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Structures with negative index of refraction

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We review recent progress in the investigation of man made composites which exhibit negative refraction of the electromagnetic waves. Results for both left-handed materials (LHM) and photonic crystals are presented. Experiments on LHM are mostly done in the microwave regime, while on photonic crystals can be performed in the optical or the far-infrared regime.

1 Introduction

Rapidly increasing interest in the left-handed materials (LHM) started after Pendry et al. predicted that certain man-made composite structure could possess, in a given frequency interval, a negative *effective* magnetic permeability $\mu_{\rm eff}$ [1]. Combination of such a structure with a negative effective permittivity medium – for instance the regular array of thin metallic wires [2–7] – enabled the construction of meta-materials with both *effective* permittivity and permeability *negative*. This was confirmed by experiments [8, 9].

Structures with negative permittivity and permeability were named "left-handed" by Veselago [10] over 30 years ago to emphasize the fact that the intensity of the electric field $\boldsymbol{E}$, the magnetic intensity $\boldsymbol{H}$ and the wave vector $\boldsymbol{k}$ are related by a left-handed rule. This can be easily seen by writing Maxwell's equation for a plane monochromatic wave:

$$\boldsymbol{k} \times \boldsymbol{E} = \frac{\omega\mu}{c}\,\boldsymbol{H} \quad \text{and} \quad \boldsymbol{k} \times \boldsymbol{H} = -\frac{\omega\epsilon}{c}\,\boldsymbol{E}\,. \tag{1}$$

Once ϵ and μ are both positive, then $\boldsymbol{E}$, $\boldsymbol{H}$ and $\boldsymbol{k}$ form a right set of vectors. In the case of negative ϵ and μ, however, these three vectors form a left set of vectors.

In his pioneering work, Veselago described the physical properties of LH systems: Firstly, the direction of the energy flow, which is given by the Poynting vector

$$\boldsymbol{S} = \frac{c}{4\pi}\boldsymbol{E} \times \boldsymbol{H} \tag{2}$$

does not depend on the sign of the permittivity and permeability of the medium. Then, the vectors $\boldsymbol{S}$ and $\boldsymbol{k}$ are parallel (anti-parallel) in the right-handed (left-handed) medium, respectively. Consequently,

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the phase and group velocity of an electromagnetic wave propagate in *opposite* directions in the left-handed material. This gives rise to a number of novel physical phenomena, as were discussed already by Veselago. For instance, the Doppler effect and the Cherenkov effect are reversed in the LHM [10].

If both ϵ and μ are negative, then also the index of refraction n is negative [10, 11]. This means the negative refraction of the electro magnetic wave passing through the boundary of two materials, one with positive and the second with negative n (negative Snell's law). Observation of negative Snell's law, reported experimentally [12] and later in [13], is today a subject of rather controversially debate [14–18]. Analytical arguments of the sign of the refraction index were presented in [11]. Numerically, negative phase velocity was observed in FDTD simulations [19]. Negative refraction index was calculated from the transmission and reflection data [20]. Finally, negative refection on the wedge experiment was demonstrated by FDTD simulations [21].

Negative refraction allows the fabrication of a flat lens [10]. Maybe the most challenging property of the left-handed medium is its ability to enhance the evanescent modes [22]. Therefore a flat lens, constructed from left-handed material with $\epsilon = \mu = -1$ could in principle work as perfect lens [22] in the sense that it can reconstruct an object without any diffraction error.

The existence of the perfect lens seems to be in contradiction with fundamental physical laws, as was discussed in a series of papers [14, 23, 24]. Nevertheless, more detailed physical considerations not only showed that the construction of "almost perfect" lens is indeed possible, but brought some more insight into this phenomena [25–32].

