

## ARTICLES

## Theoretical investigation of off-plane propagation of electromagnetic waves in two-dimensional photonic crystals

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We study the transmission of electromagnetic waves propagating in two-dimensional photonic crystals having triangular structure. The transmission has been calculated using the transfer matrix method. We find that for dielectric constant ratios higher than 12.25, there is a full photonic band gap for both polarizations and for out-of-plane incident angle as high as  $85^\circ$ . [S0163-1829(98)03835-1]

### I. INTRODUCTION

It is now well known that the propagation of electromagnetic (EM) waves in periodic dielectric arrays can be completely forbidden for a certain range of frequencies, the so-called photonic band gap (PBG).<sup>1-3</sup> These two-dimensional (2D) or 3D photonic crystals offer the potential to engineer the properties of the EM waves in these structures.<sup>1-3</sup> The initial interest in this subject came from the proposal to use PBG crystals to inhibit spontaneous emission in photonic devices, leading to more efficient light emitters like thresholdless semiconductor lasers and single-mode light-emitting diodes.<sup>1-3</sup> There are several other applications of the photonic crystals in those frequency regions, such as efficient antennas, filters, sources, and waveguides.<sup>1-3</sup> In particular, 2D photonic crystals<sup>4-14</sup> have been extensively studied because they are relatively easier to be fabricated and operated in the optical wavelengths.<sup>10,11</sup>

In this paper, we study the transmission properties of 2D triangular photonic crystals consisting of air cylinders. We are particularly interested in off-plane incidence. Although the in-plane wave propagation (i.e., waves with wave vector  $k$  perpendicular to the axis of the cylinders) have been studied extensively,<sup>4-14</sup> there is little work for off-plane incidence<sup>3,11,12</sup> (i.e., waves with  $k$  having a component parallel to the axis of the cylinders). None of these works studied the transmission for off-plane incidence.

We use the transfer matrix method (TMM), introduced by Pendry and MacKinnon,<sup>15</sup> to calculate the EM transmission through a photonic crystal with defects. In the TMM, the total volume of the photonic crystal is divided in small cells and the fields in each cell are coupled with those in the neighboring cells. Then the transfer matrix can be defined by relating the incident fields on one side of the photonic crystal with the outgoing fields on the other side. Using the TMM, the band structure of an infinite periodic system can be calculated, but the main advantage of this method is the calculation of transmission and reflection properties of EM waves of various frequencies incident on a finite thickness slab of PBG material. In that case, the material is assumed to be

periodic in the directions parallel to the interfaces. The TMM has previously been applied to defects in 2D PBG structures,<sup>16</sup> photonic crystals with complex and frequency-dependent dielectric constants,<sup>17</sup> metallic PBG materials,<sup>18,19</sup> and angular filters.<sup>20</sup> In all these examples, the agreement between theoretical calculations and experimental measurements was very good.

### II. RESULTS AND DISCUSSION

We study a 2D crystal consisting of infinitely long cylinders with their axis along the  $y$  axis and forming a triangular lattice (see Fig. 1). The lattice constant is  $a = 2 \mu\text{m}$  and the rods diameter  $1.8 \mu\text{m}$ . Cylinders have a dielectric constant of 1 and they are surrounded by a dielectric constant of 12.25. The system is finite along the  $z$  axis with thickness  $L$ .  $E$ -polarized waves have their  $E$  fields along the axis of the cylinders ( $y$  direction) and  $H$ -polarized waves have their  $H$  fields along the  $y$  axis. We assume periodic boundary conditions at the edges of the system in the  $x$  direction. We used

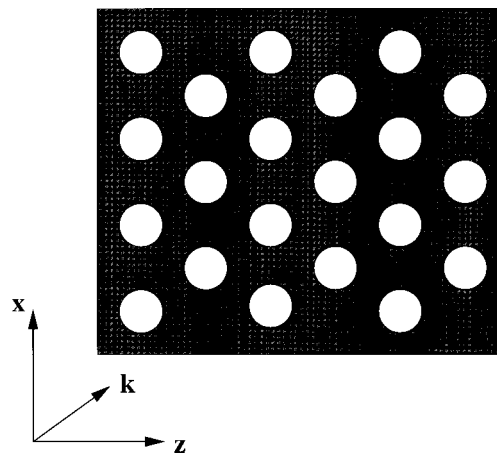


FIG. 1. The cross section of a triangular lattice. Rods (white circles) are along the  $y$  axis.  $k$  is the wave vector.

