Asteroseismology

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observed pulsations

- operate on the **dynamical** time scale
- accessible on convenient time scale
- probe **global** and **local** structure
- periods change on 'evolutionary' time scale (thermal or nuclear) - depend on global properties
- amplitudes change on ~ 'local' thermal time scale

dynamical stability

- "stable" configuration represents a stable ***mean*** configuration
- on short time scale, oscillations occur, but the mean value is fixed on longer time scales
- simple example: *a* ***pendulum*** (single mode)
  - *most likely* position - extrema
  - *mean* position is at zero displacement
  - with no *damping* would oscillate forever
- more complex example: *a* ***vibrating string***
  - multiple modes with different frequencies
  - enumerated by number of nodes

a more complex example: **a star**

- multiple oscillation modes
- ***radial*** modes - enumerated by number of nodes between center and surface
- ***non-radial*** modes - nodes also across surface of constant radius
- modes frequencies determined by solution of the appropriate **wave equation**

stability, damping, and driving

- zero energy change:
  constant amplitude oscillation
- energy loss via pulsation:
  oscillation amplitude drops with time
- if net energy *input*:
  amplitude increases with time
  (if properly phased)

Okay, start your engines...

- **PG 1159: light curve**
  - what kind of star might this be?
  - what kind of star can this not possibly be?
  - what about the amplitude over the run?
- **PG 1336 light curve**
  - huh? what time scale(s) are involved
  - what kind of star (or stars)?
  - tell us *everything* you can about this!

Multimode pulsation

- Oscillations at "normal mode" frequencies
- mode = specific eigensolution of equations of motion within the confines of a stellar structure
- normal mode frequencies parallel structural properties
- simple example: radial fundamental is one mode, 1st overtone (a node within) is another mode

towards the wave equation I

- continuity equation:

$$\frac{\partial M_r}{\partial r} = 4\pi r^2 \rho$$

- equation of motion (HSE when RHS=0):

$$\frac{\partial^2 r}{\partial t^2} = -\frac{GM_r}{r^2} - 4\pi r^2 \frac{\partial P}{\partial M_r}$$

- perturb *r, P, and* ρ:

$$x(t, M_r) = x_o(M_r)\left[1 + \frac{\delta x(t, M_r)}{x_o(M_r)}\right]$$

- and assume δx << x so we can linearize

towards the wave equation II

- replace x with x+δx in the two equations, subtract off the equilibrium equations, and keep only 1st-order terms to find:
- linearized continuity equation

$$\frac{\delta\rho}{\rho_o} = -3\frac{\delta r}{r_o} - r_o\frac{\partial(\delta r/r_o)}{\partial r_o}$$

- linearized equation of motion

$$\rho_o r_o \frac{d^2\delta r/r_o}{dt^2} = -\left(4\frac{\delta r}{r_o} + \frac{\delta P}{P_o}\right)\frac{\partial P_o}{\partial r_o} - P_o\frac{\partial(\delta P/P_o)}{\partial r_o}$$

towards the wave equation III

- assume adiabatic relationship between *P* and ρ

$$\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{\rm ad} \quad \text{so} \quad \frac{\delta P}{P_o} = \Gamma_1\frac{\delta\rho}{\rho_o}$$

- combine continuity and equation of motion:

$$\frac{\partial^2\eta}{\partial t^2} = \frac{1}{\rho r^4}\frac{\partial}{\partial r}\left(\Gamma_1 P r^4\frac{\partial\eta}{\partial r}\right) + \eta\frac{1}{r\rho}\left\{\frac{\partial}{\partial r}\left[(3\Gamma_1 - 4)\,P\right]\right\}$$

- assume exponential (complex) time dependence

$$\frac{\delta r(t,r_o)}{r_o} = \frac{\delta r(r_o)}{r_o}e^{i\sigma t} = \eta(r_o)e^{i\sigma t}$$

towards the wave equation IV

- substitute to yield the Linear Adiabatic Wave Equation (LAWE):

$$\mathbf{L}(\eta) = -\frac{1}{\rho r^4}\frac{\partial}{\partial r}\left(\Gamma_1 P r^4\frac{\partial\eta}{\partial r}\right) - \eta\frac{1}{r\rho}\left\{\frac{\partial}{\partial r}\left[(3\Gamma_1 - 4)\,P\right]\right\} = \sigma^2\eta$$

- This is a wave equation: $\mathbf{L}(\eta) = \sigma^2\eta$ in the displacement η.
- the eigenvalue σ^2 corresponds to the oscillation frequency

the LAWE: a simple case

- assume Γ_1 and η both constant throughout the star (homologous motion)
- LAWE becomes

$$-\eta\ \frac{1}{\rho r}(3\Gamma_1 - 4)\frac{\partial P}{\partial r} = \sigma^2\eta$$

- now, assume a constant density, and use HSE to replace the pressure derivative to find

$$\Pi = \frac{2\pi}{\sigma} = \frac{\sqrt{\pi}}{\sqrt{G\bar{\rho}\,\left(\Gamma_1 - \frac{4}{3}\right)}}$$

- look familiar?!

the LAWE: standing wave solutions

- boundary conditions:
  - center: zero displacement (η = 0)
  - surface: perfect wave reflection [d(δP/P)/dr = 0]
- asymptotic analysis:
  - clever change of variables renders LAWE as:

$$\frac{d^2 w(r)}{dr^2} + \left[\frac{\sigma^2 \rho}{\Gamma_1 P} - \phi(r)\right] w(r) = 0$$

  - recognizing the sound speed when we see it:

$$\frac{d^2 w(r)}{dr^2} + \left[\frac{\sigma^2}{c_s^2} - \phi(r)\right] w(r) = 0$$

the LAWE: asymptotic solution

$$\frac{d^2 w(r)}{dr^2} + \left[\frac{\sigma^2}{c_s^2} - \phi(r)\right] w(r) = 0$$

- represent the eigenfunction as: $w(r) \propto e^{ik_r r}$ where k_r is the (local) radial wavenumber and varies slowly with radius so, locally:

$$k_r^2 = \frac{\sigma^2}{c_s^2(r)} - \phi(r)$$

- for a standing wave, we need an integral number of half-wavelengths between inner and outer reflection points:

$$\int_a^b k_r \; dr = (n+1)\pi$$

the LAWE: asymptotic solution

$$\int_a^b k_r \; dr = (n+1)\pi \quad \text{where} \quad k_r^2 = \frac{\sigma^2}{c_s^2(r)} - \phi(r)$$

- if $\frac{\sigma^2}{c_s^2} \gg \phi$ then

$$\sigma = (n+1)\pi \left[\int_a^b \frac{dr}{c_s}\right]^{-1} = (n+1)\;\sigma_o$$

- i.e. high-frequency (high overtone, n) radial modes are equally spaced in frequency, with

$$\sigma_o^2 \approx G<\rho>$$

Nonradial oscillations

- preserve angular derivatives in LAWE
- similar operator structure for radial part (as before), now along with angular part

$$\frac{d^2 \delta\mathbf{r}}{dt^2} = -\nabla\left(\frac{P'}{\rho} + \psi'\right) + \mathbf{A}\frac{\Gamma_1 P}{\rho}\nabla \cdot \delta\mathbf{r}$$

where the quantity **A** is:

$$A = \frac{d\ln\rho}{dr} - \frac{1}{\Gamma_1}\frac{d\ln P}{dr} = \frac{1}{\lambda_P}\frac{\chi_T}{\chi_\rho}\left[\nabla - \nabla_{ad}\right]$$

A < 0 when radiative
A > 0 when convective
(the 'Schwarzschild A')

Decompose into Spherical Harmonics

- position perturbation decomposition

$$\delta\mathbf{r} = \delta r\ \mathbf{e_r} + r\delta\theta\ \mathbf{e}_\theta + r\sin\theta\delta\phi\ \mathbf{e}_\phi$$

- produces (after some work):

$$\nabla\cdot\delta\mathbf{r} = \frac{1}{r^2}\frac{\partial(r^2\delta r)}{\partial r} - \frac{1}{\sigma^2}r^2L^2\left(\frac{P'}{\rho}+\psi'\right)$$

where the operator L^2 (the Legendrian) is:

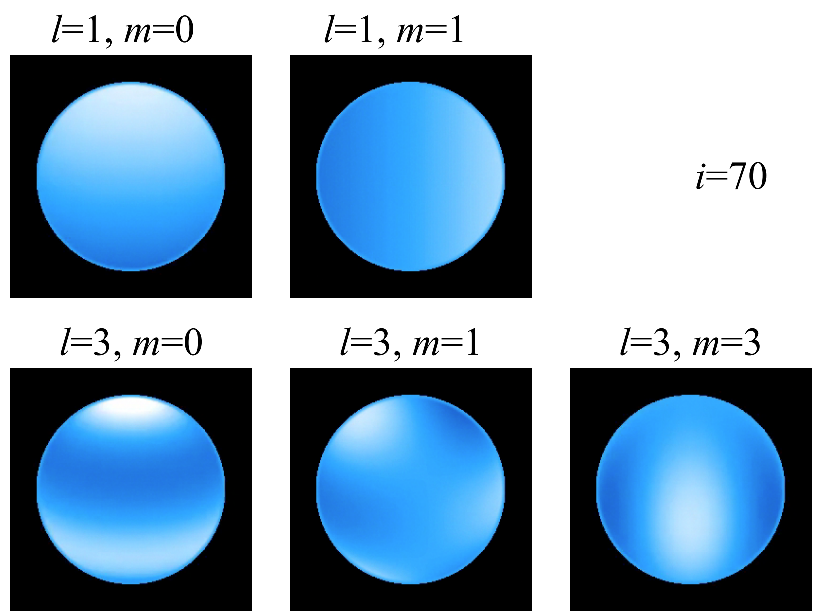
$$L^2 = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

