

## Localization of the mean-field superconducting order parameter on random wire networks

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Studies of the magnetic-field dependence of the mean-field transition temperature  $T_c(H)$  of square superconducting wire networks from which bonds have been removed randomly reveal a washing out of the cusps in the phase boundary curve with decreasing bond occupation probability  $p$ . These results are consistent with computations implying localization of the mean-field superconducting order parameter.

### I. INTRODUCTION

The localization of electrons<sup>1</sup> and classical waves<sup>2</sup> in disordered media, and the interplay between localization and superconductivity<sup>3</sup> have been studied intensively. In this article we report experiments and numerical computations which suggest that the initial transition to the superconducting state on square, two-dimensional, lattices in which some wires joining adjacent sites have been removed at random involves a localized mean-field superconducting order parameter.

The superconductivity of wire lattices with submicrometer feature sizes, which is the subject of the present investigations, has been studied in a variety of geometries.<sup>4</sup> The mean-field theory  $T_c(H)$  exploits an isomorphism between the linearized Ginzburg-Landau (GL) equation and the Schrödinger equation for a particle of charge  $2e$ .<sup>5</sup> The consequence of solving the linearized Ginzburg-Landau equation for  $T_c(H)$  on square networks<sup>5</sup> is a structure of cusps in  $T_c(H)$ .

The motivation for studying square wire networks with bonds removed was the fact that they could serve as better models of the infinite cluster of percolation<sup>6</sup> than previously studied Sierpinski gaskets,<sup>7</sup> as in addition to having a fractal geometry, they incorporated randomness and dead-end bonds.<sup>8</sup> For samples with bond occupation probabilities  $p = 0.54, 0.56,$  and  $0.60$ , which were studied initially, neither the anticipated fractal behavior<sup>9</sup> in  $T_c(H)$  nor cusps in  $T_c(H)$  were found. The absence of the structure in  $T_c(H)$  for random percolation networks has also been discussed by Steinmann and Pannetier for the case of a specific network with a bond occupation probability  $p = 0.6$ .<sup>10</sup>

### II. EXPERIMENT

Samples were prepared using electron-beam lithographic techniques described elsewhere.<sup>7</sup> The networks consisted of  $800 \times 800$  square lattices of wire bonds which were present with a probability  $p$ . The geometry of a particular structure was fixed in the fabrication process with the configuration of bonds determined using a random number generator. The wire widths were approximately  $0.3 \mu\text{m}$  and they were  $0.05 \mu\text{m}$  thick. The unit cell size was about  $1.7 \mu\text{m}$ . Normal-state resistances varied from 1 to  $100 \Omega$  and the zero-temperature superconducting coherence length  $\xi(0)$  in all samples was the order of  $0.2 \mu\text{m}$ .

Sample resistances were measured with a four-probe ac technique with a current bias of about  $1 \mu\text{A}$ . The electrical leads to the samples were evaporated Al pads at the corners. The sample resistance was used as the sensor to determine  $T_c(H)$ . The mean-field transition temperature was taken to be that at which the sample resistance was slightly less than half its normal state value  $R_N$ . The magnetic field was then swept in a semicontinuous fashion, and the changes in temperature were followed using the Ge resistance thermometer in thermal contact with the sample holder. The process was carried out over a two-hour period for a given trace so as to ensure both thermal and magnetic equilibrium. Temperature measurements were reproducible between sweeps and were accurate to better than  $50 \mu\text{K}$  in a single sweep. The detailed structure in  $T_c(H)$  versus  $p$  was unaffected when the resistance set point varied from  $0.1$  to  $0.5R_N$ , although the patterns were sharper at the lower resistances.

### III. THEORY

The eigenvalue equation which follows from the linearized Ginzburg-Landau equation that is used to compute  $T_c(H)$  for simple superconducting lattices is of the form

$$-\Delta_i z_i \cos\theta + \sum_j \Delta_j \exp(i\gamma_{ij}) = 0, \quad (1)$$

where  $\Delta_i$  is the order parameter at node or site  $i$ , and the sum  $j$  is over the  $z_j$  nearest neighbors of the  $i$ th node of the network. The quantity  $\theta = L/\xi(T_c(H))$ ,  $L$  is the distance between the nodes divided by the coherence length,

