Practice exam Math 414

1) Let $a_n = \frac{n}{n-1/2}$. Prove directly from the definition of convergence that $\{a_n\}$ is convergent.

2) Let $x_k = \sin(k\pi/2) + \frac{(-1)}{k}$

a) Find limsup $\{x_k\}$ and liminf $\{x_k\}$. (Explain)

b) Give an example of a convergent subsequence $\{x_{k_j}\}$. What are the possible values L that a subsequence of $\{x_k\}$ could converge to?

c) Prove that $\{x_k\}$ is divergent.

3) Use the definition to divergence to infinity to prove

$$\{\frac{n^2-n}{n+1}\} \to \infty$$

4) Use a limit theorem to help prove that $\{\frac{n+\cos n}{n}\} \to 1$.

5) Let $E = \{1 + \frac{1}{n} : n \in \mathbf{N}\}$.

Find $\inf(E)$ and $\sup(E)$ (and justify your answer)

6) Use limit theorems to prove the convergence of $\{x_n\}$, where

$a)$ $x_n = \dfrac{n^2-3n+2}{n^2+n}$ $\qquad$ $b)$ $x_n = \pi^{-n}\cos(n^2-2n)$ $\qquad$ c $x_n = \dfrac{3^n+2^n}{n!}$

7) If $\{a_n\}$ is bounded and $\{b_n\}$ convergent, prove $\{a_n b_n\}$ is bounded.