

## Transmission losses in left-handed materials

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We numerically analyze the origin of the transmission losses in left-handed structures. Our data confirms that left-handed structures can have very good transmission properties, in spite of the expectable dispersion of their *effective* permeability and refraction index. The large permittivity of the metallic components improves the transmission. High losses, observed in recent experiments, could be explained by the absorption of the dielectric board.

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Left-handed (LH) materials is a common name for the manmade structures which possess, in a given frequency region, both negative *effective* electrical permittivity and magnetic permeability. Although such materials are in general not available in nature, their experimental fabrication became possible after the suggestion of Pendry *et al.* [1]. They predicted, that a lattice of metallic split rings resonators (SRR) may exhibit, in a resonance frequency region, negative *effective* permeability  $\mu_{\text{eff}}$ . It is also well known that a periodic lattice of thin metallic wires behaves as an effective medium with negative *effective* permittivity  $\epsilon_{\text{eff}}$  [2]. By combining a lattice of metallic wires with a lattice of SRRs, Smith *et al.* [3] created for the first time left-handed structures.

At present, LH materials attract a growing interest of both theoretical and experimental research. Various interesting physical properties of LH structures were discussed in Refs. [4] and [5]. Pendry [6] suggested that LH materials enable the construction of the perfect lens. Smith *et al.* [7] proved, on the basis of the numerical data, that the LH structure indeed possesses negative refraction index. The negative refraction of the electromagnetic (EM) waves was experimentally observed in Ref. [8]. These unusual results [3–7] have raised objections both to the interpretation of the experimental data and to the realization of negative refraction [9–11].

In spite of the considerable progress in the studies of the LH materials, a lot of questions remained unanswered. One of the most important questions is whether the LH structures have propagating solutions. LH systems must be disperse [4]. The frequency dependence of the *effective* permittivity and permeability of the LH materials is [1]

$$\epsilon_{\text{eff}}(f) = 1 - \frac{f_e^2}{f^2 + if\gamma} \quad (1)$$

and

$$\mu_{\text{eff}}(f) = 1 - \frac{f_m^2 - f_{m0}^2}{f^2 - f_{m0}^2 + if\gamma}. \quad (2)$$

In Eqs. (1) and (2),  $f_e$  ( $f_m$ ) is the electronic (magnetic) plasma frequency, respectively,  $f_{m0}$  is magnetic resonance frequency, and  $\gamma$  represents the losses of the system. Due to the strong dispersion in the resonance interval, the absorption is assumed to be large [12]. This seems to be consistent with experimental data [3,8] which reported that the transmission of the LH samples was only -20 dB. However, absorption cannot be estimated from these measurements because reflection was not measured. Garcia and Nieto-Vesperinas [9] analyzed the experimental data [8] and concluded that the imaginary part of the effective permittivity  $\epsilon_{\text{eff}}$  [Eq. (1)] dominates its real part. They concluded that the measured transmission of the EM wave through the LH structures is dominated by the large imaginary part of the effective permittivity  $\epsilon_{\text{eff}}$  [13].

Our aim in the present paper is to show that LH structures could possess very high transmittance. Our recent numerical simulations [14] already showed that the transmission of a LH system could be as good as for a right-handed system.

To analyze the transmission properties of LH structures in more detail, we first study the system length dependence of the transmission power  $T$  for the LH structure. The structure is described in detail in Ref. [14]. The unit cell of the LH structure is shown in the inset of Fig. 1(a). Permittivity of the metallic components (SRR and wires) is  $\epsilon_m = 10^5 \times (-3 + i 5.88)$ . We will present below also the frequency dependence of  $T$  for different values of  $\epsilon_m$ . Figure 1(a) shows the frequency dependence of the transmission for various system sizes. This data was obtained by the use of the transfer matrix (TM) technique [14]. A resonance interval of  $9.8 \leq f \leq 11$  (in GHz), in which transmission is close to one, is clearly visible. Fig. 1(b) shows the transmission peak for a homogeneous LH model with an effective permittivity and permeability given by Eqs. (1) and (2), respectively. In Eqs. (1) and (2) we choose parameters which fit our numerical data, shown in Fig. 1(a). Note that the value of  $\gamma = 6 \times 10^{-5}$  GHz is three orders of magnitude smaller than that used in Ref. [8] to interpret the experimental data. This means that there are almost no losses in our structure [15].

