{bmatrix}. \quad (5)$$

According to Bloch's theorem [22,23], we have

$$\begin{bmatrix} u_{11} - e^{-iK_B d} & u_{12} \\ u_{21} & u_{22} - e^{-iK_B d} \end{bmatrix} \begin{bmatrix} E_{n+1} \\ H_{n+1} \end{bmatrix} = 0, \quad (6)$$

where  $K_B$  is the Bloch wave vector. Then the dispersion relation of the PC can be obtained by solving [24]

$$\cos(K_B d) = (u_{11} + u_{22})/2. \quad (7)$$

From Eq. (6), the phase coefficient  $e^{-iK_B d}$  can also be solved and written as

$$e^{-iK_B d} = \frac{(u_{11} + u_{22}) \pm \sqrt{(u_{11} + u_{22})^2 - 4}}{2}. \quad (8)$$

The Bloch impedance is defined as

$$Z_B = \frac{E_{n+1}}{H_{n+1}} Z_0. \quad (9)$$

Combining Eq. (6), we have

$$Z_B = -\frac{u_{12}}{u_{11} - e^{-iK_B d}} Z_0. \quad (10)$$

By Substituting Eq. (8) into Eq. (10), we can obtain the expression

$$Z_B = \frac{\pm 2u_{12}}{u_{11} - u_{22} + \sqrt{(u_{11} + u_{22})^2 - 4}} Z_0. \quad (11)$$

According to the Fresnel reflectance equation, the reflectance of a semi-infinite PC can be written as

$$R = \left| \frac{1 - Z_B/Z_0}{1 + Z_B/Z_0} \right|^2, \quad (12)$$

where  $Z_0$  is the intrinsic impedance of the incident medium.

In lossless cases, the solutions of Bloch wave vector  $K_B$  in Eq. (6) determine the band structure of the considered PC with allowed zones separated by bandgaps. Real solution

for  $K_B$  can be found at frequencies within the allowed bands. If the frequency of the wave locates inside a forbidden gap,  $K_B$  becomes pure imaginary and  $R$  equals to 1. In fact, the band structure of PCs can also be obtained from Bloch impedance in lossless situations [25,26]. In these cases, Bloch impedance  $Z_B$  is complex within the passband while it becomes pure imaginary inside the bandgap. However, when losses are taken into account, the real and imaginary parts of  $K_B$  and  $Z_B$  will both be nonzero.

### 3. BLOCH IMPEDANCE AND ITS RELEVANCE TO ABSORPTION LOSS

Impedance has been applied in reducing the insertion loss for traditional waveguides and transmission lines. Investigations of the impedance are also helpful for understanding the process of the energy flow for wave propagating from one medium to another. Similarly, the Bloch impedance, as an intrinsic characteristic of the PC, can be used to improve the performance of the PC waveguides by reducing their insertion losses [26–28]. However, the Bloch impedance has attracted much less attention than the Bloch wave vector. Here, we will investigate the properties of the Bloch impedance of the considered DPS/DNG PC. The relationship between the absorption loss and the Bloch impedance will also be discussed.

In Fig. 2, we plot the average refractive index  $\bar{n}$ , Bloch wave vector  $K_B$ , the complex modulus  $|Z_B|$ , and the phase angle  $\theta_{ZB}$  of the Bloch impedance versus the frequency for an infinite DPS/DNG periodic structure without considering the absorption loss ( $\gamma_e = \gamma_m = 0$ ). It was pointed out that the zero- $\bar{n}$  gap opens from a frequency where the volume averaged index  $\bar{n} = (1/d) \int_0^d n(z) dz$  equals to zero [10]. For lossy structures, the zero- $\bar{n}$  gap should originate from frequency where  $(1/d) \int_0^d \text{Re}(n(z)) dz = 0$ . As shown in Fig. 2(a),  $\bar{n} = 0$  is satisfied at 7.867 GHz, from which a PBG emerges. Such a zero- $\bar{n}$  gap exists in frequency range from 7.651 to 8.164 GHz, in which  $K_B$  is pure imaginary, as shown in Fig. 2(b). On the other hand, the Bloch impedance  $Z_B$  calculated from Eq. (11) is pure imaginary within the zero- $\bar{n}$  gap and is complex inside the passbands, as show in Fig. 2(c).