- which has eigenstates $\boldsymbol{Y_l^m}$ such that:

$$L^2Y_l^m(\theta,\phi) = l(l+1)Y_l^m(\theta,\phi)$$

Spherical Harmonics....

courtesy *asteroseismology.org* (Travis Metcafe)



using Spherical Harmonics

- now we have

$$\nabla\cdot\delta\mathbf{r} = \frac{1}{r^2}\frac{\partial(r^2\delta r)}{\partial r} - \frac{l(l+1)}{\sigma^2r^2}\left(\frac{P'}{\rho}+\psi'\right)$$

- expanding into components:

frequency^{-2}

$$r^2\frac{d\eta_r}{dr} = \left[\frac{gr}{c_s^2}-2\right]r\eta_r + r^2\frac{l(l+1)}{r^2}\left[1-\frac{\sigma^2r^2}{c_s^2}\frac{1}{l(l+1)}\right]r\eta_t$$

$$r^2\frac{d\eta_t}{dr} = \left[1+\frac{Ag}{\sigma^2}\right]r\eta_r + \left[(-Ag)\frac{r}{g}-1\right]r\eta_t$$

another frequency^2

the two characteristic frequencies

- so:

$$r^2\frac{d\eta_r}{dr} = \left[g\frac{l(l+1)}{S_l^2}-2r\right]\eta_r + l(l+1)\left[1-\frac{\sigma^2}{S_l^2}\right]r\eta_t$$

$$r^2\frac{d\eta_t}{dr} = \left[1-\frac{N^2}{\sigma^2}\right]r\eta_r + \left[N^2\frac{r}{g}-1\right]r\eta_t$$

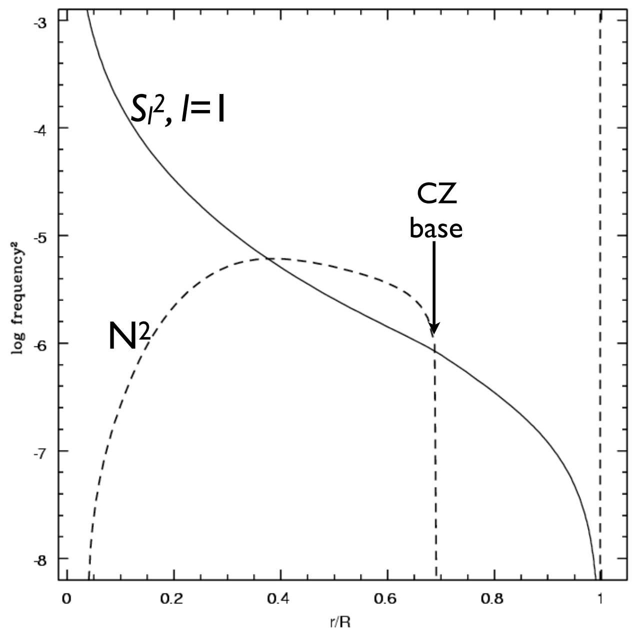
- where we've defined 2 "structural" frequencies
  - the **acoustic** (**Lamb**) frequency $\boldsymbol{S_l}$:

$$S_l^2 = \frac{l(l+1)}{r^2}c_s^2$$

  - the **Brunt-Väisälä** (**buoyancy**) frequency $\boldsymbol{N}$:

$$N^2 = -Ag = -g\left[\frac{d\ln\rho}{dr} - \frac{1}{\Gamma_1}\frac{d\ln P}{dr}\right]$$

Propagation diagram, ZAMS solar model



NRP dispersion relation

- identify the horizontal wave number(s)

$$k_t^2 = \frac{l(l+1)}{r^2} = \frac{S_l^2}{c_s^2}$$

- allows the wave equation(s) to reduce to a local dispersion relation, as with the radial case, to provide relationship between k_r and σ:

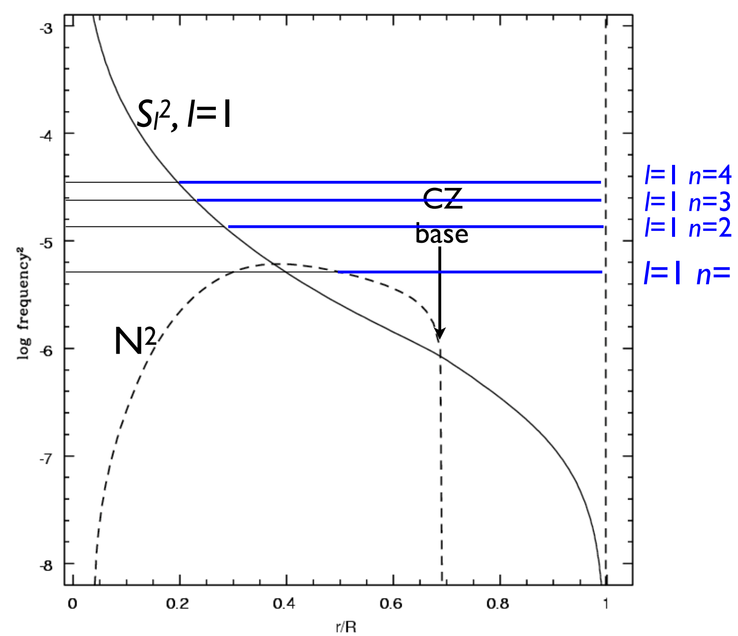
$$k_r^2 = \frac{1}{\sigma^2 c_s^2}(\sigma^2 - N^2)(\sigma^2 - S_l^2)$$

asymptotic analysis

$$k_r^2 = \frac{1}{\sigma^2 c_s^2}(\sigma^2 - N^2)(\sigma^2 - S_l^2)$$

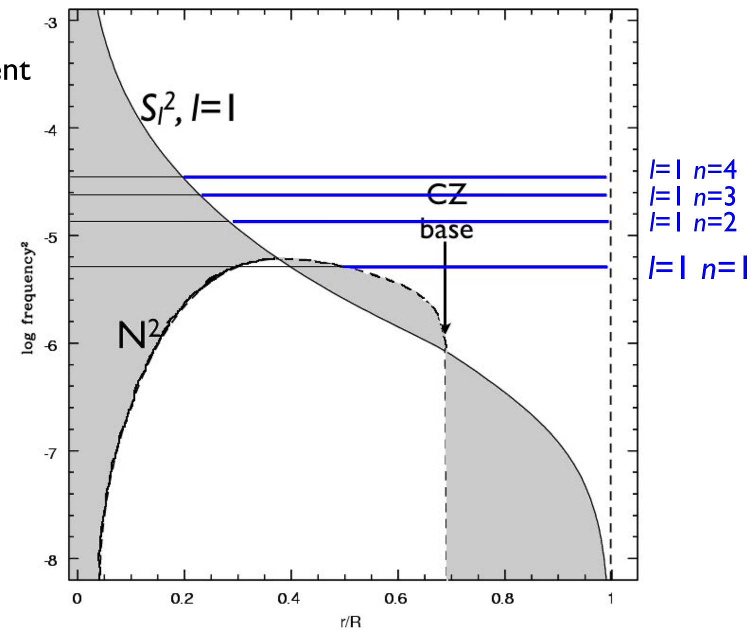
- $k_r^2 > 0$ (k_r real) when
  - **$\sigma^2 > N^2, S_l^2$** - or - **$\sigma^2 < N^2, S_l^2$**
  - k_r real means oscillatory eigenfunctions
- $k_r^2 < 0$ (k_r imaginary) when
  - $S_l^2 > \sigma^2 > N^2$ **or** $S_l^2 < \sigma^2 < N^2$
  - k_r real means evanescent (exponentially decreasing or increasing) eigenfunctions

Propagation diagram, ZAMS solar model



Propagation diagram, ZAMS solar model

evanescent zones in gray

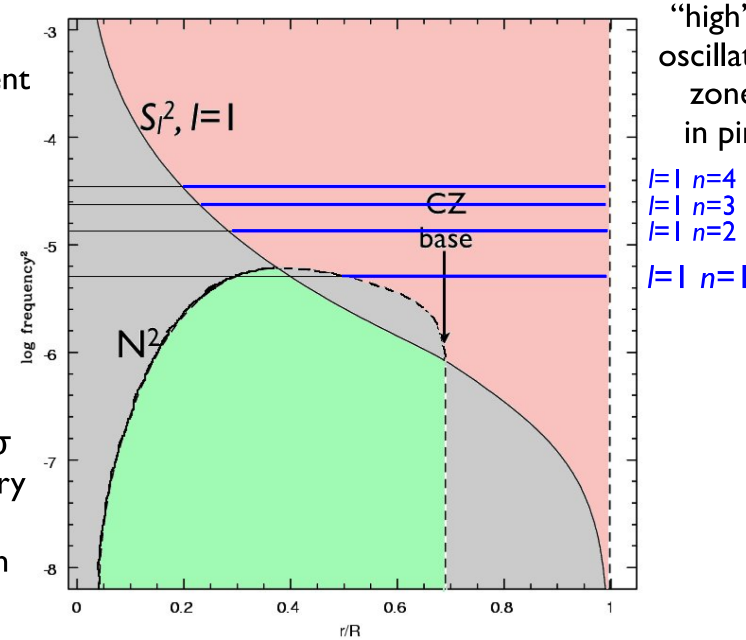


Propagation diagram, ZAMS solar model

evanescent zones in gray

"high" σ oscillatory zones in pink

"low" σ oscillatory zones in green



the NRP LAWE: asymptotic solutions

- again, integrate dispersion relation over propagation regions:

$$\int_a^b k_r \; dr = (n+1)\pi \quad \text{where} \quad k_r^2 = \frac{1}{\sigma^2 c_s^2}(\sigma^2 - N^2)(\sigma^2 - S_l^2)$$