All the experiments that showed left-handed behavior were performed in the microwave regime. It is of interest to examine if it will be possible to observe left-handed behavior at the optical or the far-infrared regime. Using the metallic structures needed in the microwave experiments might not be possible to fabricate them in such small length scales. It was therefore proposed that photonic crystals might have some frequency region, which will show left-handed behavior [33–39]. Notomi [33] studied light propagation in strongly modulated two dimensional (2D) photonic crystals. Such a photonic crystal (PC) behaves as a material having an effective refractive index n_{eff} controllable by the band structure. In these PC structures the permittivity is periodically modulated in space and is positive. The permeability is equal to one. Negative n_{eff} for a frequency range was found. The existence of negative n_{eff} was demonstrated [34, 36] by a finite difference time domain (FDTD) simulation. Negative refraction on the interface of a 3D PC structure has been experimentally observed by Kosaka et al. [38]. Very recently negative refraction and superlensing has been experimentally observed [39] in 2D photonic crystals. Similar unusual light propagation was observed [40] in 1D and 2D diffraction gratings.

As Veselago also discussed in his pioneering paper, the permittivity and the permeability of the left-handed material must depend on the frequency of the EM field, otherwise the energy density [41]

$$U = \frac{1}{2\pi} \int d\omega \left[\frac{\partial(\omega\epsilon')}{\partial\omega} |E|^2 + \frac{\partial(\omega\mu')}{\partial\omega} |H|^2 \right] \tag{3}$$

would be *negative* for *negative* ϵ' and μ' (real part of the permittivity and permeability). Then, according to Kramers–Kronig relations, the imaginary part of the permittivity (ϵ'') and of the permeability (μ'') are non-zero in the LH materials. Transmission losses are therefore unavoidable in any LH structure. Theoretical estimation of losses is rather difficult problem, and led even to the conclusion that LH materials are not transparent [17]. Fortunately, recent experiments [13, 42] confirmed the more optimistic theoretical expectation [43], that the losses in the LH structures might be as small as in conventional RH materials.

In this paper we present typical structures of the left-handed materials (Sect. 2), discuss a numerical method of simulation of the propagation of EM waves based on the transfer matrix (Sect. 3), and present some recent results obtained by this method in Sect. 4. The method of calculation of the refractive index is presented and applied to the LH structure. An unambiguous proof of the negative refraction index is given in Sect. 5. Finally, in the Section 6 we discuss some new directions of the development of both theory and experiments.

2 Structure

LHM materials are by definition composites, whose properties are not determined by the fundamental physical properties of their constituents but by the shape and distribution of specific patterns included in them. The route of the construction of the of LH structure consists from three steps:

Firstly, the split ring resonators (SRR) (see Fig. 1 for the structure of SRR) was predicted to exhibit the resonant *magnetic* response to the EM wave, polarized with ***H*** parallel to the axis of the SRR. Then, the periodic array of SRR is characterized [1] by the *effective* magnetic permeability

$$\mu_{\rm eff}(f) = 1 - \frac{F\nu^2}{\nu^2 - \nu_m^2 + i\nu\gamma}\,. \tag{4}$$

In (4), ν_m is the resonance frequency which depends on the structure of the SRR (Fig. 1) as $(2\pi\nu_m)^2 = 3L_x c_{\rm light}^2/[\pi \ln(2c/d) r^3]$. F is the filling factor of the SRR within one unit cell and γ is the damping factor $2\pi\gamma = 2L_x\rho/r$, where ρ is the resistivity of the metal.

Formula (4) assures that the *real* part of $\mu_{\rm eff}$ is *negative* at an interval $\Delta\nu$ around the resonance frequency. If an array of SRR is combined with a medium with negative *real* part of the permittivity, the resulting structure would possess *negative effective* refraction index in the resonance frequency interval $\Delta\nu$ [11]. The best candidate for the negative permittivity medium is a regular lattice of thin metallic wires, which acts as a high pass filter for the EM wave polarized with ***E*** parallel to the wires. Such an array exhibits negative effective permittivity

$$\epsilon_{\rm eff}(\nu) = 1 - \frac{\nu_p^2}{\nu^2 + i\nu\gamma}\,. \tag{5}$$

[2, 4, 6] with the plasma frequency $\nu_p^2 = c_{\rm light}^2/(2\pi a^2 \ln(a/r))$ [2]. Sarychev and Shalaev derived another expression for the plasma frequency, $\nu_p^2 = c_{\rm light}^2/(2\pi a^2[\ln(a/\sqrt{2}r) + \pi/2 - 3]$ [6]. Apart from tiny differences in both formulas, the two theories are equivalent [7] and predict that effective permittivity is *negative* for $\nu < \nu_p$.

