Conductivity in disordered systems

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(Received 19 November 1984)

By combining numerical results on wires of finite cross section with the coherent-potential approximation and the potential-well analogy, a formula for the conductivity of a three-dimensional disordered system is obtained which interpolates between the weak-scattering limit and the mobility edge.

I. INTRODUCTION

Despite the extensive attention that disordered systems have received in the last fifteen years, our ability to obtain explicit quantitative results is still limited. Recently an analogy of the localization problem with that of a bound state in a potential well was developed[1] on the basis of progress in the conductivity calculation.[2] The analogy with the potential well permits explicit calculations of the localization lengths, mobility edges,[1] etc. from quantities that can be obtained from mean-field theories such as the coherent-potential approximation (CPA). However, in order to check the results of the approximate scheme outlined above [based on the CPA and the potential-well analogy (PWA)] we need independent methods of obtaining the same quantities. Probably the most reliable such method is the strip or wire method. In this method one considers coupled one-dimensional (1D) systems. Each 1D system is described by a tight-binding Hamiltonian of the form

$$H=\sum_n |n\rangle \epsilon_n \langle n| + V\sum_{n,m}{}' |n\rangle\langle m| \ , \tag{1.1}$$

where ϵ_n are independent random variables with a common probability distribution. In our explicit results we assume this probability distribution to be a rectangular of total width W. The corresponding sites of the nearest-neighbor 1D system are coupled together by an interchain matrix element V' which we take equal to V. As the number of coupled chains approaches infinity, we recover a two-dimensional (2D) or three-dimensional (3D) disordered system depending on whether the chains have been placed on a plane with two nearest neighbors each or whether they have been placed as to form a cylinder of square cross section. In the present work we concentrate on the 3D case so that our system consists of M^2 regularly placed chains, each one having four nearest neighbors.

Then one determines through a rather sophisticated numerical technique[3–5] the largest localization length λ_M in this system of M^2 coupled chains. The largest M for which reliable numerical determination[3,4] of λ_M has been carried out is $M=8$. One finds two distinct behaviors of the function λ_M versus M. In the first case, corresponding to localized states the second derivative $d^2\lambda_M/dM^2$ is negative and λ_M seems to approach a finite limit λ as $M\to\infty$; obviously λ is the localization length of the resulting 3D disordered system. In the second case, $d^2\lambda_M/dM^2$ is positive and $\lambda_M\to\infty$ as $M\to\infty$, implying that the states in the resulting 3D systems are extended. Here we consider this second case.

It was found numerically that the function λ_M versus M obeys a simple scaling relation of the form

$$\frac{\lambda_M}{M}=f\left[\frac{M}{\xi}\right] , \tag{1.2}$$

where $f(x)$ is a universal function of its argument, ξ is a quantity which depends on the properties of the system (but it is independent of M as long as $M>4$, and $M>4l/a$), where a is the spacing between the chains and l is the mean free path in the resulting 3D system (as $M\to\infty$). Hereafter all lengths will be given in units of a. Thus a single quantity ξ determines the localization length λ_M of the system. This result is consistent with the scaling theory of localization[6] which is based upon the assumption of a single scaling quantity.

Let us consider now our system with M^2 coupled chains, each one having a finite length L. It is obvious that for $L\gg\lambda_M$ the transport properties of our system are determined by the ratio L/λ_M. However, in view of the one-parameter structure of our results, it follows that the quantity L/λ_M uniquely determines the transport properties of our system at all length scales L. In the special case $M=1$ (truly one-dimensional system) it is well known[7] that the resistance R_1 is given by

$$R_1=\frac{\pi\hbar}{e^2}(e^{2L/\lambda_1}-1), \quad M=1 \ . \tag{1.3}$$

MacKinnon and Kramer[4] have implicitly assumed that Eq. (1.3) is valid for all M by replacing $R_1 \to R_M$ and $\lambda_1 \to \lambda_M$. We show here that although this is not true, one can still write a relationship between R_M and L/λ_M similar to Eq. (1.3). This is very significant because it allows an explicit numerical determination of the conductivity, which, when combined with the potential-well analogy, provides a simple interpolation formula for the conductivity between the weak-scattering limit and the mobility edge.