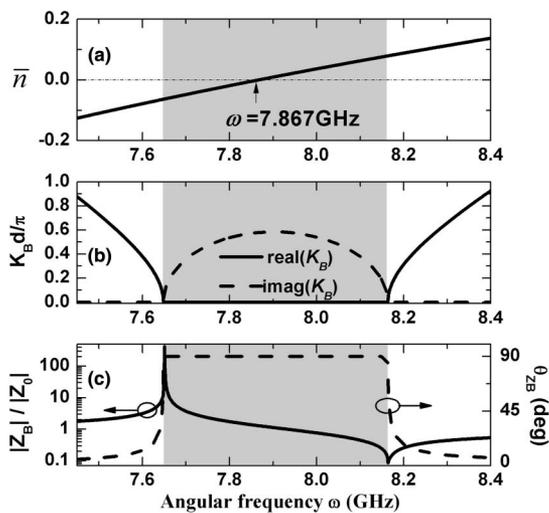


Fig. 2. (a)  $\bar{n}$ , (b)  $K_B$ , and (c)  $|Z_B|$  and  $\theta_{ZB}$  as a function of the angular frequency  $\omega$  in infinite DPS/DNG PC with  $d_1 = 6$  mm,  $d_2 = 12$  mm, and  $\gamma_e = \gamma_m = 0$ . The gray areas correspond to the zero- $\bar{n}$  gap.

We focus on the frequency range corresponding to the zero- $\bar{n}$  gap. It can be seen that, as the frequency increases across the zero- $\bar{n}$  gap, the value of  $|Z_B|/|Z_0|$  decreases, which means that the electric field component becomes weaker while the magnetic field component becomes stronger. For a more clear understanding, the electric and magnetic field distributions inside a 100-period DPS/DNG PC at  $\omega = 7.68$  GHz and  $\omega = 8.14$  GHz are simulated and shown in Figs. 3(a) and 3(b). As shown in Fig. 3(a), the relative amplitude of the electric field  $|E_r| (= |E|/|E_{in}|)$ ,  $E_{in}$  is the electric field amplitude of the incident wave) is much larger than that of the magnetic field  $|H_r| (= |H|/|H_{in}|)$ ,  $H_{in}$  is the magnetic field amplitude of the incident wave) at 7.68 GHz where a large value of  $|Z_B|$  exhibits in Fig. 2(c). On the other hand, as shown in Fig. 3(b), the relative amplitude of the magnetic field is much larger than that of the electric field at 8.14 GHz where  $|Z_B|/|Z_0|$  is much smaller than 1. Such differences of the electric and magnetic field distributions between different frequencies are due to the changes of the interference of the scattering inside the PC structure.

Furthermore, as shown in Fig. 2(c),  $\theta_{ZB}$  keeps a fixed value of  $\pi/2$  in the bandgap. Therefore, according to the definition of energy flux  $S = |E|^2 \cos(\theta_{ZB}) / (|Z_B|/|Z_0|)$ , the energy flux will be zero and a complete reflection of the wave energy appears. In fact, both  $|Z_B|$  and  $\theta_{ZB}$  are closely associated with the reflection from PCs. If  $Z_B$  equals  $Z_0$ , i.e., a perfect impedance match between the incident medium (free space) and the considered PC, a perfect transmission will be observed and the reflectance will become zero. On the other hand, as  $|Z_B|/|Z_0|$  deviates from 1 or  $\theta_{ZB}$  deviates from  $0^\circ$ , the greater the impedance mismatch, the larger the reflectance will be. When  $Z_B$  becomes pure imaginary, a complete impedance mismatch occurs, the wave cannot propagate forward and the reflectance becomes 1, which corresponds to the case in a gap.