By combining both the above structures, a left-handed structure can be created. This was done for the first time in the experiments of Smith et al. [8]. Left-handed material is a periodic structure. A typical unit cell of the left-handed structure is shown in Fig. 1. Each unit cell contains a metallic wire and one split ring resonator (SRR), deposited on the dielectric board.

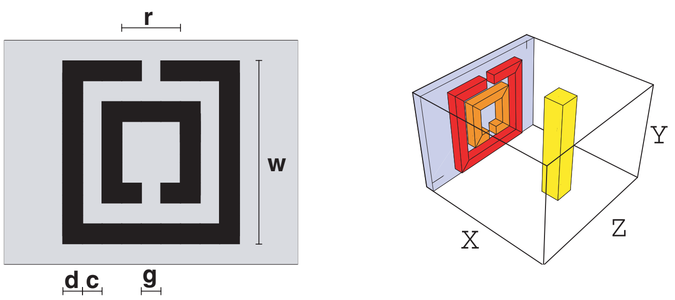

Fig. 1 (online colour at: www.interscience.wiley.com) Left: Structure of the split ring resonator (SRR). The SRR consists of two splitted metallic "rings". The SRR is characterized by the size w, width of the rings d, and two gaps: g and c. The external magnetic field induces an electric current in both rings [1]. The shape of the SRR (square or circular) is not crucial for the existence of the magnetic resonance. Right: Structure of the unit cell of the left-handed material. Each unit cell contains the split ring resonator located on the dielectric board, and one wire. Left-handed structure is created by regular lattices of unit cells. The EM wave propagates along the z direction. Periodic boundary conditions are considered in the x and the y direction, which assures the periodic distribution of the EM field.

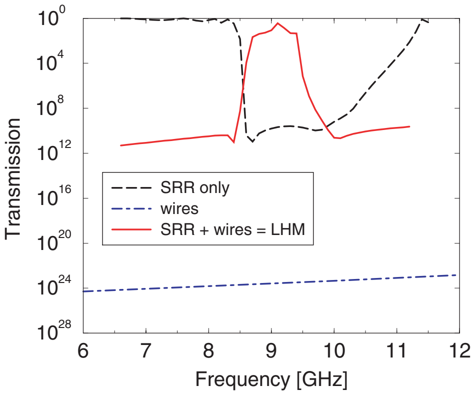

Fig. 2 (online colour at: www.interscience.wiley.com) Transmission of the EM wave, polarized with $\boldsymbol{E} \parallel y$ and $\boldsymbol{H} \parallel x$, through a periodic array of split ring resonators, wires, and of both SRR and wires.

Figure 2 shows the transmission of the EM waves through the left-handed structure discussed above. The transmission through the array of the SRR is close to unity for all the frequencies outside the resonance interval (8.5–11 GHz in this particular case) and decreases to -120 dB in this interval, because μ_{eff} is negative (Eq. (4)). The transmission of the array of metallic wires is very small for all frequencies below the plasma frequency (which is $\sim$20 GHz in this case), because ϵ_{eff} is negative (Eq. (5)). The structure created by the combination of an array of SRR and wires exhibits high transmission $T \sim 1$ within the resonance interval, where both ϵ_{eff} and μ_{eff} are negative. For frequencies outside the resonance interval, the product $\mu_{\text{eff}}\epsilon_{\text{eff}}$ is negative. The transmission decays with the system length, and is only ~ -120 dB in the example of Fig. 2. Experimental analysis of the transmission of all the three structures was performed by Smith et al. [8].

We want to obtain a resonance frequency $\nu_m \approx 10$ GHz. This requires the size of the unit cell to be 3–5 mm. The wavelength of the EM wave with frequency $\sim$ 10 GHz is $\approx$4 cm, and exceeds by a factor of 10 the structural details of the left-handed materials. We can therefore consider the left-handed material as macroscopically homogeneous. This is the main difference between the left-handed structures and the "classical" photonic band gap (PBG) materials, in which the wave length is comparable with the lattice period.

