Reply to "Comment on 'Metal-insulator transition in random superconducting networks' "

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We address the remarks of Domínguez, López, and Simonin [Phys. Rev. B **42**, 8665 (1990); preceding paper] on the determination of the normal-to-superconducting (N-S) phase boundary in random superconducting networks. We refute their claims that the disappearance of the fine structure of the N-S boundary and the change of the critical exponent k for the slope of the critical field on $(p-p_c)$ are due to the introduction of very weak links between nodes in the superconducting networks.

In a previous paper[1] we studied the normal-to-superconducting (N-S) phase boundary for a random percolating superconducting network within the context of the linearized Ginzburg-Landau equations.[2] As expected, the structure of the superconducting transition temperature $T_c(H)$ is washed out, due to the randomness of the percolating networks. By examining the nature of the eigenstates for different values of the concentration of the sites present, p, as a function of the reduced flux ϕ/ϕ_0, we were able to obtain bounds for the N-S phase boundary, where $\phi = HL^2$ is the magnetic flux through the unit cell of the superconducting network and

$$\phi_0 = hc/2e = 2.07\times 10^{-7}\ \mathrm{G\,cm}^2$$

is the supeconducting flux quantum. A very interesting mobility-edge trajectory was found for the metal-to-insulator transition which may be observable experimentally. Finally, a bound for the critical exponent k of the upper critical field was determined and found to be $k>0.84$, consistent with the experimental measurements.[3]

Domínguez, López, and Simonin have argued in the previous paper[4] that the disappearance of the fine structure of $T_c(H)$ as well as the value of k are due to the weak links ($\epsilon \neq 0$ but $\epsilon \to 0$) that we have introduced between sites, which would be normally removed in random superconducting networks in one of the three numerical techniques (the transfer-matrix technique) for calculating the localization length and then the N-S phase boundary. We believe that their two points are not correct as we will show here.

In our previous work,[1] it was clearly stated that the N-S phase boundary was calculated by three different methods: (i) by a partial diagonalization of $10\,000\times 10\,000$ matrix which involve only the infinite cluster, (ii) by a numerical calculation of density of states (DOS) from which the behavior of the band edge of the highest band could be determined, and (iii) by calculating the localization length with the finite size scaling transfer-matrix technique. In fact, only in the last case we have used a value of ϵ which was nonzero. As was stressed in Ref. 1, the results are independent of ϵ for $\epsilon < 10^{-3}$.

These three independent methods gave the same results for the N-S phase boundary. In methods (i) and (ii), all the finite clusters were removed and we only used *the infinite cluster* in the analysis. Therefore, the disappearance of the fine structure of $T_c(H)$, as more sites or bonds are removed, does not depend on the value of ϵ. It is related simply to the fact that the superconducting long-range order is destroyed as sites are removed. In fact, our calculations have shown that the band edge states of the *infinite cluster* for $p\neq 1$ are *localized* and this localized nature of the superconducting wave functions is responsible for the disappearance of the sharp structure of $T_c(H)$. A recent experimental study[5] of superconducting wire networks on a square lattice from which bonds have been removed randomly reveal a rapid washing out of the rich structure of cusps in $T_c(H)$ with decreasing bond occupation probabilities p. These results are consistent with our numerical results which suggest that the disappearance of the structure of $T_c(H)$ is associated with the localization of the mean-field supeconducting order parameter on the network.

We also disagree with their statement[4] on the critical exponent k. The critical exponent k is derived from the low H dependence of $T_c(H)$. As we have mentioned above, $T_c(H)$ was calculated by all three independent methods and all gave similar results provided that $\epsilon \leq 10^{-3}$. However, by examining Fig. 4 of Soukoulis, Grest, and Li[1] and Fig. 2 of Simonin and López,[6] one clearly sees that the values of the parameter A, which is proportional to $(dH_{c2}/dT)_{T_c}$, close to p_c obtained by the different techniques of Refs. 1 and 6 are equivalent. On the other hand, far from p_c our results do differ somewhat from the results given in Ref. 6, which we suspect is due to the continued function method used in Ref. 6. We found that for the band-edge state, $k=1.50$, while the numerical estimations of Ref. 6 give a value of $k=0.93$. However, both of these estimates depend somewhat on the range of p values used in the fit. However, we also find[1] in agreement with Simonin and López[6] that the band-edge states are strongly localized for $p \leq 0.96$. This strong localization of the supeconducting wave function of the infinite network is only related to the site dilution i.e., to the introduction of disorder in the superconducting network. It does not have anything to do with the small clusters since Simonin and López[6] as well as Soukoulis, Grest, and Li,[1] diagonalized the infinite network and found that the superconducting wave function at the band edge are strongly localized. This is exactly the reason why we undertook[1] a systematic study of the extended or localized nature for all the eigenstates as a function of energy starting from the band-edge state, which is localized. The energy at which the first extended solution appears will determine a lower limit for the N-S phase boundary. Using these extended wave functions, we found[1] that the exponent $k = 0.84$, from which we derive a lower bound.

In conclusion, our previous work[1] has correctly shown that rapid disappearance of the fine structure of $T_c(H)$ with increasing site[1] or bond[5] diulution is due to the localization of the mean-field superconducting order parameter of the *infinite* network. It has nothing to do with the inclusion or not of a small nonzero value of ϵ. We find in agreement with Simonin and López[6] that there is strong localization of the band-edge superconducting wave functions for the infinite superconducting network.

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[1] C. M. Soukoulis, G. S. Grest, and Qiming Li, Phys. Rev. B **38**, 12 000 (1988). There is a typographical error in this paper. The exponent k of the upper critical field for the band edge is equal to 1.50 and not 2.50.

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[3] J. M. Gordon, A. M. Goldman, and B. Whitehead, Phys. Rev. Lett. **59**, 2311 (1987).

[4] D. Domínguez, A. López, and J. Simonin, preceding paper, Phys. Rev. B **42**, 8665 (1990).

[5] F. Yu, A. M. Goldman, R. Bojko, C. M. Soukoulis, Q. Li, and G. S. Grest (unpublished).

[6] J. Simonin and A. López, Phys. Rev. Lett. **56**, 2649 (1986).