II. RESULTS OF THE WIRE METHOD

MacKinnon and Kramer[4] (MK) have studied numerically the localization length at the center of the band: $E=0$. They have numerically determined the form of the function f and the dependence of ξ on the disorder W. They found that $f(x)$ is an increasing function of x with

$$f(x) \to cx \quad \text{as } x \to \infty \ , \tag{2.1}$$

and

$$f(x) \to 0.6 \quad \text{as } x \to 0 \ . \tag{2.2}$$

Note that the scaling requirement determines ξ up to multiplicative constant. To uniquely determine ξ, one needs an additional condition. One such condition is to demand that c in Eq. (2.1) equals one. The corresponding parameter will be denoted by $\bar{\xi}$, i.e., $\bar{\xi}$ is an appropriate scaling variable such that

$$\lambda_M \to M^2/\bar{\xi} \quad \text{as } M \to \infty \ . \tag{2.3}$$

In other words, $1/\bar{\xi}$ is the slope of the straight line λ_M/M versus M for sufficiently large M. As can be seen from Fig. 1, the linear relation between λ_M/M and M is reasonably well obeyed down to $M=3$, giving $\bar{\xi}=4.92$ as opposed to $\xi_{\mathrm{MK}}=1.143$ obtained in Ref. 4 for the same disorder $W=10$. Thus, we conclude that

$$\bar{\xi}=4.30\xi_{\mathrm{MK}} \ . \tag{2.4}$$

To further check the accuracy of the proportionality constant, in Eq. (2.4) we examined the case $W=12$ and we found $\bar{\xi} \approx (11 \pm 1)$ as opposed to $\xi_{\mathrm{MK}}=2.53$ yielding a ratio of 4.35 ± 0.4 in reasonable agreement with Eq. (2.4).

Another quite common way to determine the multiplicative uncertainty of the length ξ is by demanding that ξ blows up in exactly the same way as the localization length at the mobility edge, i.e., if $\lambda \to b/(E_c - E)^{\nu}$ as $E \to E_c^-$, then ξ is determined by the requirement that

$$\xi \to b/(E - E_c)^{\nu} \quad \text{as } E \to E_c^+ \ . \tag{2.5}$$

By examining the data of MacKinnon and Kramer we find that ξ_{MK} is about 20% larger than their localization length on the other side of the critical point. However, the localization length of MacKinnon and Kramer seems to be about 10% less than the actual localization which for $W=30$ we found to be $\lambda=2.05$ (see Fig. 2) as opposed to $\lambda=1.867$ found in Ref. 4. The localization length, λ, is the inverse of the slope of the straight line M/λ_M versus M. Thus, we conclude that