In Fig. 2 we plot the transmission as a function of the system length for different frequencies  $f$ . The transmission decreases exponentially with the system length, when  $f$  lies outside the resonance interval. However, for EM waves with frequencies within the resonance interval only a small decrease of the transmission is observed. This unambiguously

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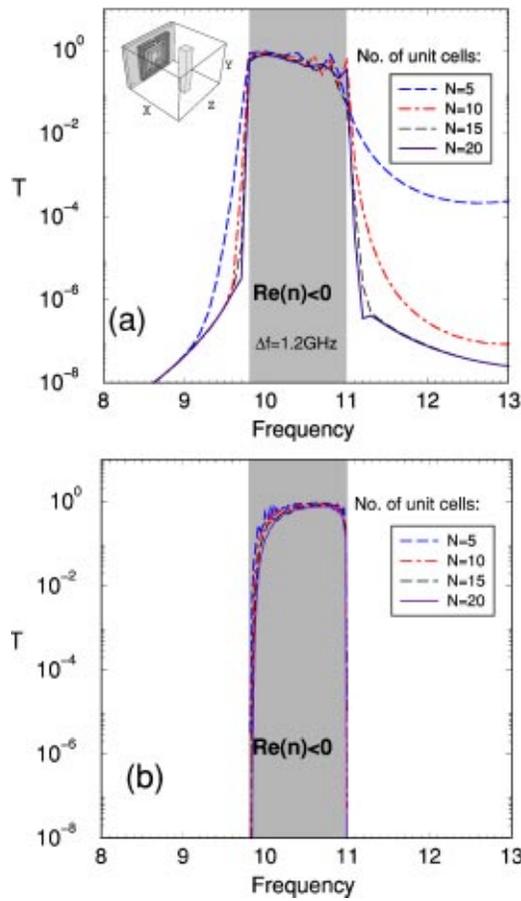


FIG. 1. Transmission power  $T$  of the EM wave through the LH structure of various lengths along the propagation direction. (a) Result of transfer matrix simulations. The inset shows the structure of the unit cell. The size of the unit cell is  $3.3 \times 3.67 \times 3.67$  mm. The simulated system consists of a regular three dimensional array of unit cells, infinite in the  $x$  and  $y$  directions. The polarized EM wave with  $E \parallel y$  and  $H \parallel x$  propagates along the  $z$  direction. (b) Transmission of a homogeneous LH slab with  $\epsilon_{\text{eff}}$  and  $\mu_{\text{eff}}$  given by Eqs. (1) and (2) with parameters  $f_{m0}=9.8$ ,  $f_m=11$ ,  $f_e=12.8$ , and  $\gamma=6 \times 10^{-5}$  (all frequencies are given in GHz).

shows that the transmission is really high in LH materials with realistic parameters for the permittivity of metal. This is correct despite the fact that  $\text{Im } \epsilon_m$  is of the order of  $10^5$ . The EM field is excluded from the metallic component when the  $\text{Im } \epsilon_m$  is high. As the metal absorbs EM waves, an increase of  $\text{Im } \epsilon_m$  reduces absorption losses and improves the transmission of the LH structure.

Figure 2(a) shows also that the transmission never decreases below a certain limit. This effect was observed already in Ref. [14]. Due to the anisotropy of the structure there is namely a nonzero probability  $t_{yx}$  that the EM wave, polarized with  $E \parallel y$  is converted into the polarization  $E \parallel x$ . The total transmission  $t$  can be therefore written as

$$t = t_0 + t_{xy}t't_{yx} + \dots \quad (3)$$

Here,  $t_0$  is the transmission of the  $E \parallel y$  wave. It decreases exponentially with the system length. The second term rep-

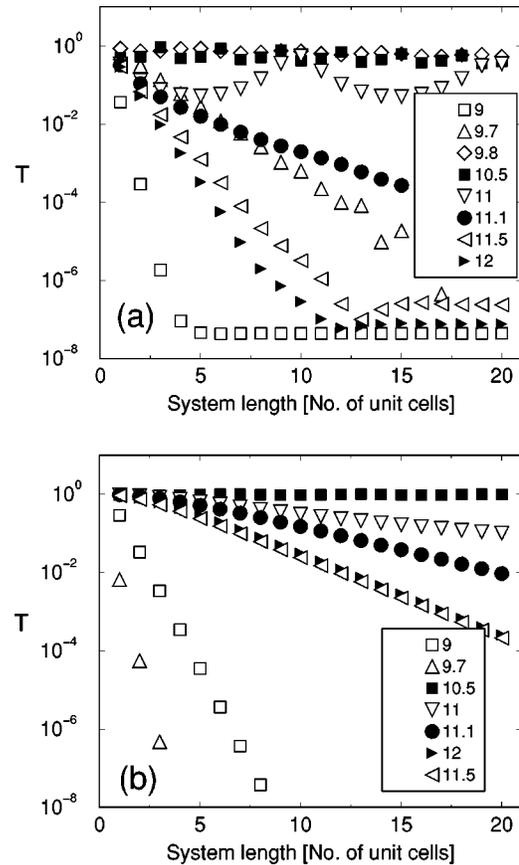


FIG. 2. System length dependence of the transmission power  $T$  of the EM waves for various frequencies. (a) Data from transfer matrix simulations. Note that  $T$  never decreases below a certain limit. (b) Data for the homogeneous model with effective permittivity and permeability given by Eqs. (1) and (2) with parameters listed in Fig. 1.

resents the process of transmission of the  $E \parallel y$  wave into  $E \parallel x$  ( $t_{yx}$ ), its transmission through the structure ( $t'$ ), and conversion back into the original polarization ( $t_{xy}$ ). As  $t' \approx 1$  for any system length,  $t$  is bounded from below:  $t > t_{xy}t_{yx}$ .