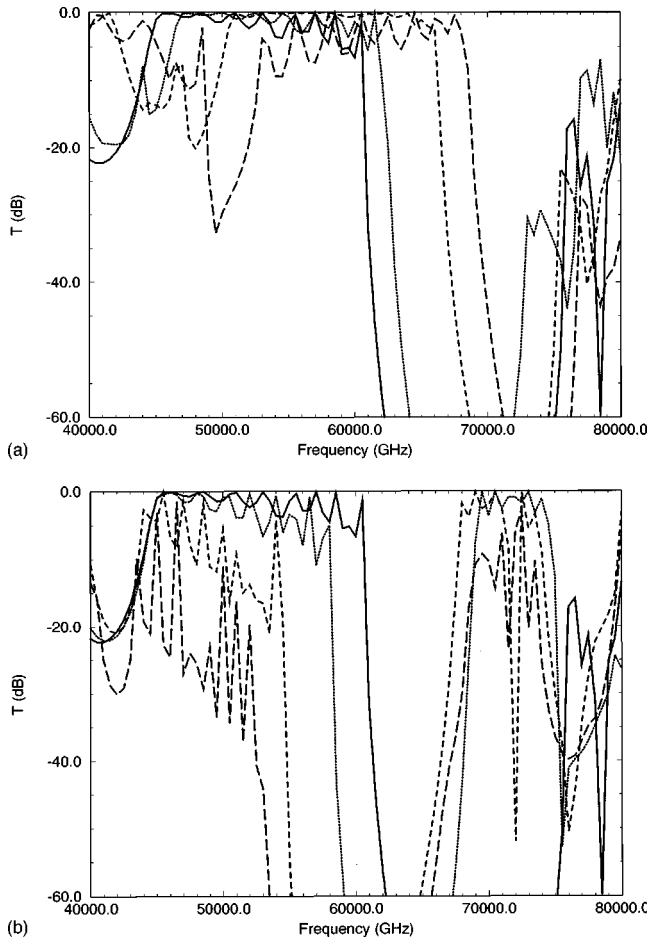


FIG. 2. The transmitted intensity ( $T$ ) of  $E$ -polarized EM waves propagating in a 2D triangular lattice consisting of cylindrical air rods with filling ratio  $f=0.734$  surrounded by a dielectric with  $\epsilon = 12.25$ . The lattice constant is  $a=2 \mu\text{m}$  and the thickness of the system is  $L=17 \mu\text{m}$ . Solid, dotted, dashed, and long dashed lines correspond to incident angles of  $0^\circ$ ,  $30^\circ$ ,  $60^\circ$ , and  $85^\circ$ . In panel (a)  $k$  is in the  $yz$  plane and in panel (b),  $k$  is in the  $xz$  plane.

these particular parameters because in that case there is a complete band gap for both polarizations and for incident  $k$  vectors in the  $xz$  plane.

Figure 2(a) shows the transmission of  $E$ -polarized waves with the incident  $k$  in the  $yz$  plane (off-plane propagation). As the incident angle increases, the lower edge of the gap moves to higher frequencies. However, the upper edge moves initially to lower frequencies at  $30^\circ$  and then, for even higher angles, moves to higher frequencies. In Fig. 2(b) ( $E$ -polarized waves with  $k$  in the  $xz$  plane), the gap shifts to lower frequencies as the incident angle increases. Band-structure calculations show that the lower band edge has the highest frequency at the  $\Gamma$  point and decreases in frequency as the wave vector increases, i.e., as the incident angle increases.

For  $H$ -polarized waves, there is a wide gap at normal incidence from 42.5 to 73.5 THz (solid line in Fig. 3). As the incident angle increases and for  $k$  vectors in the  $yz$  plane (off plane), both edges of the gap move to higher frequencies and the width of the gap decreases [see Fig. 2(a)]. Notice in Fig. 3(a) the interesting structure of the transmission that appears between 50–75 THz for off-plane incidence. For  $H$ -