When absorption is taken into account, the reflectance of the PC can still be obtained via the Bloch impedance by using Eq. (12). Here, we try to find out the relationship between the absorption of the PC and the Bloch impedance. For an infinite lossy PC structure, the transmittance is zero, the absorptance can be obtained from

$$A = 1 - R = 1 - \left| \frac{1 - Z_B/Z_0}{1 + Z_B/Z_0} \right|^2. \quad (13)$$

Using Eq. (13), the dependence of the absorption on  $Z_B$  is calculated and plotted in Fig. 4. It can be seen from Fig. 4 that,

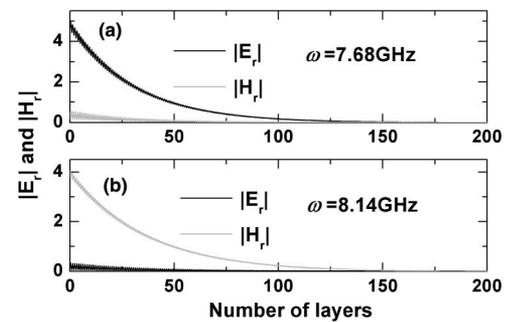


Fig. 3. EM field distributions in lossless DPS/DNG PC with  $d_1 = 6$  mm, and  $d_2 = 12$  mm at (a)  $\omega = 7.68$  GHz and (b)  $\omega = 8.14$  GHz near the lower and upper gap edge, respectively.

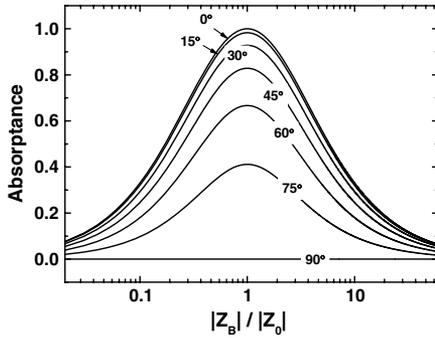


Fig. 4. Dependence of absorbance on  $|Z_B|$  under different values of  $\theta_{ZB}$ .

as  $|Z_B|$  deviates from  $|Z_0|$  or  $\theta_{ZB}$  increases, the absorbance decreases. This can be understood as the greater impedance mismatch leads to the larger reflection and the smaller absorption. On the other hand, when the impedance of the incident space and that of the considered PC are well matched, i.e.,  $Z_B$  equals to  $Z_0$ , most EM energy will transmit into the PC and finally be absorbed by DNG materials, the absorption of the considered system becomes maximum. As shown in Fig. 4, when  $|Z_B|/|Z_0|$  is close to 1, the absorbance becomes insensitive to the changes of  $|Z_B|$ . We can also see that the absorbance becomes quite sensitive to  $\theta_{ZB}$  when  $|Z_B| \rightarrow |Z_0|$  and  $\theta_{ZB}$  is large. Such results, attributed to the difference between the influence of  $|Z_B|$  and that of  $\theta_{ZB}$  on the impedance mismatch, are useful for understanding the properties of the absorption loss in the DPS/DNG PC.

#### 4. INFLUENCE OF THE STRUCTURE PARAMETERS ON BLOCH IMPEDANCE AND ABSORPTION LOSS

In this section, we will investigate how the structural parameters of the DPS/DNG PC influence the Bloch impedance and the corresponding absorption. In Fig. 5, we plot  $|Z_B|$ ,  $\theta_{ZB}$  and the absorbance versus the frequency for the DPS/DNG PC under different damping coefficients: (a) and (b)  $\gamma_e/\gamma_m = 10$ ; (c) and (d)  $\gamma_e/\gamma_m = 1$ ; (e) and (f)  $\gamma_e/\gamma_m = 0.1$ . It can be seen that, for all cases, the Bloch impedance are complex numbers,  $|Z_B|$  decreases from a maximum to a minimum value as the frequency increases across the zero- $\bar{n}$  gap.