$$\gamma_{ij} = (2\pi/\Phi_0) \int_i^j \mathbf{A} \cdot d\mathbf{s}$$

is the circulation of the vector potential  $\mathbf{A}$  along the link  $ij$ , and the flux quantum  $\Phi_0 = hc/2e$ . Notice that Eq. (1), with the right-hand side given by  $\lambda\Delta_i$  instead of zero, would be very similar to the tight-binding Hamiltonian of the electronic problem on the network. However, here, since we are concerned with superconductivity, we are interested only in the zero-eigenvalue case, i.e.,  $\lambda=0$ . For a network with bonds removed randomly, there is diagonal disorder which depends upon  $z_i$ , and  $\cos(L/\xi)$  is taken to be  $E$ .<sup>11</sup> Then the off-diagonal term  $e^{i\gamma_{ij}}$  is correlated with the diagonal through the applied field  $H$ , and  $T_c(H)$  or  $E = \cos(L/\xi)$  can be determined from the largest value of  $E$  for which Eq. (1) has a solution. In order to find the eigenvalues and eigenvectors  $\Delta_i$  of Eq. (1), one can either diagonalize the Hamiltonian for a given  $H$  and  $p$ , or use the transfer-matrix method and finite-size scaling. This latter procedure is useful since it can be used to study the nature of the different eigenstates  $\Delta_i$ , in particular, whether they are localized or extended. Direct diagonalizations or a tridiagonalization procedure could also be used to study the nature of the eigenstates, but is known not to give accurate results.

In the transfer-matrix method<sup>12,13</sup> one considers coupled one-dimensional (1D) systems. Each is described by a tight-binding Hamiltonian of the same form as Eq. (1). The corresponding sites of the nearest-neighbor system are coupled together by an interchain matrix element  $t_{ij}e^{\gamma_{ij}}$  that depends on the strength of the magnetic field  $H$  and the bond occupation probability  $p$ . In particular, we choose a gauge such that  $A$  is parallel to the 1D chains and  $t_{ij}=1$  if both sites are present,  $\epsilon$  if one site is missing, and  $\epsilon^2$  if both sites are missing. Then  $M$  chains are coupled together into a 2D array with interchain coupling  $t_{ij}$  and  $z_i = \sum_j t_{ij}$ . The additional term  $t_{ij}$  is necessary to ensure that the lattice is connected. For the  $M$  connected chains of length  $N$ , one determines the largest localization length  $\lambda_M$  as  $N \rightarrow \infty$ . From a plot of  $\lambda_M$  versus  $M$ , one can obtain the localization properties of the system.<sup>12,13</sup>

The localization length  $\lambda_c$  is obtained from the relation  $M/\lambda_M = M/\lambda_c$ , while the correlation length  $\xi_c$  for extended states is obtained from  $\lambda_M/M = M/\xi_c$ . So by studying the scaling plots  $\lambda_M/M$  versus  $M$ , one obtains a reasonable estimate of the mobility edge trajectory. Exactly at the mobility edge, we also find  $\lambda_M/M \approx 0.6$ , in agreement with previous work on electronic localization,

while for extended and localized states we have that  $\lambda_M/M$  versus  $M$  increases or decreases, respectively. For our studies here we used  $M$  equal to 2–64 with  $N$  up to 10000 and  $\epsilon=10^{-4}$ . We found that our results were independent of  $\epsilon$  provided that  $\epsilon \leq 10^{-3}$ . The phase boundary  $T_c(H)$ , or the largest eigenvalue  $E = \cos(L/\xi)$ , which satisfies Eq. (1), is obtained by the calculation of the integrated density of states (DOS). The technique used is described in Ref. 14.

The amplitude of the oscillation of  $T_c$  with magnetic field depends on the value of the zero-temperature coherence length  $\xi(0)$ , a material-dependent parameter. The quadratic background in  $T_c$  versus  $H$ , which is superimposed on the  $T_c$  oscillations, and is a consequence of the critical field of the wires themselves, was used to determine  $\xi(0)$  from the relation  $\Delta T_c = T_{c0} - T_c = \alpha H^2$ , which is derived from the Ginzburg-Landau theory. Here

$$\alpha = \frac{\pi^2 d^2 \xi(0)^2 T_c}{3\Phi_0^2},$$

$d$  is the film thickness,  $T_{c0}$  is the zero-field transition temperature, and  $\Phi_0$  is the flux quantum. The parameter  $\theta(L/\xi)$  relevant to the theory is actually material independent. It can be computed using the relation  $\theta(L/\xi) = L/\xi(T_c(H))$ , where

$$\xi(T_c(H)) = \xi(0) / \{ [T_c(0) - T_c(H)] / T_c(0) \}^{1/2}. \quad (2)$$

### IV. RESULTS

Figure 1(a) shows the variation of  $\theta$  with magnetic field for different lattices characterized by values of  $p$ . Figure 1(b) shows the results of numerical calculations for the same  $p$ 's. The quantitative differences in the amplitudes of the curves are possibly a consequence of differences in the number of squares in the physical wire sample and in the numerical calculation. Curves in both Figs. 1(a) and 1(b) have been offset for clarity. The fundamental period of the pattern  $H_0 = 7.9$  Oe is due to the fluxoid quantization condition on the elementary squares and corresponds to an area of  $\Phi_0/H_0 = 2.62 \mu\text{m}^2$ , in good agreement with the measured area.