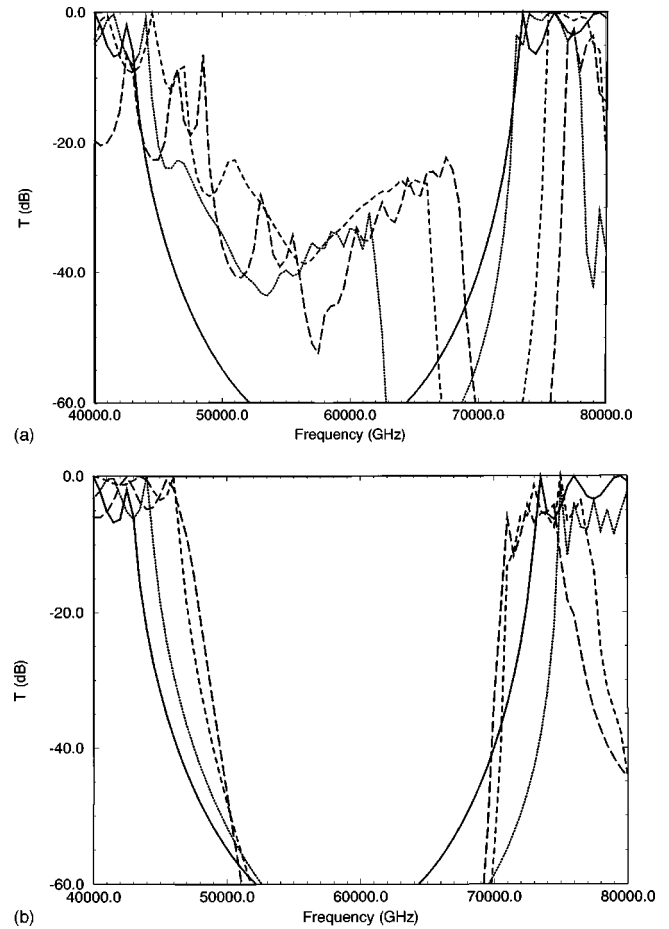


FIG. 3. The same as in Fig. 1 but for  $H$ -polarized waves.

polarization and for  $k$  in the  $xz$  plane, the width of the gap decreases slightly as the incident angle increases.

For  $k$  in the  $xz$  plane [Figs. 2(b) and 3(b)], there is a complete band gap for both polarizations and for all the angles as we expect from previous plane-wave calculations.<sup>2,10</sup> However, for off-plane propagation ( $k$  in the  $yz$  plane), the overall gap tends to decrease and eventually at  $85^\circ$  the gap is exactly zero. At this angle the lower edge of the gap for  $E$  polarization and  $k$  in the  $yz$  plane is 68 THz. This is the frequency of the higher edge of the gap for  $E$  polarization,  $k$  in the  $xz$  plane, and incident angle  $60^\circ$ . Interestingly, for  $H$  polarization (Fig. 3) the gap survives even for angles as high as  $85^\circ$ . This is because in that polarization, the full band gap for in-plane propagation is wider than for the  $E$  polarization.<sup>2,10</sup>

Coming back to the interesting structure appearing inside the gap in Fig. 3(a), we show in Fig. 4 the thickness dependence of the transmission for  $H$  polarization,  $k$  in the  $yz$  plane, and incident angle of  $85^\circ$ . At around 50 and 73 THz, the transmission decreases exponentially as the thickness of the system increases, indicating that these are band gaps. However, between these two gaps there is a region of low transmission ( $T$  is between  $-50$  to  $-20$  dB). The transmission in this region does not change significantly as the thickness of the system changes. This is due to the presence of antisymmetric modes in this region that do not couple well with the incident plane waves.<sup>14</sup>

For even higher dielectric constants, the pseudogap be-

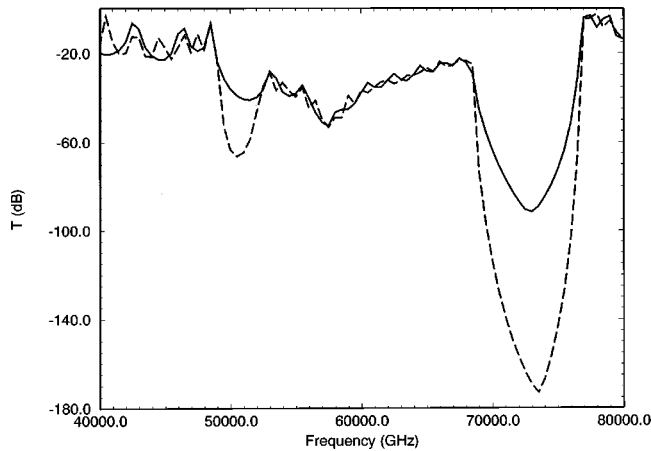


FIG. 4. The same as in Fig. 2(a). The thickness of the system is  $L=17$  and  $34 \mu\text{m}$  (solid and dashed lines).

comes more prominent. Figures 5 and 6 show results for the case where the dielectric constant of the background material is 16. For the  $E$  polarization, the lower edge of the gap is at 58 THz for  $k$  in the  $yz$  plane and  $85^\circ$ , while the upper edge of the gap is at 61 THz for  $k$  in the  $xz$  plane and  $85^\circ$ . For the  $H$  polarization, the gap is even wider. So, there is a gap for both polarizations for incident angles as high as  $85^\circ$  out of normal.