We focus on the case with  $\gamma_e/\gamma_m = 10$ , where the electric damping is more pronounced than the magnetic damping. As shown in Fig. 5(a), at frequencies near the lower edge of the zero- $\bar{n}$  gap,  $|Z_B|$  has a large value, meaning that the relative amplitude  $|E_r|$  is much larger than  $|H_r|$ . Combining with a larger absorption for the electric field component of the wave (because  $\gamma_e > \gamma_m$ ) propagating through the DNG metamaterial, it can be expected that relative large absorption appears in frequency range near the lower gap edge, which is verified in Fig. 5(b). On the other hand, we can see from Fig. 5(a) that, as the frequency increases inside the zero- $\bar{n}$  gap from 7.65 to 8.16 GHz,  $|Z_B|/|Z_0|$  decreases from 2.2 to 0.5.  $\theta_{ZB}$  is close to  $90^\circ$  at frequencies near the upper gap edge. As discussed in the last section, the absorption is sensitive to  $\theta_{ZB}$  rather than  $|Z_B|$  when  $|Z_B| \rightarrow |Z_0|$  and  $\theta_{ZB} \rightarrow 90^\circ$ . In this case, the absorption inside zero- $\bar{n}$  gap is mainly determined by  $\theta_{ZB}$ . As shown in Fig. 5(b), within the zero- $\bar{n}$  gap, as  $\theta_{ZB}$  increases, greater impedance mismatch occurs, the reflection becomes larger

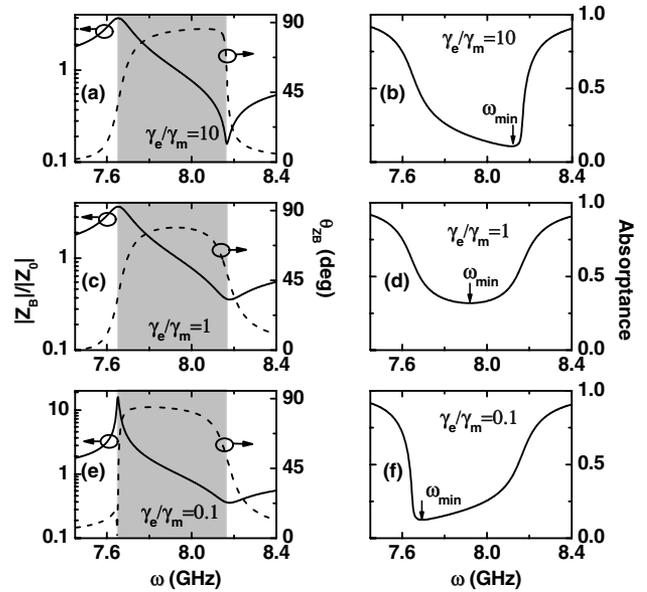


Fig. 5.  $|Z_B|$ ,  $\theta_{ZB}$  and absorbance versus frequency in DPS/DNG PC under different values of  $\gamma_e/\gamma_m$ : (a) and (b)  $\gamma_e/\gamma_m = 10$  with  $\gamma_e = 0.01$  GHz,  $\gamma_m = 0.001$  GHz; (c) and (d)  $\gamma_e/\gamma_m = 1$  with  $\gamma_e = 0.01$  GHz,  $\gamma_m = 0.01$  GHz; and (e) and (f)  $\gamma_e/\gamma_m = 0.1$  with  $\gamma_e = 0.001$  GHz,  $\gamma_m = 0.01$  GHz. The thicknesses of the layers are  $d_1 = 6$  mm and  $d_2 = 12$  mm. The gray areas represent the zero- $\bar{n}$  gap.

and the absorption becomes smaller. Such result is in accordance with that in Fig. 4. Similarly, the relationship between the properties of  $Z_B$  and those of absorption loss inside the zero- $\bar{n}$  gap can be analyzed when  $\gamma_e/\gamma_m$  is changed, as shown from Figs. 5(c) to 5(f).