The critical feature in both Figs. 1(a) and 1(b) is the disappearance into the noise of most of the features of the pattern even with  $p=0.95$ . Only the feature corresponding to the cusp at  $f=1/2$  survives, and this disappears when  $p$  falls to 0.80. As the percolation threshold for the 2D bond problem is  $p=0.5$ , these changes occur well within the connected regime. To confirm the role of randomness, a structure with  $p=0.8$  in which bonds were removed in a regular fashion was also prepared. A rich structure in  $T_c(H)$  was found as is illustrated in Fig. 2.

The cusps in the square network geometry are a manifestation of the collective behavior of the network, or the long-range coherence of the order parameter. Physically, the persistence of a feature at a submultiple with a denominator at a particular integer implies coherence over a distance at least that integer number of cells in the network. In Fig. 3 we plot the localization length corresponding to edge state eigenfunctions, obtained in the

manner described above, as a function of  $p$ . The very rapid increase from a value the order of one to two cells as  $p$  approaches unity is seen. When the localization length for the state corresponding to  $\Phi/\Phi_0 = \frac{1}{2}$  is below two cell lengths, one might expect the cusp at half a flux quantum to disappear. This occurs for  $p = 0.8$ , as seen in Fig. 3.

V. DISCUSSION

It is also important to note that the mean-field transition temperature inferred from the solution to the linear

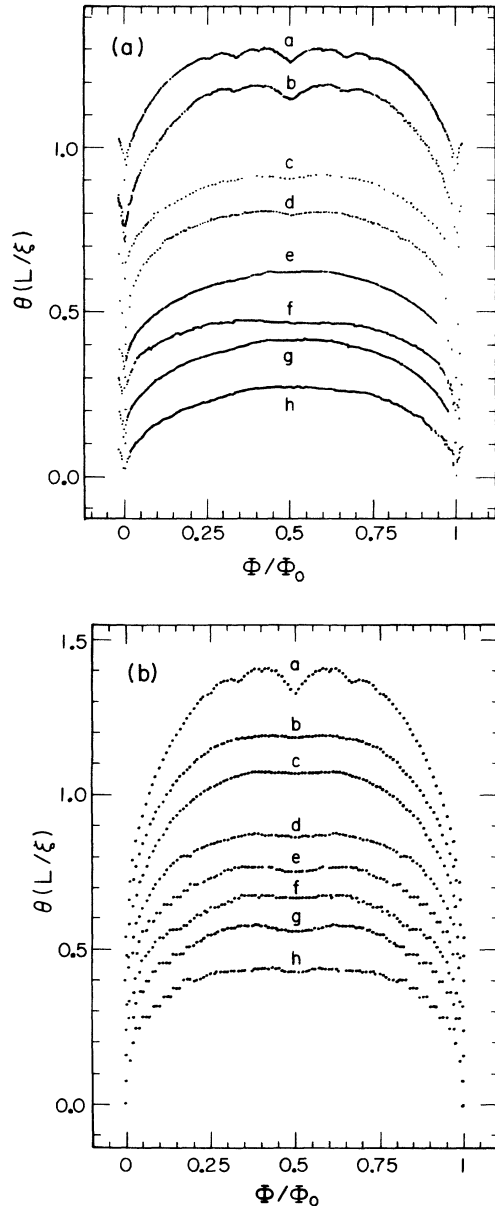


FIG. 1. (a) Plot of  $\theta = L/\xi(T)$  vs  $\Phi/\Phi_0$  of sample with different bond occupation probabilities. *a*:  $p = 1.0$ ,  $\xi(0) = 0.25 \mu\text{m}$ ; *b*:  $p = 0.98$ ,  $\xi(0) = 0.11 \mu\text{m}$ ; *c*:  $p = 0.96$ ,  $\xi(0) = 0.20 \mu\text{m}$ ; *d*:  $p = 0.90$ ,  $\xi(0) = 0.12 \mu\text{m}$ ; *e*:  $p = 0.87$ ,  $\xi(0) = 0.18 \mu\text{m}$ ; *f*:  $p = 0.85$ ,  $\xi(0) = 0.18 \mu\text{m}$ ; *g*:  $p = 0.84$ ,  $\xi(0) = 0.19 \mu\text{m}$ ; *h*:  $p = 0.81$ ,  $\xi(0) = 0.29 \mu\text{m}$ . (b) Numerical calculation of  $\theta$  for the values of  $p$  in part (a).