In Fig. 3 we present a detailed system length dependence of the transmission for  $f=10.5$  GHz, obtained by TM simulations. The length of the system was up to 300 unit cells, which corresponds to a system of length equal to 1.1 m. From the exponential decrease of the transmission amplitude we estimate the imaginary part of the refraction index to be only  $\text{Im } n = 5 \times 10^{-3}$ .

Transfer matrix data for the transmission and the reflection of EM waves provides us with the complete information needed to extract the effective parameters of the system. Inverting the equations for the transmission and reflection of the *homogeneous* slab of material with a given refraction index and impedance, we find the refraction index [7]. We present in Fig. 4 the effective refraction index as was obtained from the numerical data. For comparison, we present also data for the refraction index, calculated from the frequency dependent  $\epsilon_{\text{eff}}$  and  $\mu_{\text{eff}}$  given by Eqs. (1) and (2), and  $n = \sqrt{\epsilon_{\text{eff}}\mu_{\text{eff}}}$ . Both the numerical data and the homogeneous model give, in the resonance frequency interval, negative Re

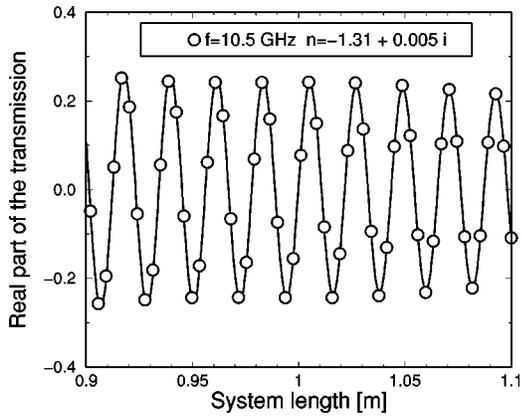


FIG. 3. System length dependence of the transmission  $t$  ( $T = t^*t$ ) of the EM wave of the frequency  $f=10.5$  GHz. Symbols represent transfer matrix data; the solid line is a fit  $a_0 e^{-\kappa x} \cos(kx + x_0)$ . Presented data correspond to the system with  $n_{\text{eff}} = -1.31 + 0.005i$ . Note that system length is in meters.

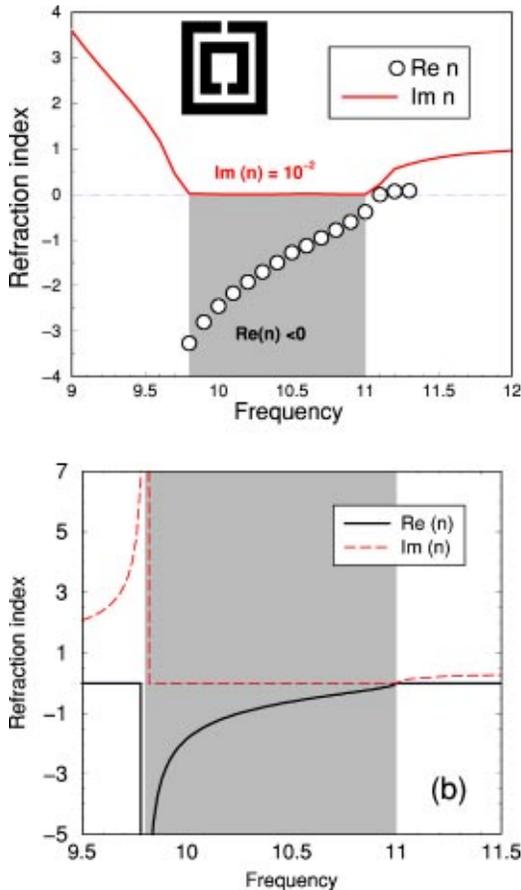


FIG. 4. Effective index of refraction (real and imaginary part) as a function of the frequency  $f$ : (a)  $n$  calculated from numerical transfer matrix data; (b)  $n$  given by Eqs. (1) and (2) (for values of the material parameters see caption of Fig. 1). Only negligible changes of this behavior have been observed when  $f_{e0} > 0$  (data not presented). Note that the  $\text{Im } n$  is very small only in the frequency interval where  $\text{Re } n$  is negative. This guarantees a good transmission of the left-handed wave.