$$\xi=0.9\xi_{\mathrm{MK}} \ . \tag{2.6}$$

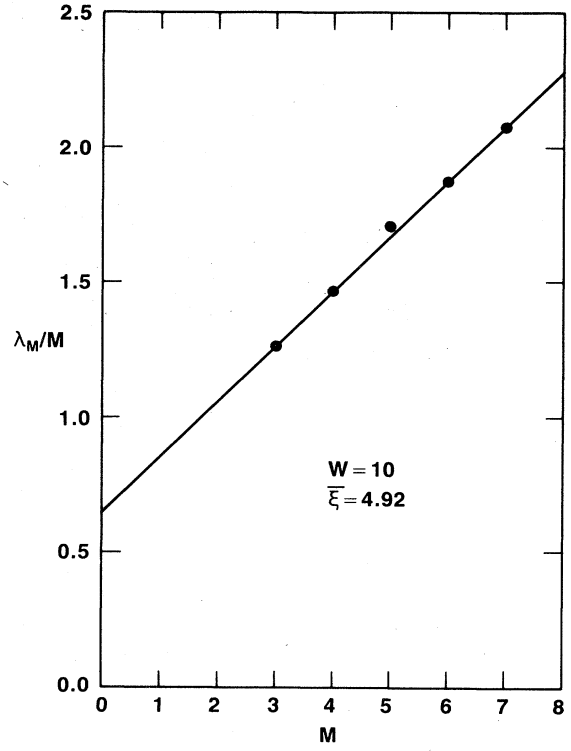

FIG. 1. Graphical determination of $\bar{\xi}$ as the inverse slope of the curve λ_M/M vs M, where λ_M is the localization length of a rod of width M for disorder $W=10$ for a simple-cubic lattice at the center of the band.

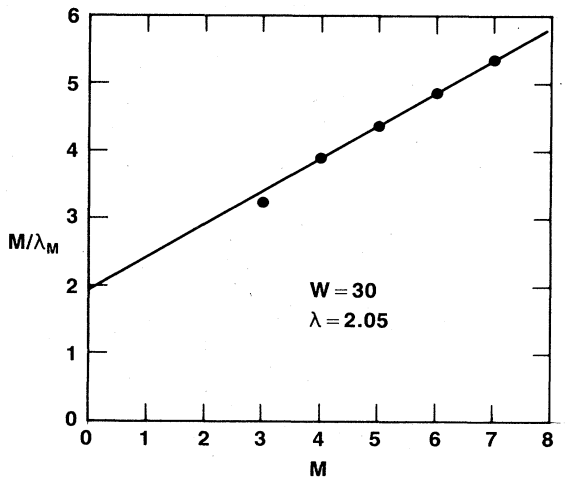

FIG. 2. Graphical determination of λ as the inverse slope of the curve M/λ_M vs M, where λ_M is the localization length of a rod of width M for disorder $W=30$ for a simple-cubic lattice at the center of the band.

TABLE I. Fluctuation lengths $\bar{\xi}$ and ξ, CPA mean free path l, and Born's approximation mean free path l_0 ($l_0 \approx 35.92/W^2$) (in units of lattice spacing) versus disorder W (in units of the transfer integral V) for the center of the band of a simple cubic lattice. The values of ξ and $\bar{\xi}$ for $W \geq 10$ were deduced from Ref. 4 (see text).

W	$\bar{\xi}$ (numerical)	ξ (numerical)	l	l_0	ξ [Eq. (4.3)]
3	0.34±0.1	0.07±0.02	4.09	3.99	0.057
4	0.55±0.05	0.114±0.01	2.40	2.245	0.11
6	1.10±0.1	0.228±0.02	1.188	0.998	0.23
8	2.0±0.15	0.41±0.03	0.76	0.561	0.42
10	4.92±0.1	1.02±0.05	0.557	0.359	0.88
11	6.95	1.44	0.493	0.297	1.39
12	11±0.1	2.28±0.1	0.443	0.249	2.33
13	18.7	3.88	0.403	0.213	4.15
14	39.6	8.22	0.370	0.183	8.01
15	95	19.7	0.343	0.160	18.2
16	456	94.6	0.321	0.140	75.3
16.45	∞	∞	0.311	0.134	∞

Comparing Eqs. (2.4) and (2.6), we find that

$$\bar{\xi} = 4.82\xi \ . \tag{2.7}$$

In Table I we present results for ξ (or $\bar{\xi}$) versus disorder W for $E=0$. For $W \geq 10$ the results were deduced from Ref. 4 by using Eqs. (2.4) and (2.6). For $W \leq 12$ we present our own results. We found it necessary to determine ξ for low disorder in order to check the correctness of the relation between the ξ or $\bar{\xi}$ and the conductivity σ to be presented in the next section. In the same table we include the values of the mean free path l as calculated by the CPA and the values of the mean free path l_0 as obtained from Born's approximation, which for the center of the band of a simple-cubic system takes the form