The previous results of the 2D triangular lattice (Figs. 5

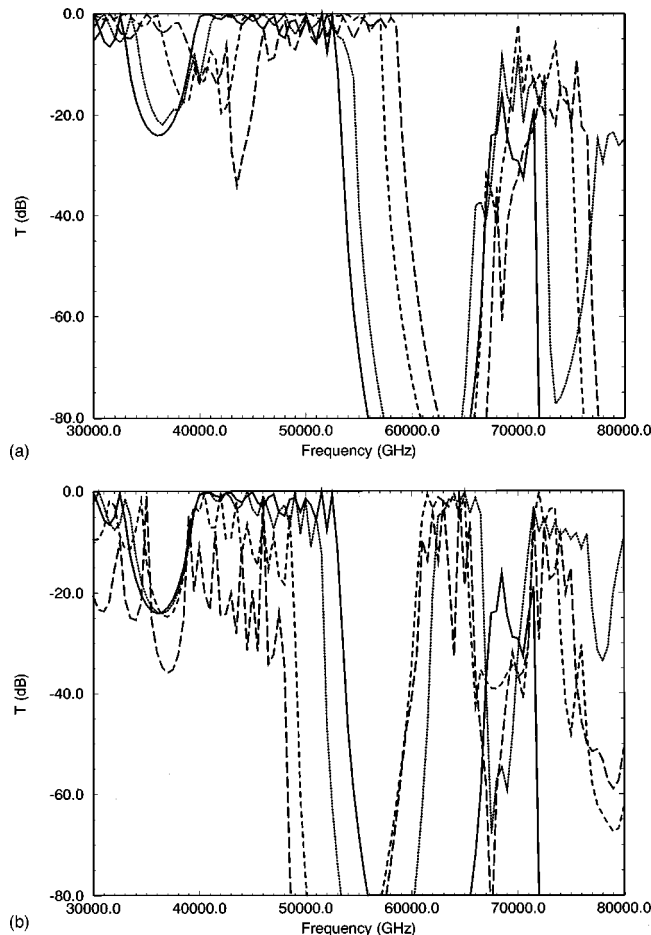


FIG. 5. The same as in Fig. 2 but now  $\epsilon=16$ .

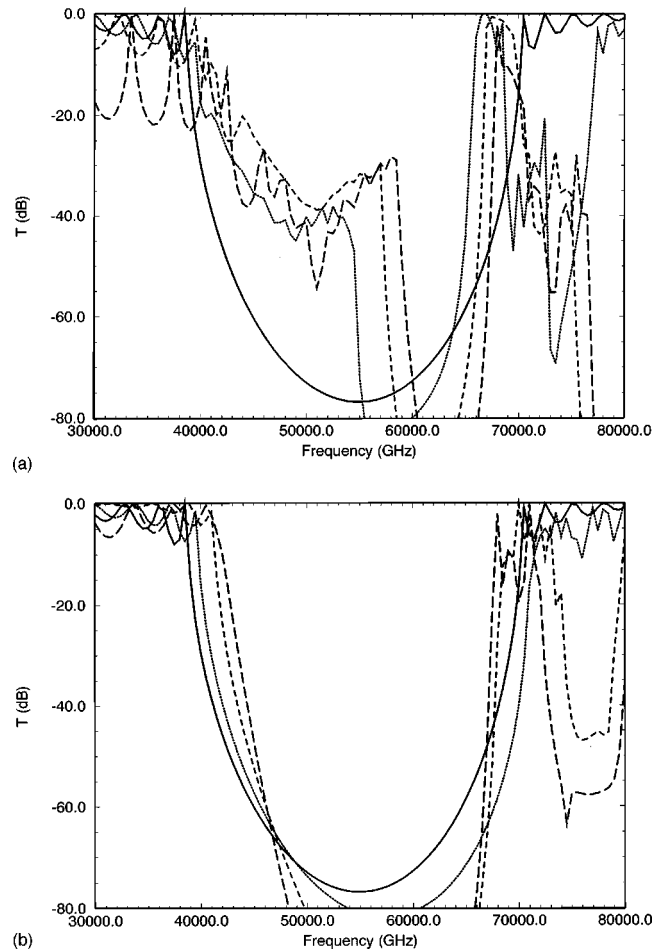


FIG. 6. The same as in Fig. 3 but now  $\epsilon=16$ .