It is important to note that the structure described in Fig. 1 is strongly anisotropic. For frequencies inside the resonance interval, the effective ϵ_{eff} and μ_{eff} are negative only for EM field with $\boldsymbol{H} \parallel x$ and $\boldsymbol{E} \parallel y$. The left-handed properties appear only when a properly polarized EM wave propagates in the z direction. The structure in Fig. 1 is therefore *effectively* one-dimensional. Any EM waves, attempting to propagate either along the x or along the y direction would decay exponentially since the corresponding product $\epsilon_{\text{eff}}\mu_{\text{eff}}$ is negative. This structure is therefore not suitable for the realization of the perfect lens. To test the negative Snell's law experimentally, a wedge type of experiment must be

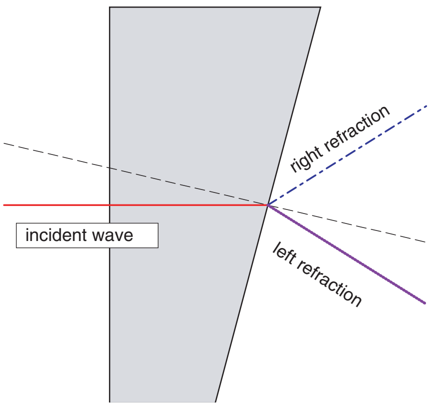

Fig. 3 (online colour at: www.interscience.wiley.com) Refraction experiment on the Left-handed material [12, 42]. The incident EM wave propagates from the left and hits perpendicularly the left boundary of the wedge. The angle of refraction is measured when the EM wave passes the right boundary of the inspected material and propagates for some time in the air. Two possible directions of the propagation of the refracted wave are shown: the right refraction for the conventional right-handed (RH) material, and left refraction for the left-handed material. This experimental design enables to use also strongly anisotropic one-dimensional LH samples, since the angle of refraction is measured *outside* the sample.

considered [12] (Fig. 3) in which the angle of refraction is measured *outside*, the left-handed medium (in air) [12, 13].

Two dimensional structures have also been constructed. For instance, the anisotropy in the $x-z$ plane is removed if each unit cell contains two SRR located in two perpendicular planes [9, 12]. No three dimensional structures have been experimentally prepared yet.

It is also worth mentioning that some other one dimensional structures were prepared, in which the wires were deposited on the same side of the dielectric board with the SRRs. The wires could be located either on the opposite side of the dielectric board, as it was done in [9, 12, 42], or put next to the SRRs on the same side of board [44]. More complicated one-dimensional structure was suggested by Ziolkowski [45]. Recently, cut wires were used instead of continuous wires [46].

3 Numerical simulation

Various numerical algorithms were used to simulate the propagation of EM waves through the LH structure. We concentrate on the transfer matrix algorithm, developed in a series of papers by Pendry and co-workers [47]. The transfer matrix algorithm enables us to calculate the transmission, reflection, and absorption as a function of frequency [48, 49]. Others use commercial software: either Microwave studio [13, 42, 50] or MAFIA [8], to estimate the position of the resonance frequency interval. Time-dependent analysis, using various forms of FDTD algorithms are also used [13, 19, 45, 51].

In the transfer matrix algorithm, we attach in the z direction, along which EM wave propagates, two semi-infinite ideal leads with $\epsilon = 1$ and $\mu = 1$. The length of the system varies from 1 to 300 unit cells. Periodic boundary conditions along the x and y directions are used. This makes the system effectively infinite in the transverse directions, and enables us to restrict the simulated structure to only one unit cell in the transverse directions.

A typical size of the unit cell is 3.66 mm. Because of numerical problems, we are not able to treat very thin metallic structures. While in experiments the thickness of the SRRs is usually 17 μm, the thickness used in the numerical calculations is determined by the minimal mesh discretization, which is usually ≈ 0.33 mm. In spite of this constrain, the numerical data are in qualitative agreement with the experimental results. This indicates that the thickness of the SRR is not a crutial parameter, unless it decreases below or is comparable with the skin depth δ. As $\delta \approx 0.7\,\mu$m at GHz frequencies of interest, we are far from this limitation.