$$l_0 = 35.919 \frac{V^2}{W^2} \ . \tag{2.8}$$

III. CONNECTION BETWEEN ξ AND THE CONDUCTIVITY

Anderson[8] has examined the problem of the resistance in a multichain system such as ours in the quasi-one-dimensional limit where $\lambda_M \gg M \gg l, 1$. In this case he found that a well-behaved quantity which can serve as the single parameter characterizing the system and which is also *additive* with respect to the length is the following:

$$A \equiv \rho_s \ln\left[1 + \frac{\bar{R}}{\pi\rho_s}\right] , \tag{3.1}$$

where $\bar{R}$ is the dimensionless resistance

$$\bar{R} = \frac{e^2}{\hbar} R = R/4108\ \Omega \ , \tag{3.2}$$

and ρ_s is a slowly varying function of the resistance with the following limits:

$$\rho_s \to \tfrac{1}{2} \quad \text{as } \bar{R} \to 0 \ , \tag{3.3a}$$

$$\rho_s \to \frac{1}{1.764} \quad \text{as } \bar{R} \to \infty \ . \tag{3.3b}$$

However, we have already argued that L/λ_M is a well-behaved quantity which can serve as the single parameter of the system and which is obviously additive in L. It follows then that A must be proportional to L/λ_M,

$$A = \gamma \frac{L}{\lambda_M} \ , \tag{3.4}$$

which in terms of $\bar{R}$ becomes

$$\bar{R} = \pi\rho_s \left[\exp\left[\frac{\gamma}{\rho_s}\frac{L}{\lambda_M}\right] - 1\right] . \tag{3.5}$$

This is the relationship which replaces Eq. (1.3) for the present multichain system. Now in the limit of $L \gg \lambda_M$, $\bar{R} \sim \exp(2L/\lambda_M)$ from which it follows that

$$\gamma = 2\rho_s(\infty) = 1.13 \ . \tag{3.6}$$

In the semiclassical regime $L \ll \lambda_M$, Eq. (3.5) becomes

$$\bar{R} = \gamma\pi \frac{L}{\lambda_M}\left[1 + \frac{\rho_s(\infty)}{\rho_s(0)}\frac{L}{\lambda_M} + \cdots\right] . \tag{3.7}$$

The conductivity σ is defined in terms of R by the relationship

$$R = \frac{1}{\sigma}\frac{L}{M^2} \tag{3.8a}$$

or

$$\bar{R} = \frac{e^2}{\hbar\sigma}\frac{L}{M^2} \ . \tag{3.8b}$$

On the other hand, we can always write that

$$\lambda_M \equiv M^2/\bar{\xi}_M \ , \tag{3.9}$$

where $\bar{\xi}_M$ approaches its limiting value $\bar{\xi}$ as follows:

$$\frac{1}{\bar{\xi}_M} \to \frac{1}{\bar{\xi}} + \frac{1}{c}\frac{1}{M} \quad \text{as } M \to \infty \ . \tag{3.10}$$

For a thin wire ($L \gg M$), Eq. (3.7) can be written as follows:

$$\frac{\delta R}{R}=\frac{1}{2\pi\rho_s(0)}\overline{R}=\frac{\overline{R}}{\pi}\ , \tag{3.11}$$

where on the right-hand side of Eq. (3.11) $\overline{R}$ can be written as $(e^2/\hbar)(L/\sigma M^2)$. In real materials the role of L is played by the inelastic diffusion length $\Lambda\equiv(D\tau_{\rm in})^{1/2}$ or the diffusion length $L_\omega\equiv(D/\omega)^{1/2}$, where D is the diffusion coefficient, $\tau_{\rm in}$ is the inelastic collision time, and ω is the frequency of an external electric field, if any. The exact relationship between L and Λ or L_ω is not known. Here we assume, as Giordanno did,[9] that

$$L\rightarrow\sqrt{2}L_\omega \tag{3.12a}$$

$$\rightarrow\sqrt{2}\Lambda\ , \tag{3.12b}$$

in which case (3.11) takes the form

$$\frac{\delta R}{R}=\frac{R'}{9126}\ , \tag{3.13}$$

where $R'\equiv\Lambda/\sigma M^2a^2$ expressed in Ω. Giordanno[9] used 25 813 in the denominator and White *et al.*[10] 16 433, instead of the present result of 9126. Note that the absence of the factor $\sqrt{2}$ in (3.12) would increase 9126 to 12 906.