and 6) are now compared with the results of a 1D photonic crystal (Fig. 7). The 1D crystal has a thickness of 5 unit cells. Each unit cell consists of a uniform layer of air with thickness  $1.6 \mu\text{m}$  and a uniform dielectric layer of  $\epsilon=16$  and thickness  $0.4 \mu\text{m}$ . The thicknesses of the layers are chosen in such a way in order to maximize the gap at normal incidence (solid lines in Fig. 7). We assume that the  $z$  axis is along the stacking direction of the layers. Because of the symmetry of the problem the results for  $k$  in the  $xz$  plane are identical with the results for  $k$  in the  $yz$  plane. Figure 7(a) shows results for the  $s$  polarization ( $E$  perpendicular to the plane of incidence). As the incident angle increases, the lower edge of the gap remains unchanged but the upper edge of the gap moves to higher frequencies, so the width of the gap increases for the  $s$  polarization. For  $p$ -polarized waves [ $E$  in the plane of incidence; see Fig. 7(b)], the width of the gap decreases and the transmission at the midgap increases as the incident angle increases. In addition, the gap moves to higher frequencies, so the lower edge of the gap at  $85^\circ$  (67 THz) coincides with the upper edge of the gap at  $0^\circ$ . As we mentioned previously, for the 2D case (Figs. 4 and 5), the gap for both polarizations survives for angles as high as  $85^\circ$ . Also, in the 2D case and for in-plane propagation [Figs. 5(b) and 6(b)] the gap survives even for  $90^\circ$  as it is known from previous studies.<sup>2,10</sup>

### III. CONCLUSION

We have studied the transmission properties of electromagnetic waves propagating along two-dimensional photo-

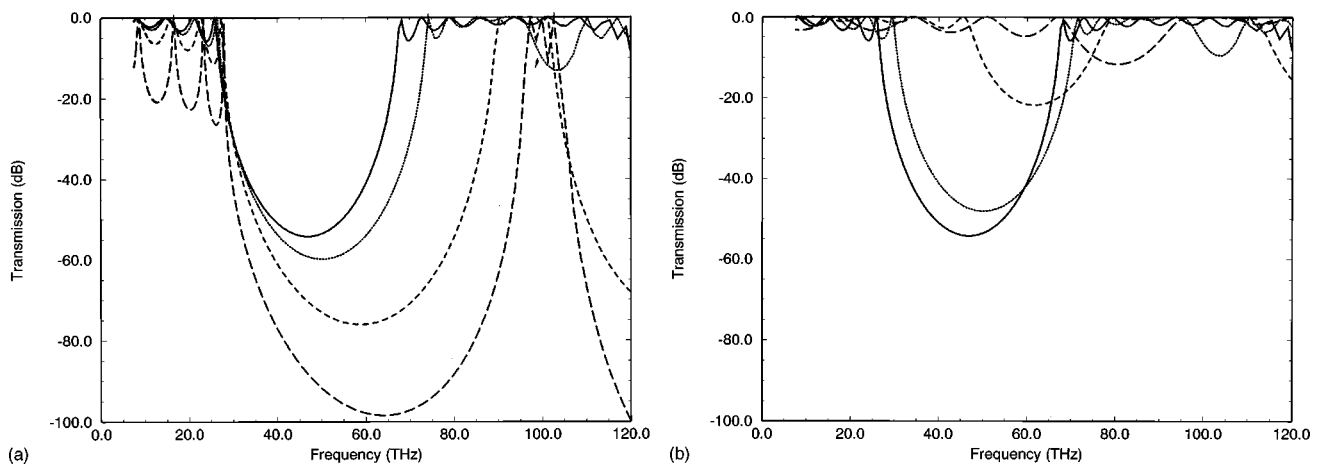


FIG. 7. The transmission profile for  $s$  and  $p$  polarized [(a) and (b) panels, respectively] waves incident on a 1D photonic crystal. The incident angle is  $0^\circ$ ,  $30^\circ$ ,  $60^\circ$ ,  $85^\circ$  (solid, dotted, dashed, and long-dashed lines, respectively).

nic crystals having the triangular lattice. Although it is well known that there is a gap for both polarizations and for  $k$  perpendicular to the axis of the rods, there are no transmission studies for off-plane propagation (where  $k$  has a component along the axis of the cylinders). We found that for background medium with dielectric constant  $\epsilon=12.25$ , the gap for both polarizations just disappears at  $85^\circ$  off normal incidence, while for  $\epsilon=16$ , there is a gap for both polarizations even for angles as high as  $85^\circ$  off normal incidence. Taking into account the easy fabrication of such 2D structures relative to 3D photonic crystals, it is interesting to fur-

ther study the effect of such structures on improving performances of dipole antennas and applications to light-emitting diodes.

#### ACKNOWLEDGMENTS

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