4 Transmission

As discussed in Section 2 the polarization of the EM waves is crucial for the observation of the LH properties. The electric field E must be parallel to the wires, and the magnetic field H must be parallel to the axis of the SRRs. In the numerical simulations, we treat simultaneously both polarizations, $\boldsymbol{E} \parallel x$ as well as $\boldsymbol{E} \parallel y$. Due to the non-homogeneity of the structure, these polarizations are not separated: there is always non-zero transmission t_{xy} from the x to y polarized wave. As we will see later, this effect is responsible for some unexpected phenomena. At present, we keep in mind that they must be included into the formula for absorption

$$A_x = 1 - |t_{xx}|^2 - |t_{xy}|^2 - |r_{xx}|^2 - |r_{xy}|^2 \tag{6}$$

and in the equivalent relation for A_y.

Figure 4 shows typical data for the transmission in the resonance frequency region. A resonance frequency interval, in which the transmission increases by many orders of magnitude is clearly visible. Of course, high transmission does not guarantee negative refraction index. The sign of n must be obtained by other methods, which will be described in Sect. 5.

In contrast to the original experimental data, numerical data show very high transmission, indicating that LH structures could be as transparent as the "classical" right-handed ones. As an example, we show in right Fig. 4 the transmission as the function of the system length for the frequency inside the

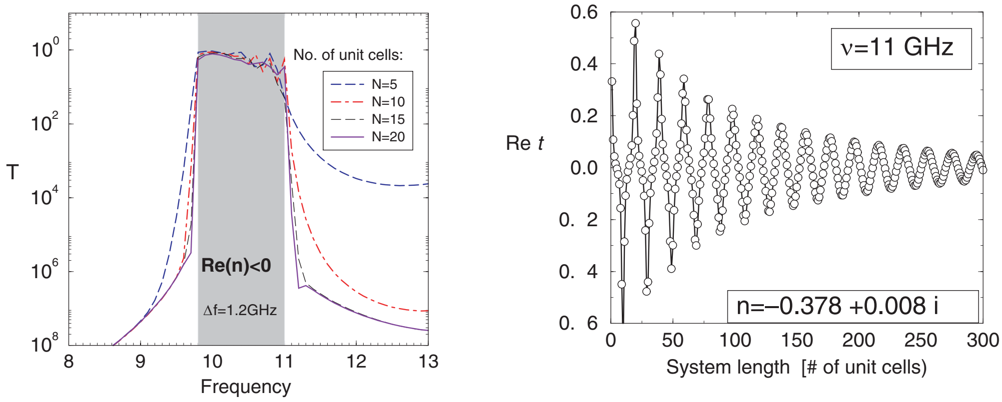

Fig. 4 (online colour at: www.interscience.wiley.com) Left: Transmission power $T = |t|^2$ through the Left-handed meta material of various lengths. Right: Length dependence of the real part of the transmission t for the frequency $\nu = 11$ GHz.

resonance frequency interval. We see that the transmission is quite good also for samples of the length of 300 unit cells. Refractive index is $n = -0.378 + 0.008i$. This is surprising, because due to the dispersion, high losses are expected. More detailed numerical analysis of the transmission losses was given in [43, 53].

Figure 4 shows also that outside the resonance interval the transmission never decreases below a certain limit. Due to the non-homogeneity of the structure there is a non-zero probability t_{yx} that the EM wave, polarized with $\boldsymbol{E} \| y$, is converted into the polarization $\boldsymbol{E} \| x$. The total transmission t_{yy} consists therefore not only from the "unperturbed" contribution $t_{yy}^{(0)}$, but also from additional terms, which describe the conversion of the y-polarized wave into x-polarized and back to y-polarized:

$$t_{yy}(0, L) = t_{yy}^{(0)}(0, L) + \sum_{z,z'} t_{xy}(0, z)\, t_{xx}(z, z')\, t_{yx}(z'L) + \dots \,. \tag{7}$$

$t_{yy}^{(0)}(0, L)$ decreases exponentially with the system length L, while the second term, which represents the conversion of the y-polarized wave into x-polarized wave and back, remains system-length independent, because $t_{xx}(z, z') \sim 1$ for any distance $|z - z'|$.