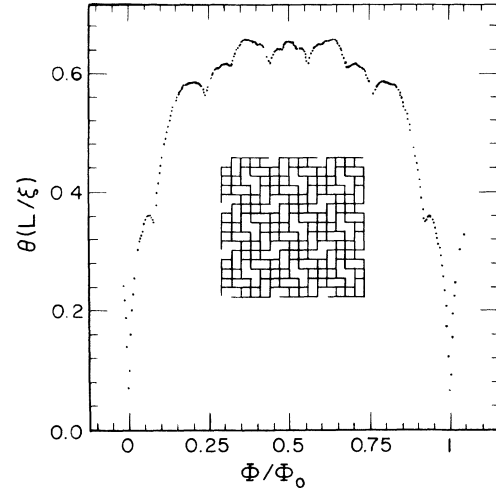


FIG. 2. Plot of  $\theta$  vs  $\Phi/\phi_0$  of a sample with bond occupation of 0.8, but with a regular pattern of bond removal. The inset shows the sample design.

GL equations with localized solutions may be neither the mean-field transition temperature nor the temperature at which the resistance falls to zero.<sup>15</sup> The former may require the solution of the nonlinear rather than the linear GL equations, which is beyond the scope of this work. The nonlinear term can couple different localized states which might give a different mean field  $T_c(H)$ . In general, the energy at which the first solution appears, independent of whether it is extended or localized, gives an upper bound for  $T_c(H)$ , while the energy for the first extended solution gives the lower bound for  $T_c(H)$ .

The modeling of the transition to zero resistance might

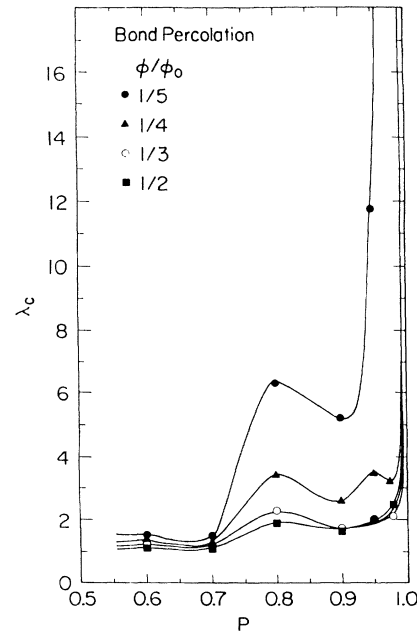


FIG. 3. Localization length  $\lambda_c$  as function of bond occupation probability  $p$  for  $\phi/\phi_0 = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ .

involve treating an array like a model granular superconductor. Instead of real grains, coupled by tunneling junctions or weak links, there would be "patches" of the localized order parameter the size of the localization length. In this context the localization of the mean-field order parameter at the transition might be viewed as a type of inhomogeneous nucleation of the order parameter. The transition to zero resistance might then result from phase locking of the patches.

It should be noted that the consequences of removing bonds in a network appear different from the consequences of introducing positional disorder. In the theory,<sup>16</sup> experiments,<sup>17</sup> and simulations<sup>18</sup> on positionally disordered arrays, all structure in  $R(H)$  disappears above a critical magnetic field whose magnitude is inversely proportional to the disorder. Oscillations in  $R(H)$  are, however, found at low fields. For the bond-removal case, the structure fades away as  $p$  decreases, with the overall periodicity persisting to substantial values of magnetic field.

In the case of positional disorder, the fading might be thought of as arising from a superposition of patterns of different periods associated with the different areas of the plaquettes. In the case of networks with bonds removed, the areas are also random, but are all multiples of the fundamental cell's area, as was first pointed out by

Simonen and Lopez.<sup>19</sup> This probably accounts for the survival of the single-flux quantum oscillations. However, in the bond-removal case there are dead ends and side chains not present in the case of positional disorder.

In summary, we have studied  $T_c(H)$  of square wire networks. The structure, other than that at multiples  $\phi_0$ , fades away with reduced bond occupation probability  $p$ , a result also found in numerical simulations based on mean-field theory, where it is also accompanied by the localization of the mean-field order parameter in linear theory.

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