In the case of a true 3D system where $M=L\rightarrow\infty$, Eqs. (3.7), (3.8b), (3.9) and (3.10) yield

$$\sigma=\frac{e^2}{\hbar}\frac{1}{\gamma\pi}\frac{1}{\overline{\xi}}=\frac{e^2}{\hbar}\frac{1}{4.82\gamma\pi}\frac{1}{\xi}\ . \tag{3.14}$$

Equation (3.14) is very important because it allows the determination of the conductivity from $\overline{\xi}$ (or ξ) which can be determined reliably numerically. Equation (3.14) permits us to check the correctness of Anderson's expression (3.1) on which (3.14) is based by going to the weak disorder limit (small W) where the conductivity can be obtained from the CPA and ξ can be obtained numerically. There is a lower limit for the disorder at which we can calculate numerically ξ imposed by the restriction $l\ll M$ and $M=8$ (due to numerical limitations). We have chosen $W=4V$ for our test for which we found that $\sigma_{\rm CPA}=0.5746$, $l=2.4$ and $\overline{\xi}=0.55$ from which it follows that $\gamma=0.99\pm0.1$ in good agreement with the value $\gamma=1.13$ predicted on the basis of Anderson's analysis. Given the numerical uncertainties, hereafter we simplify the situation and we consider ρ_s to be a constant equal to $\frac{1}{2}$; thus γ will be taken as

$$\gamma=1 \tag{3.15}$$

and the basic equation (3.14) will become

$$\sigma=\frac{e^2}{\hbar}\frac{1}{\pi}\frac{1}{\overline{\xi}}=\frac{e^2}{\hbar}\frac{0.066}{\xi}\ . \tag{3.16}$$

Combining Eqs. (3.7), (3.9), and (3.10) with $\rho_s(\infty)=\rho_s(0)=\frac{1}{2}$ for the truly 3D case of $M=L\gg\xi$, we obtain

$$\sigma=\frac{e^2}{\pi\hbar}\left[\frac{1}{\overline{\xi}}+\frac{1-c}{c}\frac{1}{L}\right],\quad L\gg\xi \tag{3.17}$$

which is the same dependence as predicted by other approaches.[2] Unfortunately the numerical results are too scattered to allow an independent determination of the constant c, i.e., of the coefficient of the $1/L$ contribution in contrast to the coefficient of $1/\overline{\xi}$ which we just determined with an estimated accuracy of about 10%. If we take the results of weak scattering[11] and make the correspondence (3.12a), we find that $(1-c)/c=1/2\pi$ so that Eq. (3.17) can be written as

$$\sigma=\frac{e^2}{\hbar}\left[\frac{0.066}{\xi}+\frac{0.051}{L}\right],\quad L\gg\xi\ . \tag{3.18}$$

It is interesting to find the conductivity close to the mobility edge where $L\ll\xi$. Equation (3.5) permits us to examine this limit. However, as was pointed out before, the derivation of Eq. (3.1) [on which Eq. (3.5) is based] assumes that $\lambda_M\gg M\leftrightarrow\xi\ll M=L$. Thus, it is doubtful whether one can use Eq. (3.5) to examine the critical region $L\ll\xi$. Nevertheless, if we just proceed, we find (using the MK result of $\lambda_M/M=0.6$ for $M\ll\xi$) that the critical resistance is

$$\overline{R}_c=42.46\ \text{ or }\ R_c=174\,436\ \Omega\ , \tag{3.19a}$$

or that the critical conductance is

$$G_c=0.0236\frac{e^2}{\hbar}\ , \tag{3.19b}$$

or that the conductivity in the critical region ($L\ll\xi$) is

$$\sigma=\frac{e^2}{\hbar}\frac{0.0236}{L},\quad L\ll\xi\ . \tag{3.20}$$

Comparing Eq. (3.20) with Eq. (3.18), we can conclude (as Vollhardt and Wölfle[2] did) that the conductivity for a 3D system has the form