5 Effective index of refraction

As was discussed above, the structural inhomogeneities of the LH materials are approximately ten times smaller than the wavelength of the EM wave. It is therefore possible, within a first approximation, to consider the slab of the LH material as an homogeneous material. Then we can and use the textbook formulas for the transmission $t\,e^{ikL}$ and reflection r for the homogeneous slab:

$$t^{-1} = \left[\cos\,(nkL) - \frac{i}{2}\left(z + \frac{1}{z}\right)\sin\,(nkL)\right] \tag{8}$$

and

$$\frac{r}{t} = -\frac{i}{2}\left(z - \frac{1}{z}\right)\sin\,(nkL)\,. \tag{9}$$

Here, z and n are the *effective* impedance and the refraction index, respectively, k is the wave vector of the incident EM wave in vacuum, and L is the length of the LH slab. We only consider perpendicular incident waves, so that only the z components of the effective parameters are important. To simplify the calculations, we neglect also the conversion of the polarized EM wave into another polarization, discussed in the Section 4. More accurate analysis should treat both t and r as 2×2 matrices. Here we assume that the off-diagonal elements of these matrices are negligible:

$$|t_{xy}| \ll |t_{yy}| . \tag{10}$$

In the present analysis, we use the numerical data for the transmission and the reflection obtained by the transfer matrix simulation. The expressions for the transmission and the reflection can be inverted as

$$z = \pm\sqrt{\frac{(1+r)^2 - t^2}{(1-r)^2 - t^2}} \tag{11}$$

$$\cos(nkL) = X = \frac{1}{2t}\left(1 - r^2 + t^2\right) . \tag{12}$$

The sign of z is determined by the condition

$$z' > 0 \tag{13}$$

which determines z unambiguously. The obtained data for z enables us also to check the assumption of the homogeneity of the system. We indeed found that z is independent of the length of the system L.

The second relation, (12), is more difficult to invert since $\cos^{-1}$ is not an unambiguous function. One set of physically acceptable solutions is determined by the requirement

$$n'' > 0 \tag{14}$$

which assures that the material is passive. The real part of the refraction index, n', however, suffers from the unambiguity of $2\pi m/(kL)$ (m is an integer). To avoid this ambiguity, data for various system length L were used. As n characterizes the material property of the system, it is L independent. Using also the requirement that n should be a continuous function of the frequency, the proper solution of (12) was found and the resonance frequency interval, in which n' is *negative* was identified [20]. Here, we use another method for the calculation of n and z: Equation (12) can be written as

$$e^{-n''kL}\left[\cos(n'kL) + i\sin(n'kL)\right] = Y = X \pm \sqrt{1 - X^2} . \tag{15}$$

Relation (15) enables us to find unambiguously both the real and the imaginary part of the refraction index from the linear fit of $n''kL$ and $n'kL$ *vs.* the system length L. The requirement (14) determines the sign in the r.h.s. of Eq. (15), because $|Y| < 1$. Then, the linear fit of $n'kL$ *vs.* L determines unambiguously the real part of n'.

Figure 5 shows the L dependence of $n'kL$ for three frequencies inside the resonance interval. The numerical data proves that the real part of the refraction index, n', is indeed *negative* in the resonance interval. For comparison, we present also $n'kL$ *vs.* L for the x polarized wave outside the resonance interval. As expected, the slope is positive and gives that $n' = 1.13$, which is close to unity.

Besides the sign of the real part of the refraction index, the value of the imaginary part of n, n'' is important, since it determines the absorption of the EM waves inside the sample. Fortunately, n'' is very small, it is only of the order of 10^{-2} inside the resonance interval. As is shown in Fig. 4, quite good transmission was numerically obtained also for samples with length of 300 unit cells (which corresponds to a length of the system 1.1 m). This result is very encouraging and indicates that left-handed structures could be as transparent as right-handed materials.