It can be also seen from Fig. 5 that,  $\theta_{ZB}$  reaches maximum at a frequency  $\omega_{\min}$  inside the zero- $\bar{n}$  gap, where a minimum absorbance can therefore be found. As  $\gamma_e/\gamma_m$  varies, the frequency dependence of  $\theta_{ZB}$  changes, hence  $\omega_{\min}$  changes as well. The dependence of  $\omega_{\min}$  on  $\gamma_e/\gamma_m$  was investigated and shown in Fig. 6. It can be seen that  $\omega_{\min}$  moves from the lower to the upper gap edge as  $\gamma_e/\gamma_m$  increases from 0.01 to 100. Such results are helpful for predicting when low absorption loss takes place inside the zero- $\bar{n}$  gap of the lossy PC structure.

Next, we investigate the dependences of the impedance and the absorption on  $\gamma_m$  when  $\gamma_e/\gamma_m$  is fixed. Taking  $\gamma_e/\gamma_m = 1$

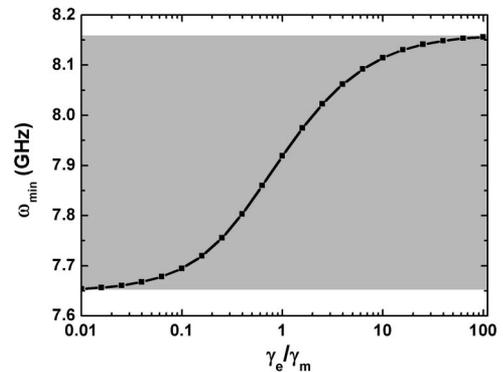


Fig. 6.  $\omega_{\min}$  as a function of  $\gamma_e/\gamma_m$  when  $\gamma_m$  is fixed to be 0.001 GHz. The gray area corresponds to the zero- $\bar{n}$  gap. The thicknesses of the layers are  $d_1 = 6$  mm and  $d_2 = 12$  mm.

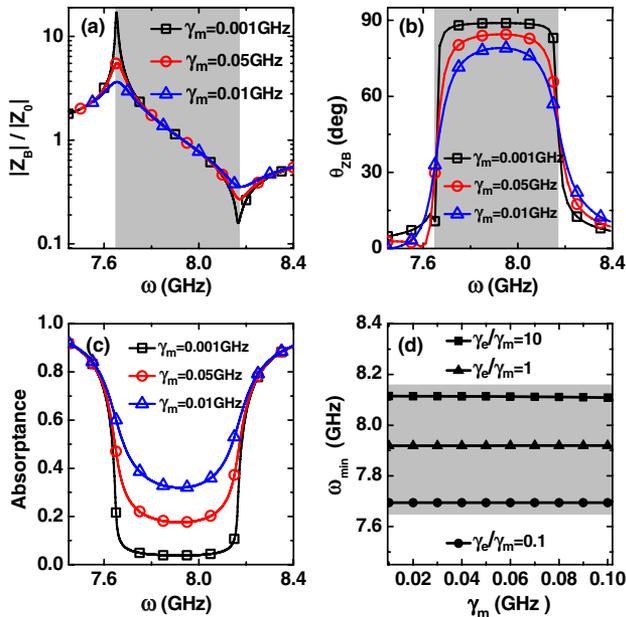


Fig. 7. (a)  $|Z_B|$ , (b)  $\theta_{ZB}$ , and (c) absorptance versus  $\omega$  under different  $\gamma_m$  in DPS/DNG PC with  $\gamma_e/\gamma_m = 1$ . (d)  $\omega_{\min}$  as a function of  $\gamma_m$  under different values of  $\gamma_e/\gamma_m$ . The thicknesses of the layers are  $d_1 = 6$  mm and  $d_2 = 12$  mm. The gray area corresponds to the zero- $\bar{n}$  gap.