$$\sigma=\frac{e^2}{\hbar}\left[\frac{0.066}{\xi}+\frac{b}{L}\right] \tag{3.21}$$

for all ranges of disorder, starting with weak disorder all the way to the mobility edge. The coefficient of ξ is accurate to about 10%, while the coefficient b is more uncertain and seems to vary from $b=0.05$ for weak disorder to about half this value at the critical point. It is possible to extend Eq. (3.21) into the region of localized states, but there are greater uncertainties there about which physical quantity will play the role of L. A better understanding of the role of the electron-phonon interaction is required before one can attempt to obtain a reliable general formula for the conductivity in the localized regime.

Equation (3.14) allows us to define the energy E_u beyond which the conductivity is adequately described by the CPA result σ_0. Since for weak disorder $\xi\sim l^{-1}\sim W^2\rightarrow0$ while for strong disorder $\xi\rightarrow\infty$, there is always an intermediate point where $l=\xi$. We choose this point as a definition of the onset of the reduction of σ due to fluctuations in amplitude (of course this reduction is a continuous process and the above definition is only for orientation purposes).

IV. A FORMULA FOR THE CONDUCTIVITY

By employing the potential-well analogy,[1,12] we have shown[1] that the localization length λ can be expressed as a product of the mean-free-path times a function of Sl^2,

where S is the surface of constant energy for the disordered system[1] and l is the mean free path. Because as we approach the critical point both ξ and λ have the same form (apart from a sign difference), it follows from Ref. 1 that

$$\xi \rightarrow 2.72l\frac{6}{\phi-1} \quad \text{as } \phi \rightarrow 1 , \tag{4.1}$$

where $\phi = Sl^2/(Sl^2)_c$ and the critical value of Sl^2, $(Sl^2)_c$ was found to be 8.96 before.[1,13] CPA is very reliable[1] in calculating the quantity Sl^2 as our comparison with numerical work has shown us. In the limit of weak disorder, on the other hand, ξ can be easily obtained in terms of the CPA conductivity, i.e.,

$$\xi \rightarrow \frac{24.4}{Sl} \quad \text{as } Sl^2 \rightarrow \infty . \tag{4.2}$$

We have used the numerical results of Table I to find a simple interpolation function between these two limits. We found that the following function provides a reasonable fit as can be seen in Table I and Fig. 3:

$$\xi = 2.72l\left[\frac{1}{\phi}+\frac{6}{\phi^2(\phi-1)}\right] . \tag{4.3}$$

Further justification for this formula will be presented elsewhere.[12] Combining Eq. (4.3) with Eq. (3.21), we obtain a general formula for the conductivity which covers the entire range of extended states up to the critical point for localization.

In Table II and in Fig. 4 we compare our results for the conductivity: σ_{00} is the result of the Born approximation, where $\sigma_{00}=(e^2/12\pi^3\hbar)Sl$ with l given by the Born approximation [Eq. (2.8)]. In the present case where $S=92.648a^{-2}$, we obtain for σ_{00}

$$\sigma_{00} = \frac{e^2}{\hbar a}\frac{8.946}{(W/V)^2} . \tag{4.4}$$

σ_0 is the CPA result which is given as follows:

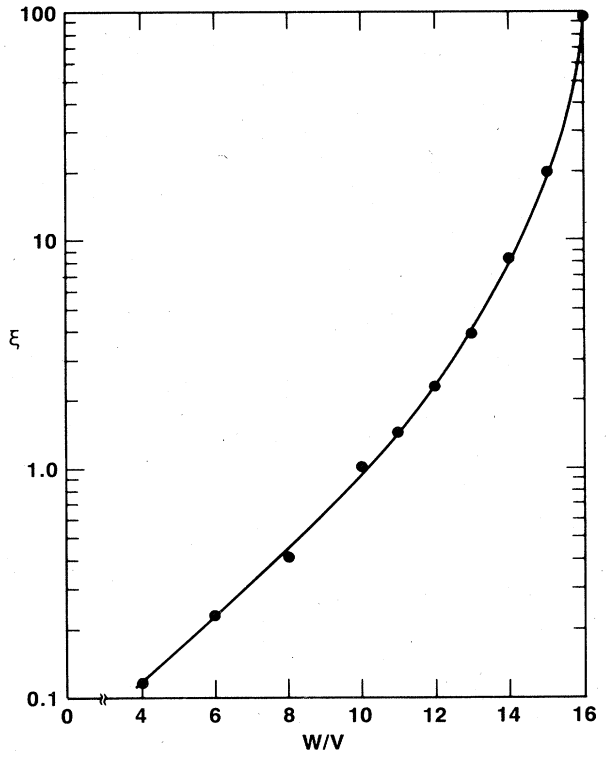

FIG. 3. Correlation length ξ (in units of lattice spacing) vs disorder W/V for a simple-cubic lattice at the center of the band. Dots are points determined numerically and the solid line is according to Eq. (4.3).

$$\sigma_0 = \frac{e^2}{12\pi^4}\int dE' S_0(E-\Sigma_1-E')v_0 \times (E-\Sigma_1-E')\frac{\Sigma_2^2}{(E'^2+\Sigma_2^2)^2} , \tag{4.5}$$

where Σ is the CPA self-energy, $\Sigma=\Sigma_1+i\Sigma_2$, S_0 is the periodic constant-energy surface in k space and v_0 is the

TABLE II. Conductivity (in units of $10^{-4}e^2/\hbar a$) versus disorder; σ_{00} is the Born approximation results, σ_0 is the CPA results, and the last two columns are the results for σ based on Eq. (3.16) with either Eq. (4.3) or the numerical determination of ξ.

W/V	σ_{00}	σ_0	σ [Eqs. (3.16) and (4.3)]	σ (numerical) [Eq. (3.16)]
3	9940	10046	10040	10180
4	5591	5746	5730	5743
6	2485	2678	2671	2894
8	1398	1592	1577	1591
10	895	1067	752	645
11	739	899	477	457
12	621	768	284	291
13	529	665	159	169
14	456	580	82.6	80
15	398	511	36.3	33.4
16	349	454	8.8	6.94
16.45	302.5	432	0	0

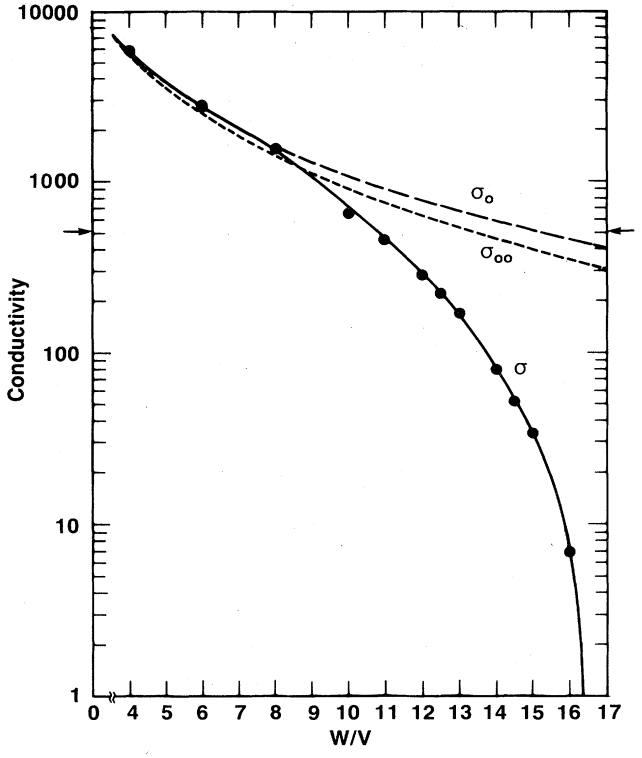

FIG. 4. Conductivity (in units of $10^{-4}e^2/\hbar a$, where a is the lattice spacing) vs disorder for the center of the band of a simple-cubic lattice; σ_{00} is the Born-approximation results, σ_0 is the CPA results; the solid line is based on Eqs. (3.16) and (4.3) and the dots are based on (3.16) and the numerical determination of ξ.