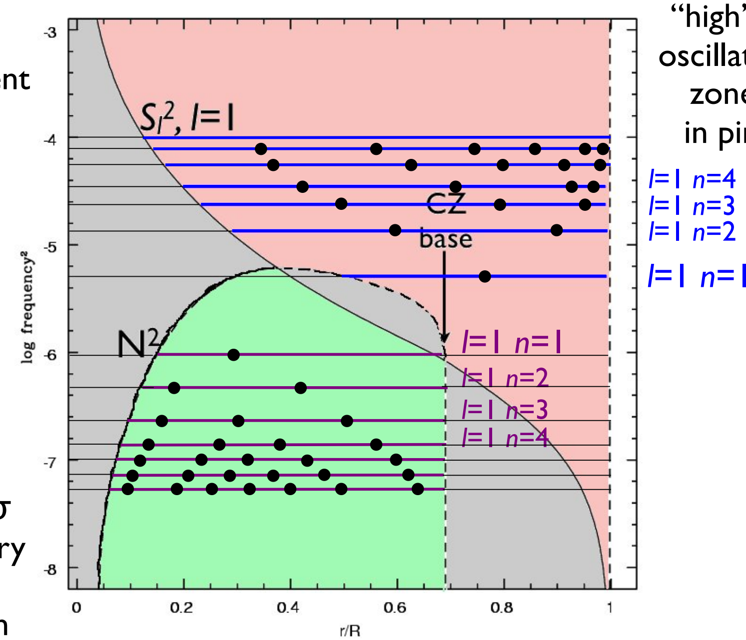
- two classes of solutions:
  - **$\sigma^2 > N^2, S_l^2$:** $\sigma_{nl} = (n + l/2)\sigma_o \;\; ; \;\; \sigma_o = \int_a^b \frac{dr}{c_s}$
    - "**p-modes**"; **pressure** as the restoring force
  - **$\sigma^2 < N^2, S_l^2$:** $\Pi_{nl} = n\frac{\Pi_o}{\sqrt{l(l+1)}} \;\; ; \;\; \Pi_o = 2\pi^2 \left[\int_a^b \frac{N}{r} dr\right]^{-1}$
    - "**g-modes**": **buoyancy** as the restoring force

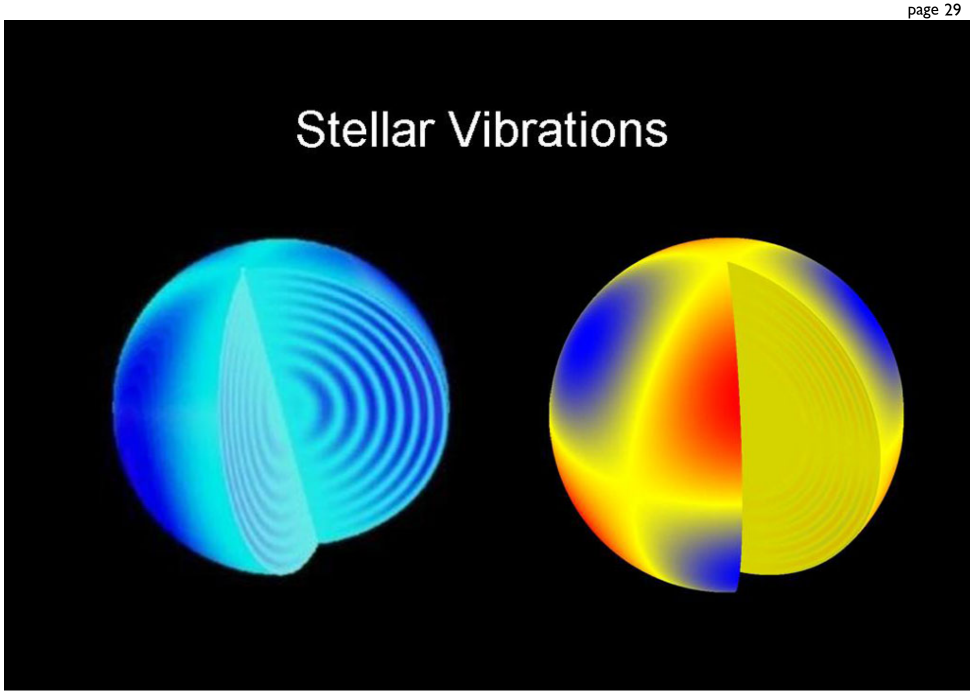
Propagation diagram, ZAMS solar model

evanescent zones in gray

"high" σ oscillatory zones in pink

"low" σ oscillatory zones in green





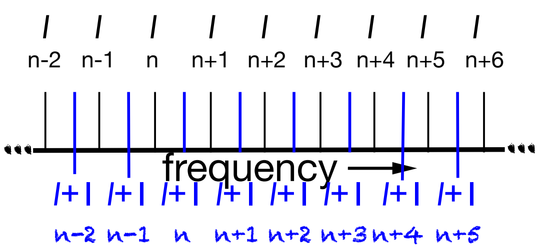
Pulsation Periods

Period of 'radial fundamental' ~ t_{ff}

	g-modes	p-modes
Periods	$\Pi > t_{ff}$	$\Pi < t_{ff}$
restoring force	buoyancy	pressure
asymptotic behavior	$\Pi \propto \Pi_o \times n$	$\sigma \propto \sigma_o \times n$
examples	white dwarfs	Cepheids, the Sun

p-modes: ~ equally spaced in frequency

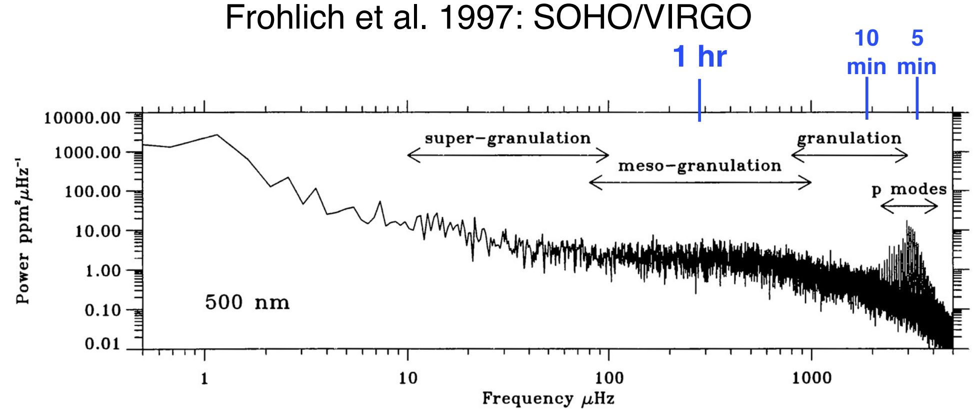
$$\sigma_{nl} = (n + l/2)\sigma_o \quad ; \quad \sigma_o = \int_a^b \frac{dr}{c_s}$$



- if modes of different l present, observed spacing ~ σ_o / 2

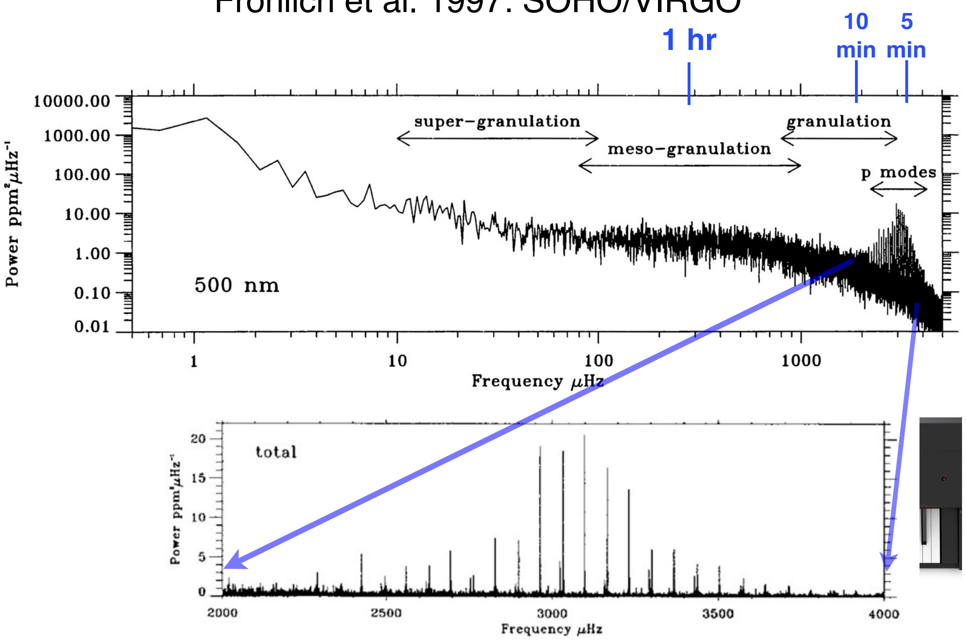
Solar Oscillations: Full-disk *photometry*