While the above method works very well in the right side of the resonance interval, we had problems to estimate n in the neighborhood of the left border of the resonance region, where n' is very large and negative. This is, however, not surprising since in this frequency region the wavelength of the propagating wave becomes comparable with the size of the unit cell, so that the effective parameters have no physical meaning. We also have serious problems to recover proper values of n outside the resonance interval. This is due to the conversion of the x polarized wave into a y polarized. As a

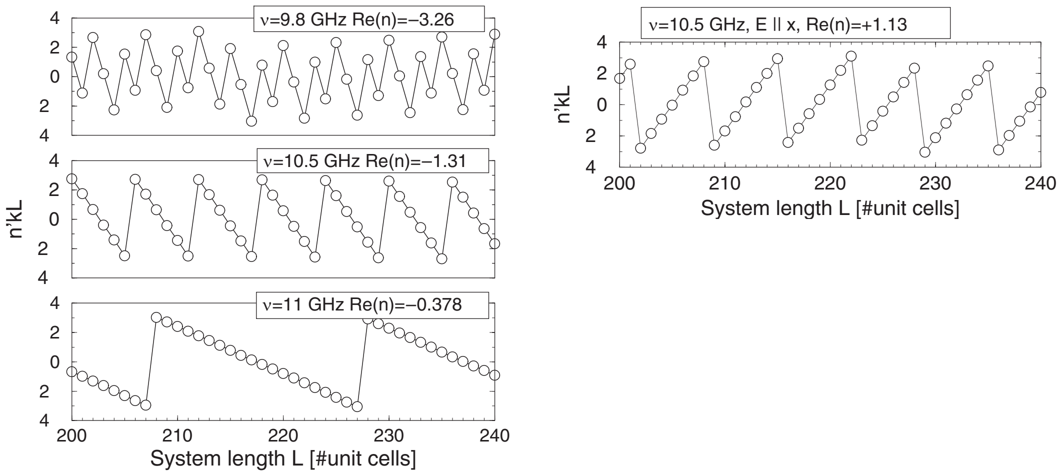

Fig. 5 Left panel shows $n'kL$ as a function of the system length L for the EM wave polarized with E parallel to the wires. The slope is negative which confirms that n' is negative. Right panel: $n'kL$ as a function of the system length L for the EM wave polarized $E \parallel x$. The slope is positive which confirms that Re n' is positive. A linear fit gives that $n' = 1.13$. There is almost no interaction of the LH structure with the EM wave.

result, we do not have enough numerical data for obtaining the L-dependence of the transmission t. The condition (10) is not any more fulfilled, and the present theory must be generalized as discussed above.

6 Further development

We reviewed some recent experiments and numerical simulations on the transmission of the electromagnetic waves through left handed structures. For completeness, we note that recently, Notomi [33] has studied the light propagation in strongly modulated two dimensional photonic crystals (PC). In these PC structures the permittivity is periodically modulated in space and is *positive*. The permeability is equal to unity. Such PC behaves as a material having an effective refractive index controllable by the band structure. For a certain frequency range it was found by FDTD simulations [33, 35, 36] that n_{eff} is negative. It is important to examine if left-handed behavior can be observed in photonic crystals at optical frequencies.

Negative refraction on the interface of a three dimensional PC structure has been observed experimentally by Kosaka et al. [38] and a negative refractive index associated to that was reported. Large beam steaming has been observed in [38], that authors called "the superprism phenomena". Similar unusual light propagation has been observed in one-dimensional and two dimensional refraction gratings. Finally, a theoretical work [34] has predicted a negative refraction index in photonic crystals.