for instance, in Fig. 7 we plot  $|Z_B|$ ,  $\theta_{ZB}$  and absorptance versus angular frequency under different  $\gamma_m$ . As shown in Fig. 7(a), the values of  $|Z_B|$  corresponding to different  $\gamma_m$  are almost the same in the central region of the zero- $\bar{n}$  gap, evident differences can only be found near the gap edges. On the other hand, it can be seen from Fig. 7(b) that  $\theta_{ZB}$  decreases significantly as  $\gamma_m$  increases, meaning that the impedance mismatch of the considered system weakens, more energy can transmit into the PC structure and eventually be absorbed. Hence, the absorption becomes larger as  $\gamma_m$  increases, as shown in

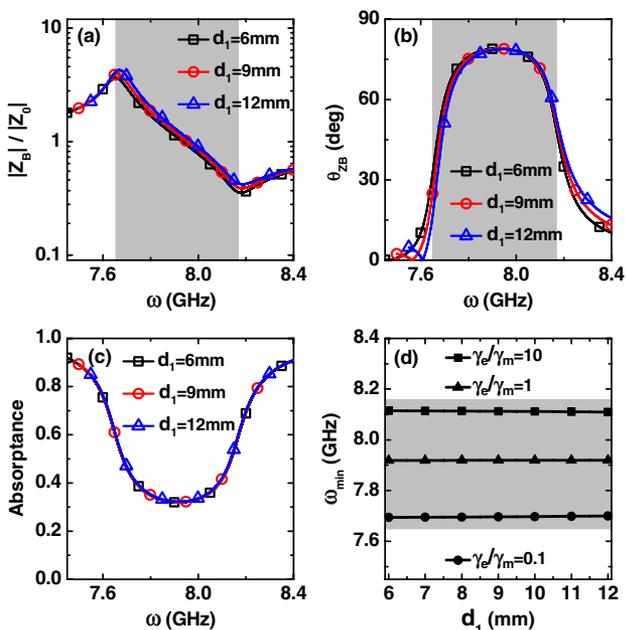


Fig. 8. (a)  $|Z_B|$ , (b)  $\theta_{ZB}$ , and (c) absorptance versus  $\omega$  under different  $d_1$  for DPS/DNG PC structures with  $d_1/d_2 = 1/2$  and  $\gamma_e = \gamma_m = 0.001$  GHz. (d)  $\omega_{\min}$  as a function of  $d_1$  in PC structures with  $d_1/d_2 = 1/2$ .

Fig. 7(c). It can also be observed from Fig. 7(b) that, for different values of  $\gamma_m$ ,  $\theta_{ZB}$  reaches maximum at almost the same frequency. Such a result causes  $\omega_{\min}$  to remain almost unchanged as  $\gamma_m$  varies, as shown in Fig. 7(c). In Fig. 7(d), we show the dependence of  $\omega_{\min}$  on  $\gamma_m$  under different values of  $\gamma_e/\gamma_m$ . We can observe that  $\omega_{\min}$  is independent of  $\gamma_m$  when  $\gamma_e/\gamma_m$  is fixed.

Then we proceed to figure out how the geometrical parameters affect the Bloch impedance and the absorption loss. In Fig. 8, we plot  $|Z_B|$ ,  $\theta_{ZB}$  and the absorptance versus angular frequency under different  $d_1$  in PC structure with  $d_1/d_2 = 1/2$  and  $\gamma_e = \gamma_m = 0.001$  GHz. It can be seen that  $|Z_B|$  and  $\theta_{ZB}$  are both insensitive to  $d_1$ , hence both the frequency range of the zero- $\bar{n}$  gap and the corresponding absorption remain invariant. The dependence of  $\omega_{\min}$  on  $d_1$  is investigated and shown in Fig. 8(d). It is seen that  $\omega_{\min}$  remains constant as  $d_1$  varies when  $\gamma_e/\gamma_m$  is fixed.