Frohlich et al. 1997: SOHO/VIRGO

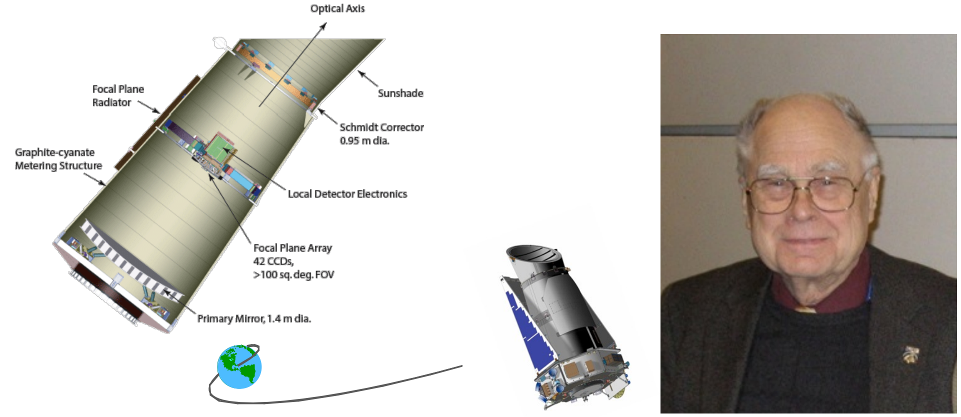


Solar Oscillations: Full-disk *photometry*

Frohlich et al. 1997: SOHO/VIRGO

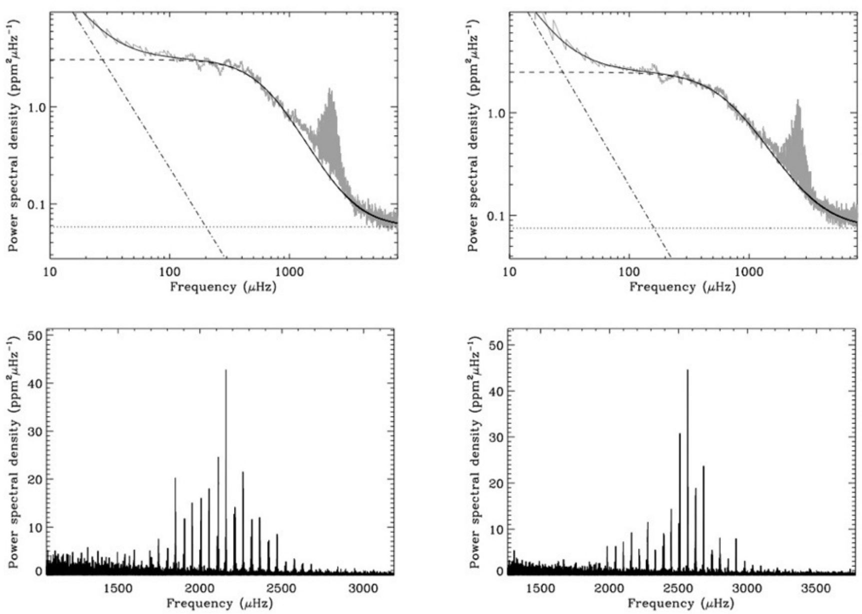


How to observe all these low amplitude modes?



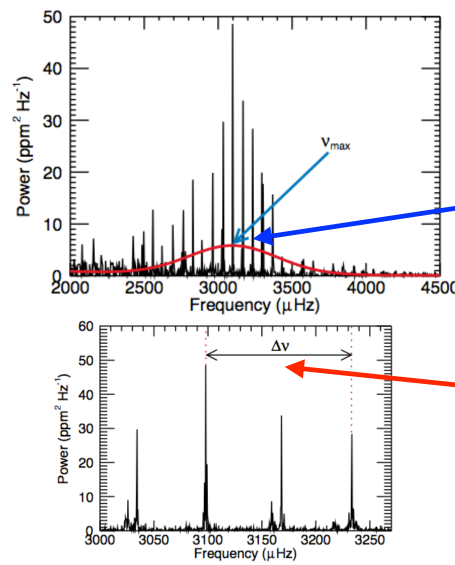
have your friends buy a $600,000,000 photometer!

16 Cyg A and B (Metcalfe et al. 2012)



asteroseismic radius determination

(i.e. Chaplin et al. 2011)



- ν_{max} scales with acoustic cutoff frequency ~ $gT_e^{-1/2}$

$$\left(\frac{\nu_{max}}{\nu_{max,\odot}}\right) \approx \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-2}\left(\frac{T_{eff}}{T_{eff,\odot}}\right)^{-0.5}$$

- $\Delta\nu$ measures mean density:

$$\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{2} \approx \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-3}$$

Kepler 93b

(Ballard et al. 2014)

Draft version May 16, 2014
Preprint typeset using LaTeX style emulateapj v. 12/16/11

KEPLER-93B: A TERRESTRIAL WORLD MEASURED TO WITHIN 120 KM, AND A TEST CASE FOR A NEW SPITZER OBSERVING MODE

Sarah Ballard[1,2], William J. Chaplin[3,4], David Charbonneau[5], Jean-Michel Désert[6], Francois Fressin[5], Li Zeng[5], Michael W. Werner[7] Guy R. Davies[3,4], Victor Silva Aguirre[4], Sarbani Basu[8], Jørgen Christensen-Dalsgaard[4], Travis S. Metcalfe[9,4], Dennis Stello[10], Timothy R. Bedding[10], Tiago L. Campante[3,4], Rasmus Handberg[3,4], Christoffer Karoff[4], Yvonne Elsworth[3,4], Ronald L. Gilliland[11], Saskia Hekker[12,13,3], Daniel Huber[14,15], Steven D. Kawaler[16], Hans Kjeldsen[4], Mikkel N. Lund[4], Mia Lundkvist[4]

Draft version May 16, 2014

ABSTRACT

We present the characterization of the Kepler-93 exoplanetary system, based on three years of photometry gathered by the *Kepler* spacecraft. The duration and cadence of the *Kepler* observations, in tandem with the brightness of the star, enable unusually precise constraints on both the planet and its host. We conduct an asteroseismic analysis of the *Kepler* photometry and conclude that the star has an average density of 1.652±0.006 g cm^{-3}. Its mass of 0.911±0.033 $M_\odot$ renders it one of the lowest-mass subjects of asteroseismic study. An analysis of the transit signature produced by the

Kepler 93b

(Ballard et al. 2014)

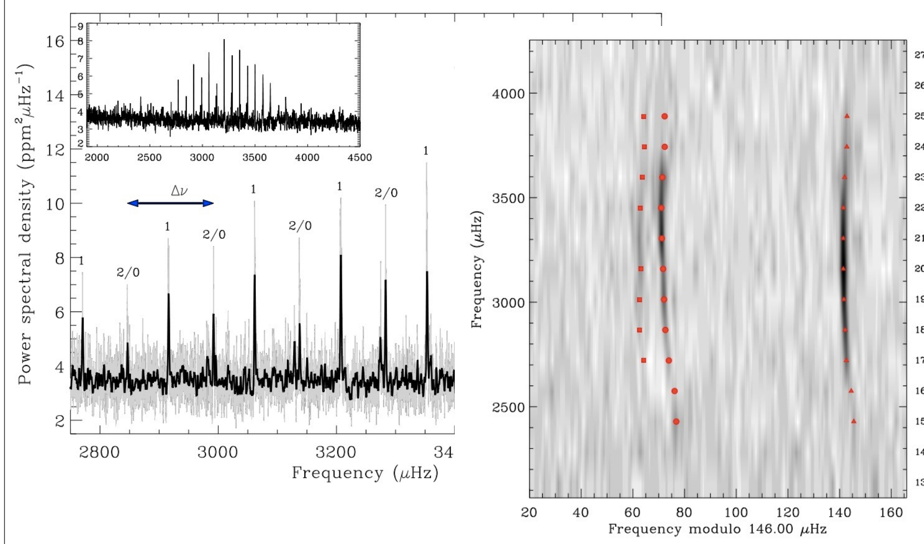
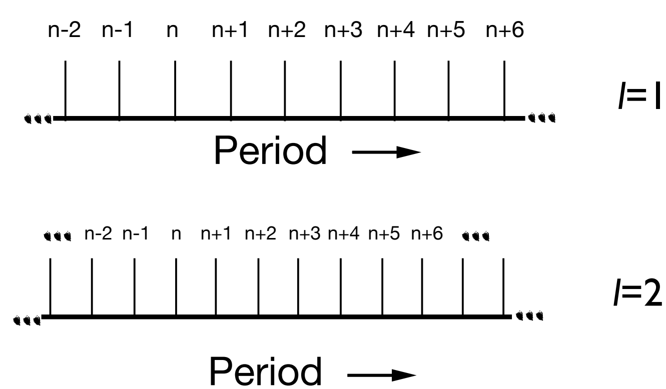


Table 2
Star and Planet Parameters for Kepler-93

Parameter	Value & 1σ confidence interval	
Kepler-93 [star]		
Right ascension[a]	19h25m40.39s	
Declination[a]	+38d40m20.45s	
T_{eff} [K]	5669±75	
$R_\star$ [Solar radii]	0.919 ± 0.011	
$M_\star$ [Solar masses]	0.911 ± 0.033	
[Fe/H]	-0.18±0.10	
log(g)	4.470 ± 0.004	
Age [Gyr]	6.6 ± 0.9	
Light curve parameters	No asteroseismic prior	With asteroseismic prior
ρ [g cm^{-3}]	$1.72^{+0.04}_{-0.28}$	1.652±0.0060
Period [days][b]	4.72673978±9.7×10^{-7}	–
Transit epoch [BJD][b]	2454944.29227 ±0.00013	–
$R_p/R_\star$	0.01474±0.00017	0.014751±0.000059
a/$R_\star$	$12.69^{+0.09}_{-0.76}$	12.496 ±0.015
inc [deg]	$89.49^{+0.51}_{-1.1}$	89.183±0.044
u_1	0.442±0.068	0.449±0.063
u_2	0.187±0.091	0.188±0.089
Impact Parameter	0.25±0.17	0.1765±0.0095
Total Duration [min]	173.42±0.36	173.39±0.23
Ingress Duration [min]	$2.52^{+0.37}_{-0.06}$	2.61±0.013
Kepler-93b [planet]	No asteroseismic prior	With asteroseismic prior
R_p [Earth radii]	1.483±0.025	1.478±0.019
Planetary T_{eq} [K]	1039±26	1037±13
M_p [Earth masses][c]	3.8±1.5	–