periodic velocity averaged over all states of constant energy. Note that σ_{00} and σ_0 are surprisingly close together even for large disorders. The reason is an accidental almost cancellation of two effects: To obtain σ_{00} from σ_0 one makes two distinct approximations. (i) S_0 and v_0 are considered energy independent and are pulled out of the integral; for the *center* of the *band* this approximation enhances the value of σ_0. (ii) l is replaced by l_0 as given by Eq. (2.8); this approximation reduces the value of l (see Table I) and hence, partially cancels the errors of the first approximation.

In Fig. 4 we plot σ_{00}, σ_0, and σ as obtained from Eqs. (3.16) and (4.3); the dots represent σ as obtained from Eq. (3.16) and the numerical determination of ξ as given in Table II. We also denote by arrows the value that Mott's formula would predict for minimum metallic conductivity. Indeed, Mott writes

$$\sigma_{\min}=\left[\frac{\rho_c}{\rho_0}\right]^2\frac{1}{3}\frac{e^2}{\hbar a}\ , \tag{4.6}$$

where ρ_c is the density of states at the critical disorder (in the present case $0.112/2V$) and ρ_0 is the corresponding density of states for zero disorder (in the present case $0.285/2V$). Thus, the result is

$$\sigma_{\min}=0.0516\frac{e^2}{\hbar a}\ . \tag{4.7}$$

Note that the unit of conductivity in both Table II and Fig. 4 has been taken as $10^{-4}e^2/\hbar a$ which for $a=2.434$ Å equals 1 $\text{cm}^{-1}\,\Omega^{-1}$. Figure 4 gives for the first time (to the best of our knowledge) a quantitate, reliable (estimated error of about 10%) result for the behavior of the conductivity versus disorder for the center of the band.

V. SUMMARY

There are two main results for the present work. First, Eq. (3.21) or (3.16), which connects a quantity of great physical importance, namely the conductivity σ, with ξ, which can be obtained from reliable numerical work. This connection allowed us to obtain a more accurate estimate of the correction to the conductivity of very thin wires, as well as a real check of Anderson's analysis[8] of the multichannel system. The second important result, Eq. (4.3) obtained through a combination of the numerical work on wires with the potential-well analogy, enables us to obtain the length ξ and consequently the conductivity σ in terms of quantities such as the mean free path l and the CPA conductivity $\sigma_0\sim Sl$, which are readily available. Equation (4.3) interpolates successfully between the two limits and thus covers the entire extended states region. Note from Fig. 3 that the ξ based on Eq. (4.3) seems to be smaller than the numerical result near the critical point. Although there is sufficient uncertainty in the numerical work to prevent us from reaching any definite conclusion, it is conceivable that the critical exponent for ξ (and consequently, for λ) is not one as in Eq. (4.3) but slightly larger than one.

It will be very interesting to investigate whether the formula for the dc conductivity of a noninteracting disordered electron system, which interpolates from the weak-coupling regime all the way to the Anderson transition, is retained in the presence of complicating factors such as off-diagonal disorder, more than one orbital per site, topological disorder, and different types of disorder or lattices.

ACKNOWLEDGMENTS

One of us (C.M.S.) wishes to thank the Research Center of Crete, where part of this work was done, for their hospitality. This work was partially supported by a North Atlantic Treaty Organization travel Grant No. RG684/84. Ames Laboratory is operated for the U.S. Department of Energy by Iowa State University under Contract No. W-7405-Eng. 82.

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