Studies of the left handed structures open a series of new challenging problems for theoreticians as well as for experimentalists. The complete understanding of the properties of left-handed structures requires the reevaluation of some "well known" facts of the electromagnetic theory. There is no formulas with *negative* permeability in classical textbooks of electromagnetism [41, 52]. Application of the existing formulas to the analysis of left handed structures may lead to some strange results. The theory of EM field has to be reexamined assuming negative μ and ϵ. We need to understand completely the relationship between the real and the imaginary parts of the permittivity and the permeability. Kramers–Kronig relations should be valid, but nobody have verified them yet in the case of the left-handed structures. The main problem is that we need ϵ_{eff} and μ_{eff} in the entire range of frequencies, which is difficult to obtain numerically. Then, due to the anisotropy of the structure as well as the nonzero transmission t_{xy} in Eq. (8), Kramers–Kronig relations should be generalized. We do not believe that today's numerical data enables their verifications with sufficient accuracy.

Both in the photonic crystals and LHM literature there is a lot of confusion about what is the correct definitions of the phase and group refractive index and what is their relations to negative refraction. In

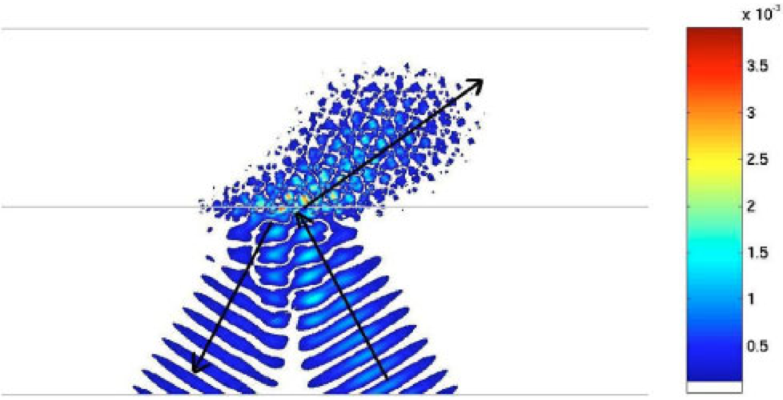

Fig. 6 An incident EM wave is propagating along a 30 degree direction from a material with an index of refraction $n = 1$ to a material with $n = -0.7$. Notice that the magnitude of the magnetic field refracts along the negative direction.

additions, it is instructive to see how the LH behavior is related with the sign of the phase and group refractive indices for the PC system. The conditions of obtaining LH behavior in PC were recently examined in [35]. It was demonstrated that the existence of negative refraction is neither a prerequisite nor guarantees a negative effective refraction index and so LH behavior. Contrary, *LH behavior can be seen only if phase refractive index* n_{phase} *is negative*. Once n_{phase} is negative, the product $\boldsymbol{S} \cdot \boldsymbol{k}$ is also negative, and the vectors $\boldsymbol{k}$, $\boldsymbol{E}$ and $\boldsymbol{H}$ form a left handed set, as discussed in the Introduction.

The following problems are currently discussed in literature. The negative Snell's law requires the understanding in more detail of the relationship between the Poynting vector, the group velocity, and the phase velocity. We believe, that there is no controversial in this phenomena [16, 18]. We need also more general relations for the energy of the EM field [54] which incorporates all the allowed signs of the real and imaginary parts of the permittivity and permeability.

We believe that further analysis, of what happens when EM wave crosses the boundary of the left-handed and right-handed systems, will bring more understanding of the negative refraction as well as perfect lensing. Numerical FDTD simulations of the transmission of the EM wave through the interface of the positive and negative refraction index [36] showed that the wave is trapped temporarily at the interface and after a long time the wave front moves eventually in the direction of negative refraction. This is clearly seen in Fig. 6, where we plot the amplitude of the magnetic field of an incident Gaussian beam that undergoes reflection and refraction for a very long time. Computer simulations of the transmission through LH wedge [21] also confirm that EM wave spends some time on the boundary before the formation of the left-handed wave front. Formation of surface waves [27, 28] can explain "perfect lensing" without violating causality. Recent experimental development [39] indicate that perfect (although not absolutely perfect) lensing might be possible. We need also to understand some peculiar properties of the left handed structures due to its anisotropy [55] and bi-anisotropy [56]. Anisotropy of real left-handed materials inspires further development of super-focusing [29].

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