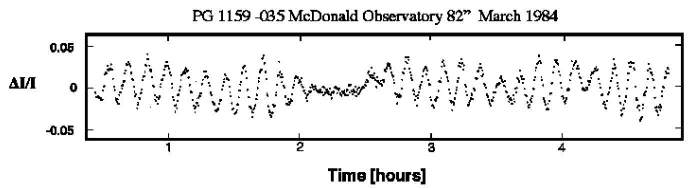
g-modes: ~ equally spaced in period

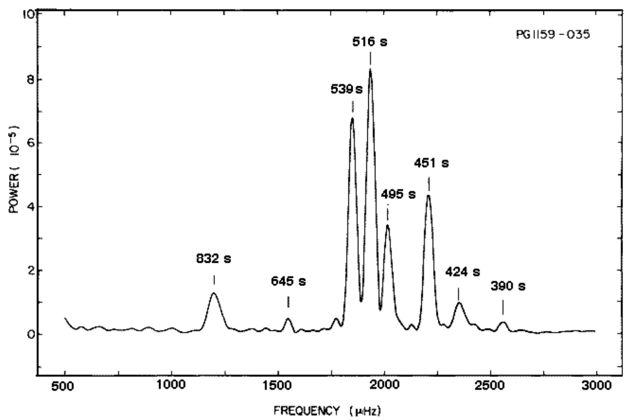
$$\Pi_{nl} = n\frac{\Pi_o}{\sqrt{l(l+1)}} \quad ; \quad \Pi_o = 2\pi^2 \left[\int_a^b \frac{N}{r} dr\right]^{-1}$$



- spacing depends on *l*

PG 1159-035: a g-mode pulsator





P	P-390	
390	0	
424	34	?
451	61	3x20.3
495	105	5x21.0
516	126	6x21.0
539	149	7x21.3
645	255	12x21.3
832	442	21x21.0

an example:

hot white dwarf PG 1159-035 (Corsico et al. [WET] 2008)

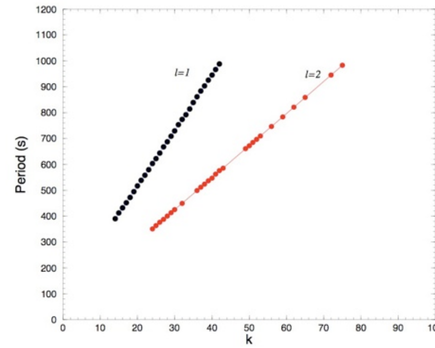
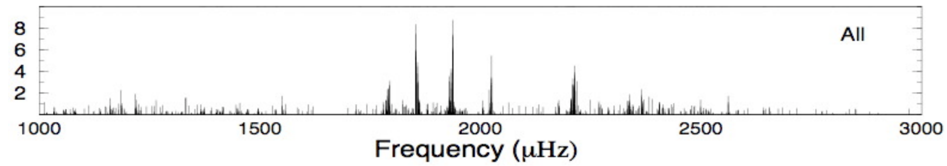
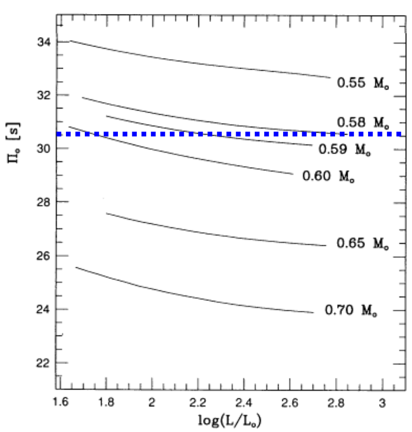


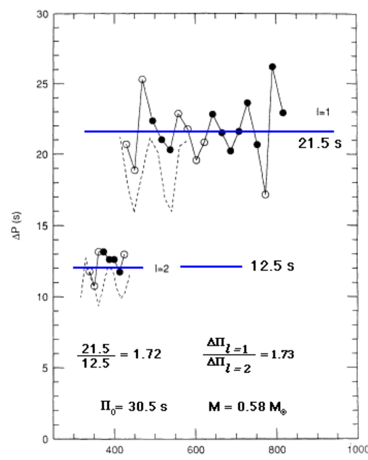
Fig. 7. Observed periods sequences for the modes $\ell = 1$ and $\ell = 2$.



in white dwarfs:

Π_o depends on total stellar mass



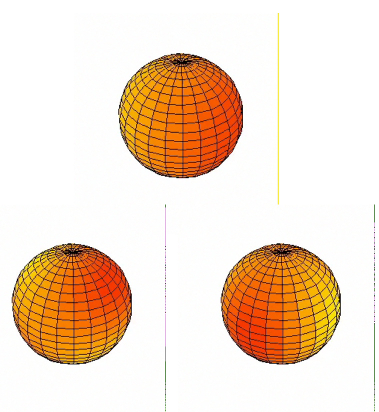


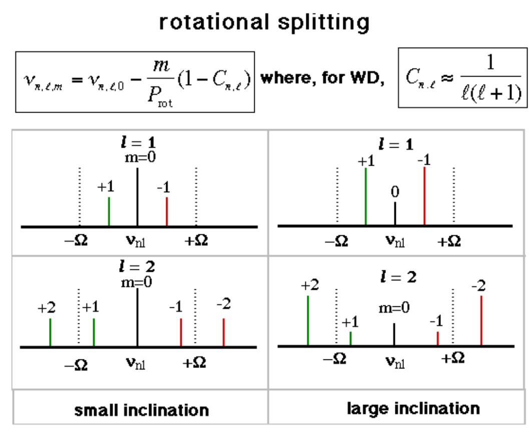
PG 1159 as an example

(Winget et al. 1991)

Rotational splitting of nonradial oscillations

(uniform, slow rotation)

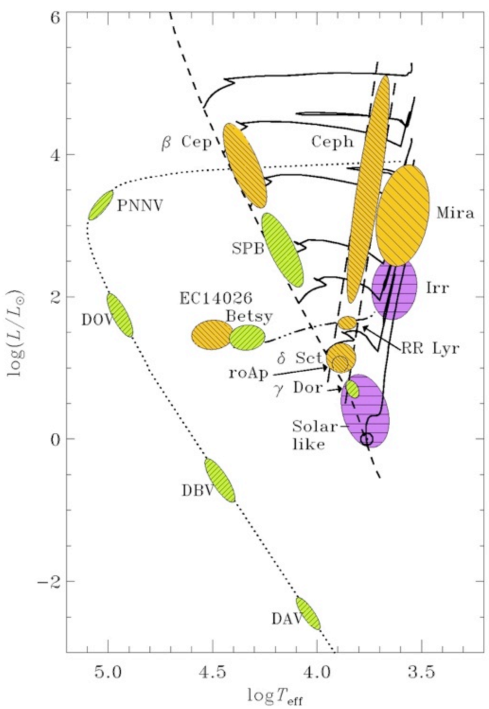


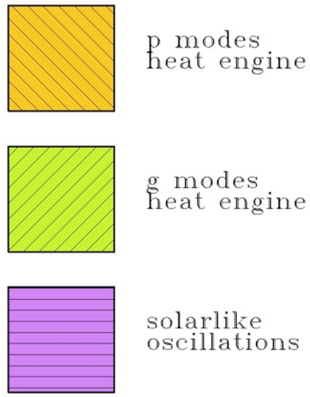


equal frequency spacing: triplets (l=1), quintuplets (l=2) etc.

page

Pulsating stars in the HR diagram





from J. Christensen-Dalsgaard

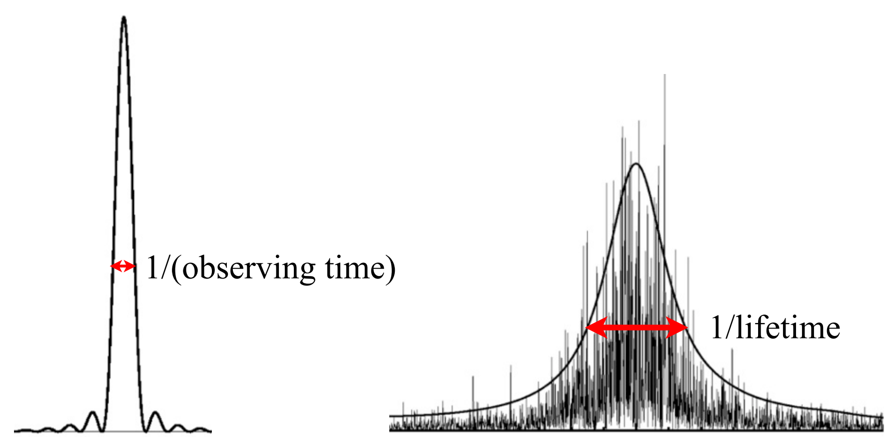
"Solar-like" oscillations

- globally stable (damped) but constantly excited
  - damping time τ generally ~ days
  - continuously re-excited by turbulence
- frequencies locked to normal modes of star
- excited mode periods ~ minutes
- broad mode selection, low amplitude
  - (integrated) velocity amplitude
    < meters / second
  - photometric amplitude
    ~ parts per million