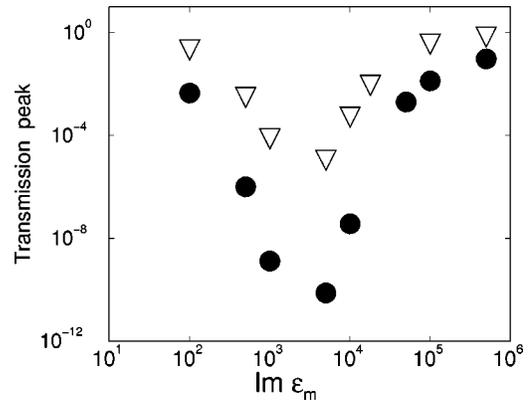


FIG. 5. (a) Transmission  $T$  of the LH structure vs  $\text{Im } \epsilon_m$ . Both “vertical” (circles) and “horizontal” (triangles) orientations of SRR were considered [18].

$n$  with typical resonance behavior in the vicinity of the left interval edge. We also obtain a very small imaginary part for the refraction index. In particular, for the wave shown in Fig. 3 we find that

$$n(f=10.5 \text{ GHz}) = -1.31 + i 0.005. \quad (4)$$

These parameters guarantee good transmission properties.

To show how the transmission of the LH structure depends on the permittivity  $\epsilon_m$  of metallic components, we have simulated LH systems with  $\text{Im } \epsilon_m$  increasing from zero up to  $5 \times 10^5$ . As it is shown in Fig. 5, an increase of the imaginary part of the metallic permittivity improves the transmission properties of LH materials, provided that  $\text{Im } \epsilon_m > 10^4$ . Although the transmission decreases to small values for  $\text{Im } \epsilon_m \approx 10^3 - 10^4$ , it starts to increase and is of the order of one for  $\text{Im } \epsilon_m \geq 5 \times 10^5$ . As the conductance of the copper  $\sigma$  is  $5.9 \times 10^7 (\Omega \text{m})^{-1}$ , the imaginary part of the permittivity of copper in GHz region is of the order of  $10^7$  [16]. We expect therefore the transmission of a realistic system to

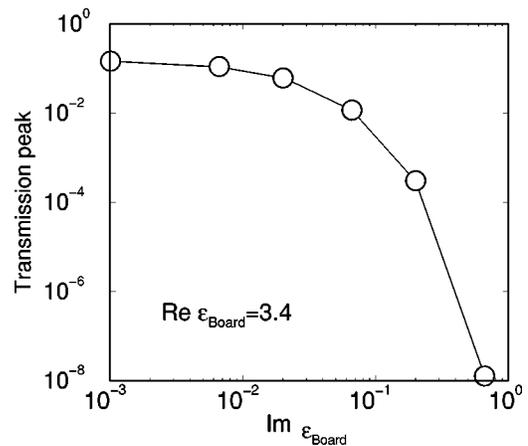


FIG. 6. Transmission peak as a function of the imaginary part of the permittivity of the dielectric board. The metallic permittivity  $\epsilon_m = (-3 + 5.88i) \times 10^5$ . The length of the system is 10 unit cells. Data represent the maximal transmission observed in the resonance peak.

be even better than the one displayed in Fig. 5 [17]. Our data clearly prove that the metallic components of the LH structures cannot be responsible for the high losses observed in the experimental studies of transmission [3,8,18]. As the LH systems are highly disperse [7], and still transparent, we believe that the dispersion is not the cause for the high losses in the LH structures.

To explain the relatively low transmission, observed in the experimental data, we have studied the dependence of the transmission on other material parameters. As the most probable mechanism of losses we consider the absorption of EM waves due to nonzero imaginary part of the dielectric board, on which the metallic components are positioned. To test this hypothesis, we repeated our numerical simulations for the same structure but with a small imaginary part to the permittivity of the dielectric board:  $\epsilon_{\text{Board}} = 3.4 + i \text{Im}\epsilon_{\text{Board}}$ . Figure 6 shows how the transmission peak decreases when the imaginary part of  $\epsilon_{\text{Board}}$  increases. Surprisingly, the transmission strongly decreases with the losses in the dielectric board.

To conclude, we present a detailed analysis of the numerical data for the transmission of the electromagnetic waves

through left-handed structures. The recovered refraction index is in agreement with the predictions of the homogeneous model with effective parameters given by Eqs. (1) and (2). Numerical simulations confirmed the excellent transmission properties of the simulated LH systems. We found that the imaginary part of the refraction index is only  $\sim 10^{-2}$ . As the value of the imaginary part of the metallic permittivity in real metals is even higher than that used in our simulations, we conclude that metallic components of the LH structures do not represent any source of absorption. Much higher losses were observed due to the absorption in the dielectric board on which SRRs are located.

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