What is the Right Form of the Probability Distribution of the Conductance at the Mobility Edge?

In a recent Letter, Slevin and Ohtsuki [1] reported finite size scaling results for the Anderson metal-to-insulator transition for the orthogonal and unitarity classes of the single electron tight-binding (TB) model. The average value of the conductance $G = (e^2/h)g$ at the mobility edge, as well as the distribution of the conductance at the critical point $p_c(g)$, was calculated. Their studies showed that $p_c(g)$ is independent of the system size. It also does not show any dip around $g = 0$, as the ϵ expansion results [2] suggest. We will present new numerical data that indeed shows that $p_c(g)$ has a dip for small g.

We have systematically studied the conductance G of the 3D TB model by using the transfer matrix technique [3], which relates the conductance G with the transmission matrix t, i.e., $G = (e^2/h)g$, with $g = 2\ \mathrm{Tr}(tt^\dagger)$. The g defined here is for both spins. In Fig. 1 we present the results of $p_c(g)$ for three different sizes of $N = 5$, 10, and 20.

The mobility edge [1] is at $W = 16.5$ and $E = 0.0$. Notice that as the size of the system increases a dip is developed at $g = 0$, which is not present in the results presented in Fig. 2 of Ref. [1]. We therefore have a size dependent $p_c(g)$, which has a dip at small g. $p(g)$ for extended states is Gaussian, while for localized states it is log normal [4]. However, it is not well known either experimentally [5] or theoretically what is the correct form of the probability distribution at the mobility edge. $p_c(g)$ obtained [2] in the ϵ expansion in the field theory has a hole at small g, in agreement with the numerical results presented here. Recent results [6] for a 2D TB model in the presence of a strong magnetic filed show that $p_c(g)$ is very broad with a dip at small g. The 2D $p_c(g)$ is very different from the 3D

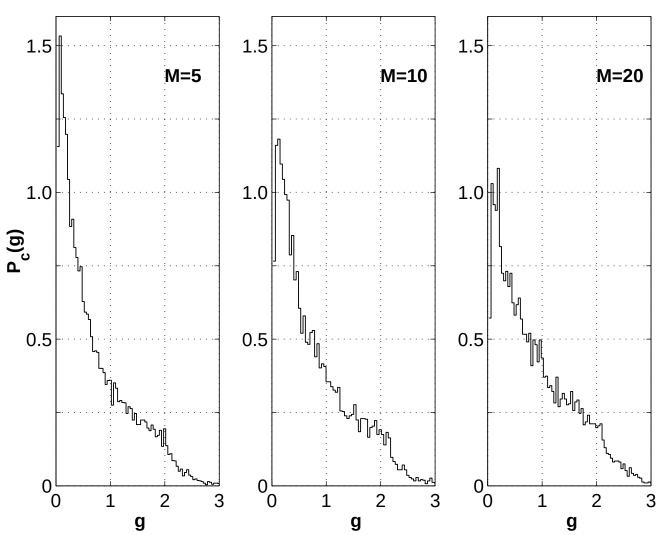

FIG. 1. The distribution of g at the critical point for three different sizes.

TABLE I. The mean and standard deviations of the critical distribution of g and $\ln g$.

N	$\langle g \rangle$	$\sigma_{\hat{g}}$	$\langle \ln g \rangle$	$\sigma_{\ln g}$
5	0.72	0.64	−0.857	1.19
10	0.78	0.66	−0.727	1.11
20	0.86	0.68	−0.587	1.09

$p_c(g)$ presented here. We have also calculated the average value of the conductance at the critical point ($E = 0.0$ and $W = 16.5$) for $N = 5$, 10, and 20 for 20 000, 10 000, and 8000 random configurations, respectively. The results are summarized in Table I.

Notice that both g and $\ln g$ have very large standard deviations, as big as their average values. Our results for both $\langle g \rangle$ and $\langle g \rangle_g = e^{\langle \ln g \rangle}$ for the $N = 10$ case $(0.78, 0.48)$ are larger than the results presented $(0.58, 0.30)$ in Table III of Ref. [1] for the same model. This difference might be due [7] to the different boundary conditions used by Ref. [1] (fixed) and ourselves (periodic). For the 2D case [6], it is shown that $\langle g \rangle = 1.00$ and $\langle g \rangle_g = 0.88$ for the infinite size system. If we extrapolate our finite size results to infinite sizes we obtain that $\langle g \rangle = 1.00$ and $\langle g \rangle_g = 0.70$. Remember that σ_g is comparable to $\langle g \rangle$.

In summary, the critical conductance distribution $p_c(g)$ in the orthogonal case does not increase without limit as $g \to 0$, contrary to the impression given by the Letter of Slevin and Ohtsuki [1]. We present evidence, based on direct numerical calculation of the conductance via the Landauer formula, that the critical conductance distribution does indeed tend to zero, in agreement with the ϵ expansion results [2].

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Received 9 June 1998 [S0031-9007(98)08161-7]
PACS numbers: 71.30.+h, 71.23.−k, 72.15.Rn

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 0031-9007/99/82(3)/668(1)$15.00