Consequences of stochasitic excitation

- lots of modes present . . . but
- coherence time of (only) days lowers peak amplitudes
  - reduces detectability
- phase instability broadens FT peaks
  - Lorentzian envelope
  - *reduces frequency accuracy*
  - confuses mode identification and rotational splitting effects

Observational Differences

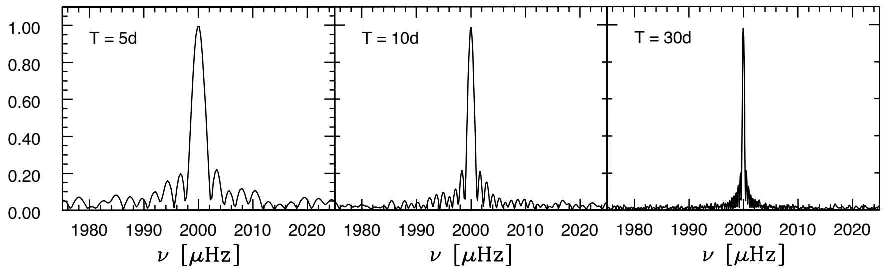


heat-engine mode

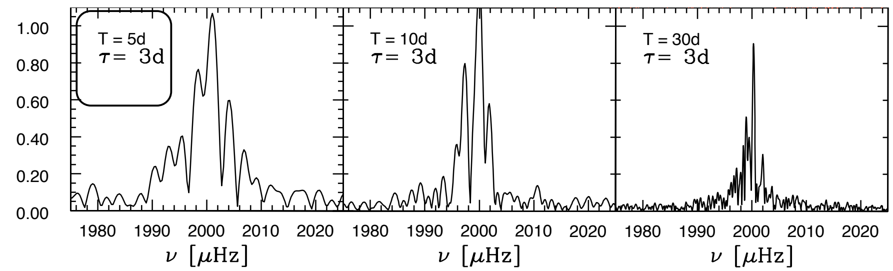
Stochastically excited mode

from J. C.-D.

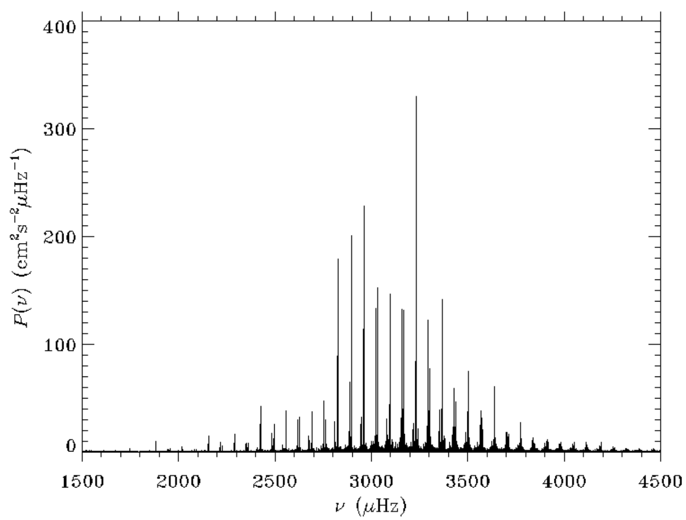
coherent pulsators: frequency precision ~ 1/T



stochastic pulsators - freq. precision poorer than 1/T



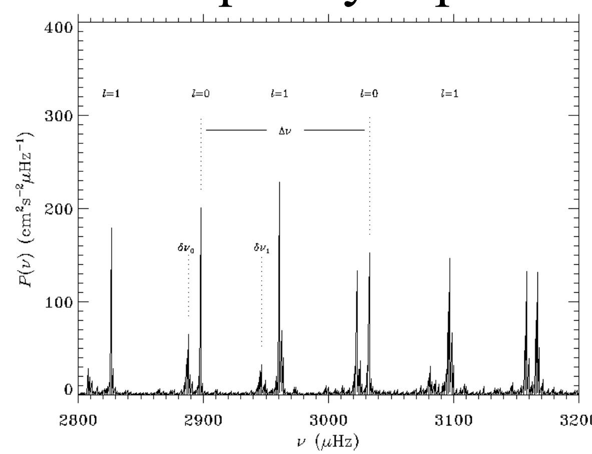
Asymptotics of low-degree p modes



$$\Delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l} \simeq \Delta\nu$$

Small frequency separations

wh

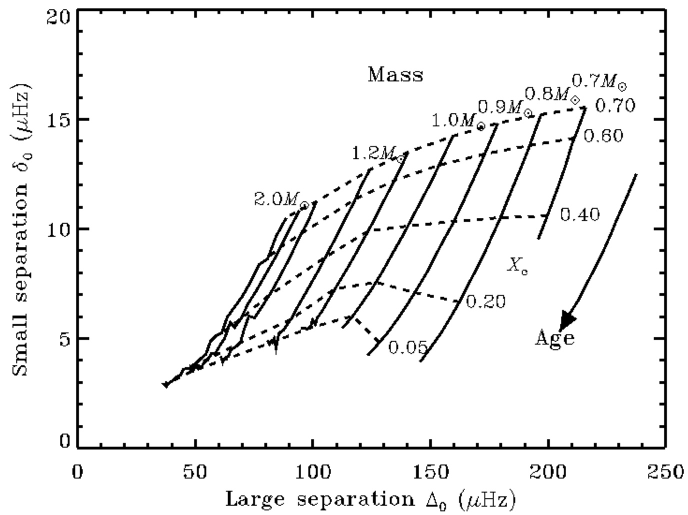


Frequency separations:

$$\delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l+2} \simeq -(4l+6)\frac{\Delta\nu}{4\pi^2\nu_{nl}}\int_0^R \frac{\mathrm{d}c}{\mathrm{d}r}\frac{\mathrm{d}r}{r}$$

$$\delta^{(1)}\nu_{nl} = (\nu_{nl}+\nu_{n+1\,l})/2 - \nu_{n\,l+1} \simeq -(2l+2)\frac{\Delta\nu}{4\pi^2\nu_{nl}}\int_0^R \frac{\mathrm{d}c}{\mathrm{d}r}\frac{\mathrm{d}r}{r}$$