Next, we change the thickness ratio  $d_1/d_2$  and study the Bloch impedance and the absorption loss of the considered system. In Fig. 9, we plot  $|Z_B|$ ,  $\theta_{ZB}$  and absorptance as a function of angular frequency in PC structure with different  $d_1/d_2$ . Here, we choose  $d_1 = 6$  mm,  $\gamma_e = 0.01$  GHz and  $\gamma_m = 0.001$  GHz. It can be seen that as  $d_1/d_2$  increases, there is a redshift of the curve of  $|Z_B|$ , meaning that the zero- $\bar{n}$  gap will shift to lower frequencies as well. Furthermore, for different values of  $d_1/d_2$ , the properties of  $|Z_B|$  and  $\theta_{ZB}$  are similar. As  $\omega$  increases across the zero- $\bar{n}$  gap,  $|Z_B|$  decreases,  $\theta_{ZB}$  increases and reaches maximum near the higher gap edge. Therefore,  $\omega_{\min}$ , corresponding to the minimum absorption, also appears near the upper gap edge for all three cases, as shown in Fig. 9(c). Figure 9(d) shows  $\omega_{\min}$  as a function of  $d_1/d_2$  in PC structure with  $d_1 = 6$  mm. It can be seen that although the zero- $\bar{n}$  gap and  $\omega_{\min}$  change significantly as  $d_1/d_2$  varies, the relative position of  $\omega_{\min}$  inside the bandgap remains invariant. From the results shown in Figs. 8 and 9, we can

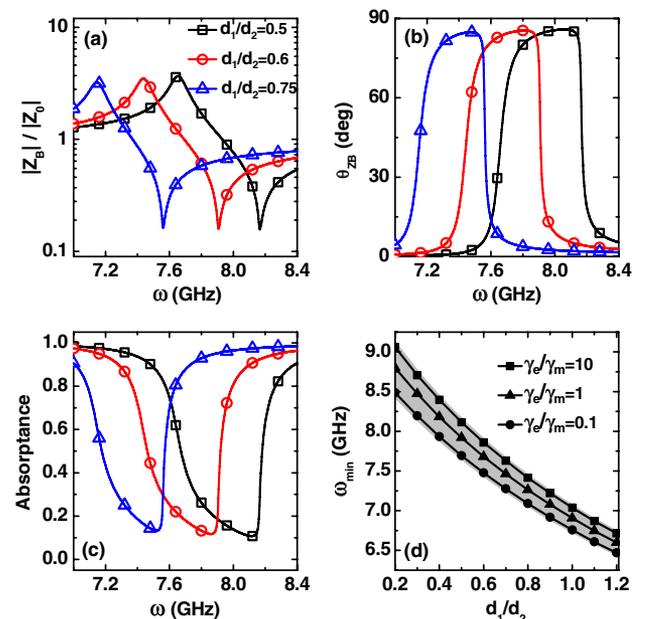


Fig. 9. (a)  $|Z_B|$ , (b)  $\theta_{ZB}$ , and (c) absorptance versus  $\omega$  under different  $d_1/d_2$  for DPS/DNG PC structures with  $d_1 = 6$  mm and  $\gamma_e = 0.01$  GHz,  $\gamma_m = 0.001$  GHz. The squares, circles, and triangles correspond to  $d_2 = 12$ ,  $10$ , and  $8$  mm, respectively. (d)  $\omega_{\min}$  as a function of  $d_1/d_2$  when  $d_1 = 6$  mm.

conclude that the properties of the absorption loss within the zero- $\bar{n}$  gap are insensitive to the geometrical parameters of the photonic structures.

## 5. CONCLUSIONS

In summary, we have investigated the properties of the one-dimensional PCs consisting of alternating layers of air and lossy DNG metamaterials from the viewpoint of the Bloch impedance. We found that the absorption loss inside the zero- $\bar{n}$  gap is mainly determined by the phase angle rather than the complex modulus of the Bloch impedance. There exists a frequency which corresponds to the maximum phase angle of the Bloch impedance and minimum absorption loss inside the zero- $\bar{n}$  gap. Such a frequency is sensitive to the value of  $\gamma_e/\gamma_m$  of the DNG metamaterials. We also found that the changes of the geometrical parameters of the considered system do not influence the main characteristics of the Bloch impedance and the absorption loss.

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