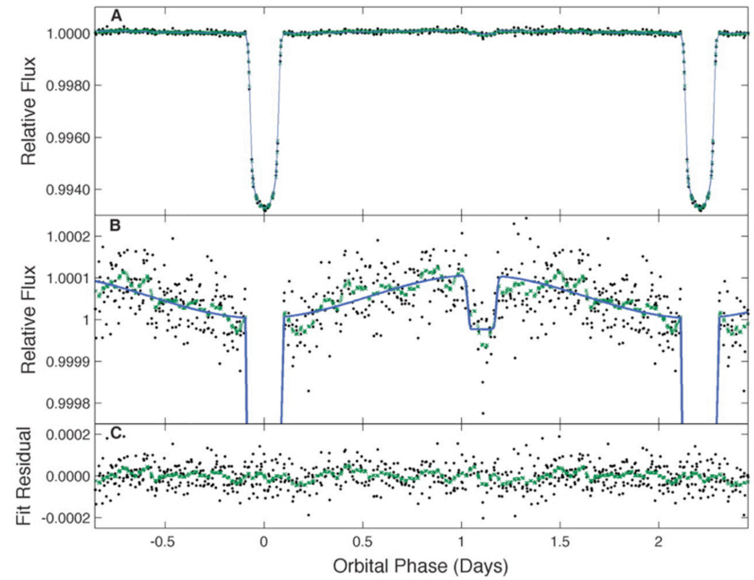
Asteroseismic HR diagram



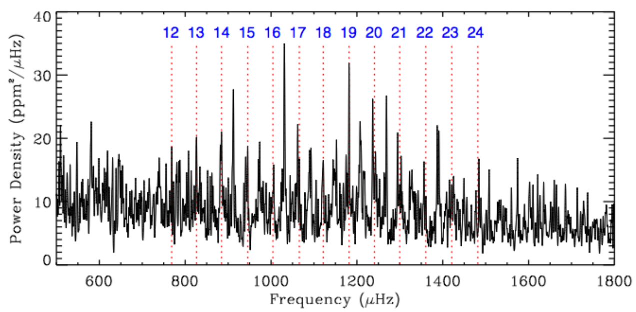
from J. Christensen-Dalsgaard

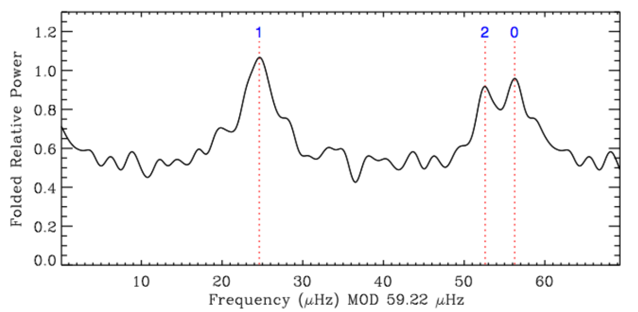
Borucki et al. 2009 -

Kepler's Optical Phase Curve of the Exoplanet HAT-P-7b

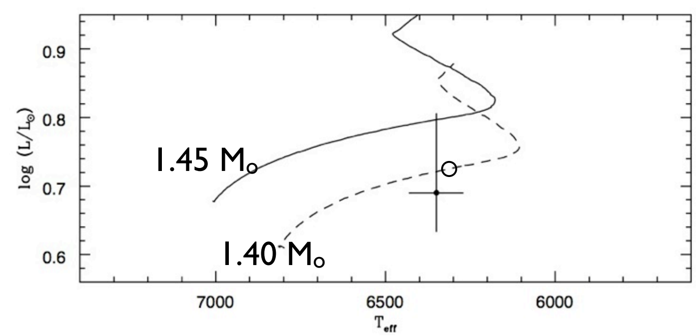


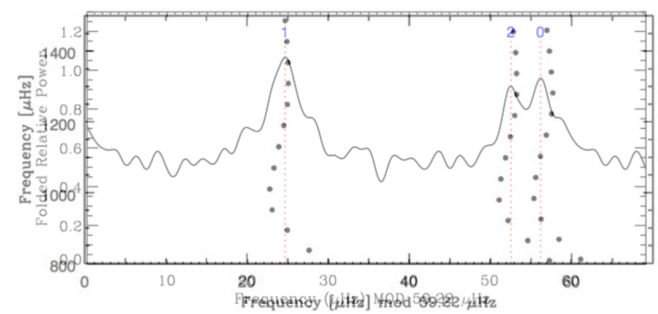
HAT-P7 asteroseismology



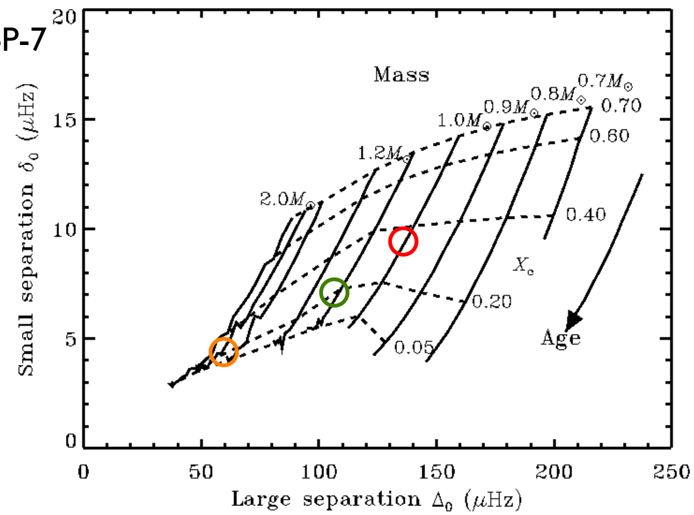


HAT-P7 asteroseismology





Asteroseismic HR diagram



from J. Christensen-Dalsgaard

HAT-P7 asteroseismology

TABLE 2
Stellar parameters for HAT-P-7

Parameter	Value	Source
$T_{\rm eff}$ (K)	6350 ± 80	SME[a]
[Fe/H]............	$+0.26 \pm 0.08$	SME
$v \sin i$ (km s^{-1}) ...	3.8 ± 0.5	SME
$M_\star$ ($M_\odot$)	$1.47^{+0.08}_{-0.05}$	Y^2+LC+SME[b]
$R_\star$ ($R_\odot$)	$1.84^{+0.23}_{-0.11}$	Y^2+LC+SME
$\log g_\star$ (cgs)	$4.07^{+0.04}_{-0.08}$	Y^2+LC+SME
$L_\star$ ($L_\odot$)..........	$4.9^{+1.5}_{-0.6}$	Y^2+LC+SME
M_V (mag)........	3.00 ± 0.22	Y^2+LC+SME
Age (Gyr)........	2.2 ± 1.0	Y^2+LC+SME
Distance (pc).....	320^{+50}_{-40}	Y^2+LC+SME

[a]SME = 'Spectroscopy Made Easy' package for analysis of high-resolution spectra Valenti & Piskunov (1996). See text.
[b]Y^2+LC+SME = Yale-Yonsei isochrones (Yi et al. 2001), light curve parameters, and SME results.

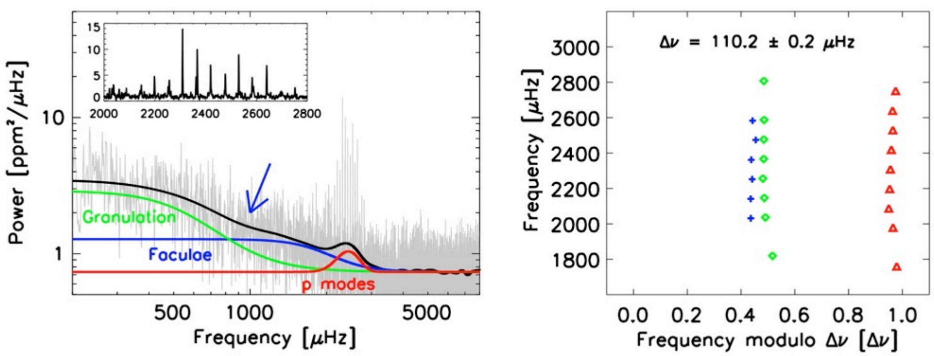
spectroscopy
Pal et al. 2008

M=1.40±0.02
t = 1.6±0.4 Gyr
r = 1.94±0.05 R_{sun}
X_c=0.19

quick seismic fit
ISUEVO

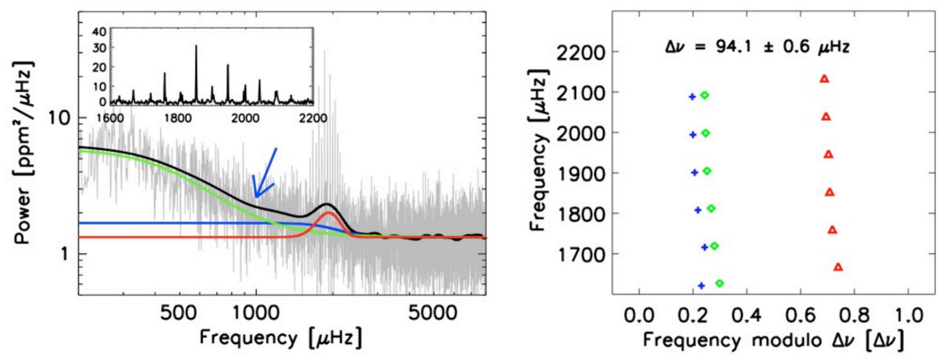
Solar-like pulsators: KIC 6603624

Chaplin et al. 2010



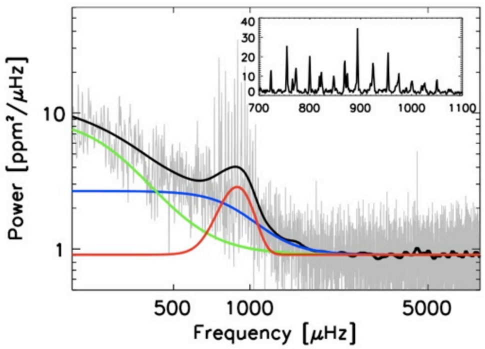
Solar-like pulsators: KIC 3656476

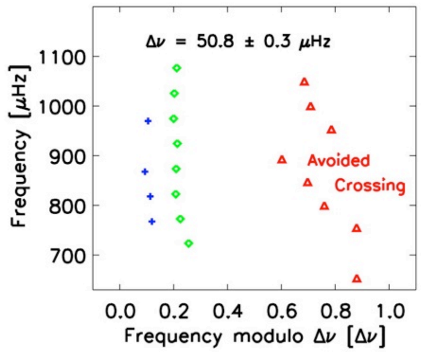
Chaplin et al. 2010



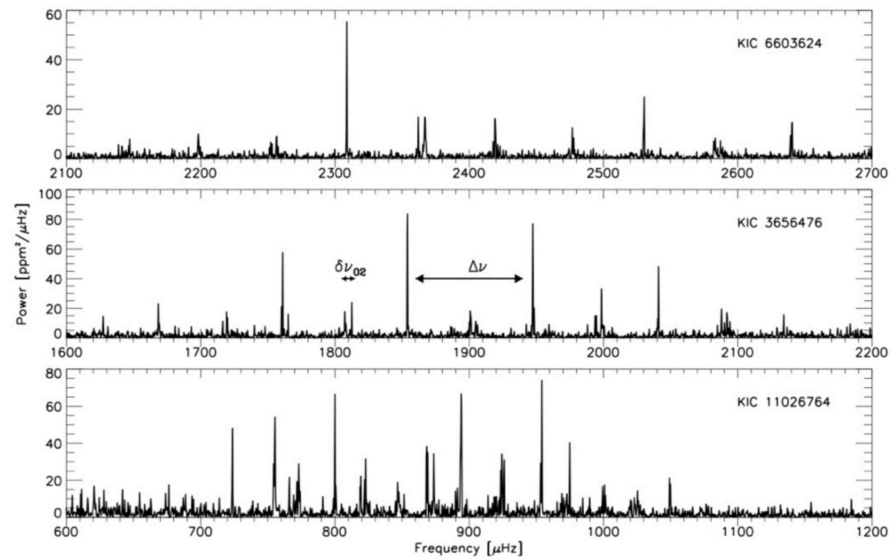
Solar-like pulsators: KIC 11026764

Chaplin et al. 2010

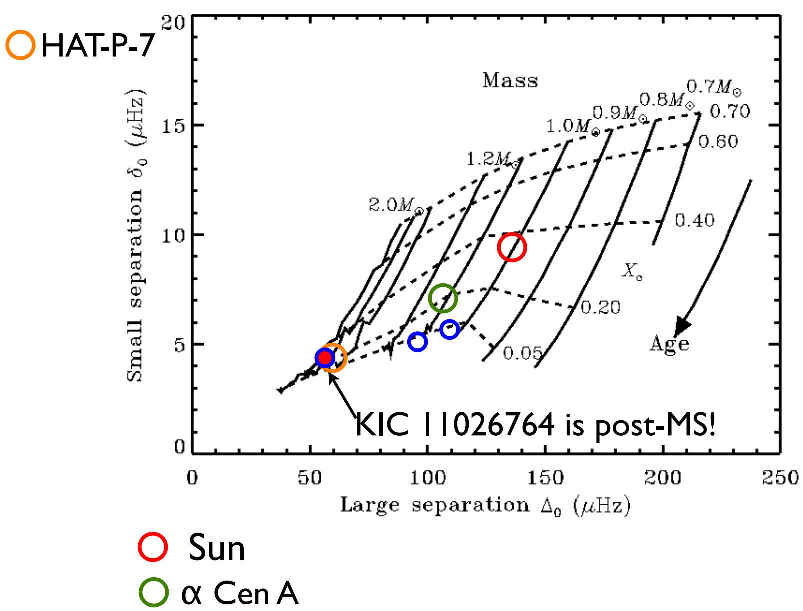




Solar-like - FTs

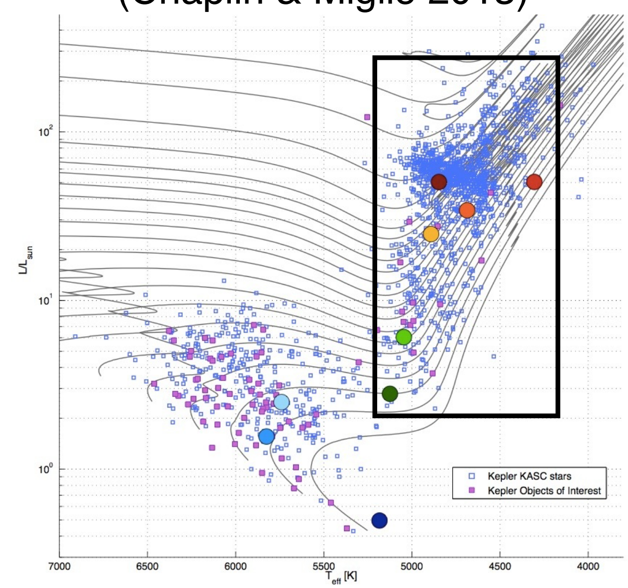


Asteroseismic HR diagram



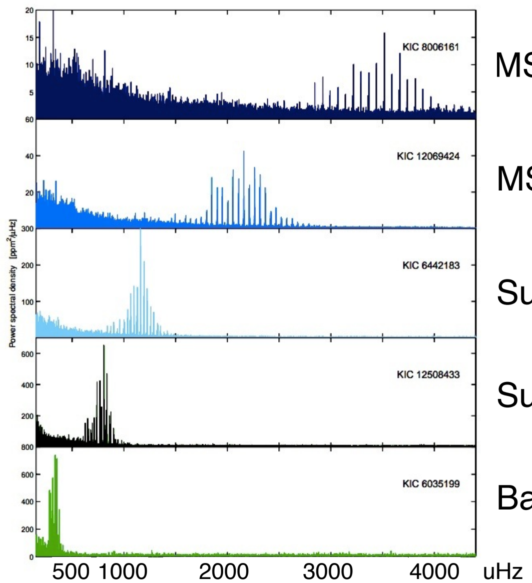
oscillations beyond the MS w/ *Kepler*

(Chaplin & Miglio 2013)



Solar-like oscillations w/ *Kepler*

(Chaplin & Miglio 2013)



MS

MS (16 Cyg A)

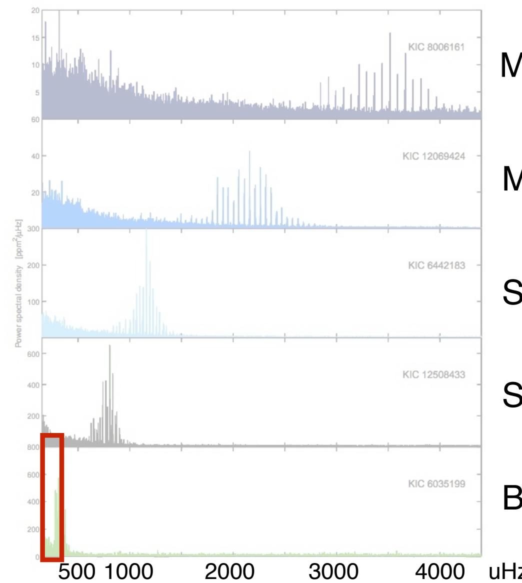
Subgiant

Subgiant

Base of RGB

Solar-like oscillations w/ *Kepler*

(Chaplin & Miglio 2013)



MS

MS (16 Cyg A)

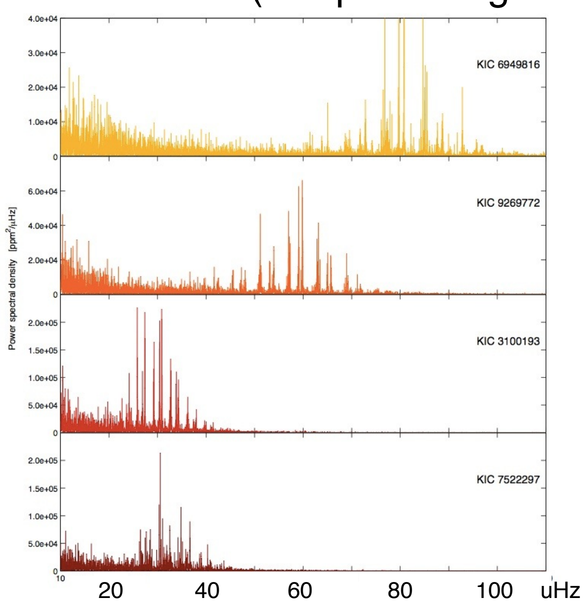
Subgiant

Subgiant

Base of RGB

oscillations beyond the MS w/ *Kepler*

(Chaplin & Miglio 2013)



First-ascent RGB

First-ascent RGB

First-ascent RGB

Red Clump

scaling relations MS to RGB

$$\Delta\nu = \Delta\nu_\odot \sqrt{\frac{M/M_\odot}{(R/R_\odot)^3}}\,\mu\text{Hz}.$$

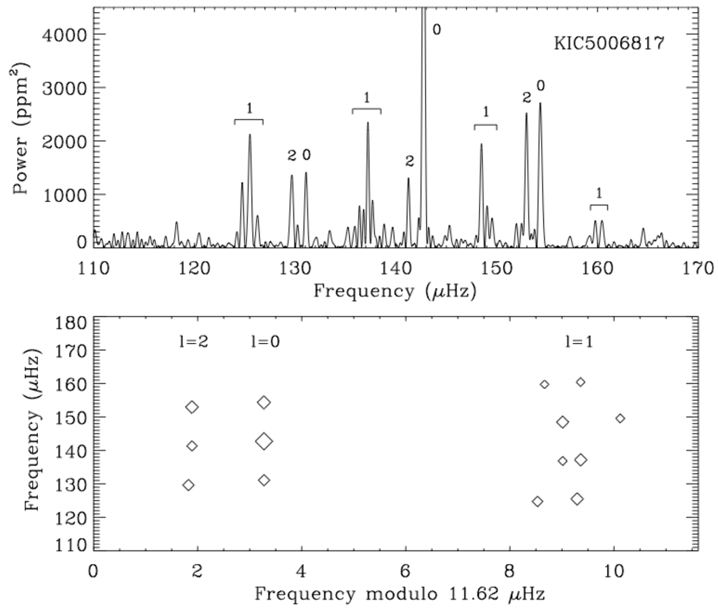
$$\nu_{\max} = \nu_{\max\odot} \frac{M/M_\odot}{(R/R_\odot)^2 \sqrt{T_{\rm eff}/T_{\rm eff\odot}}}\,\mu\text{Hz},$$

	R/Rsun	log g	Δν [uHz]	ν_{max}
MS	1	4.44	135	3300
RGB base	5	3.04	12.1	140
RGB top	20	1.84	1.5	**9**

< 1/d

1/wk

Bedding et al. (2010)

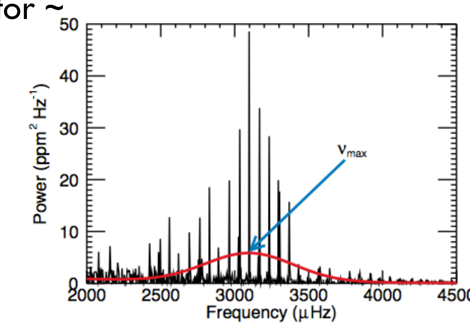


multiple
l=1 modes
per order?!

Ensemble asteroseismology of solar-type stars with Kepler

Chaplin et al. 2011 (*Science*, today!)

- determination of $\Delta\nu$ and ν_{max} for ~ 500 solar-type stars

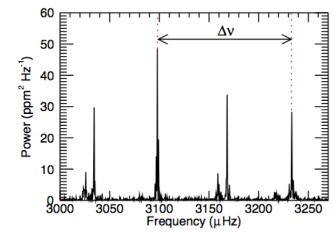


- ν_{max} scales with acoustic cutoff frequency ~ $gT_e^{-1/2}$

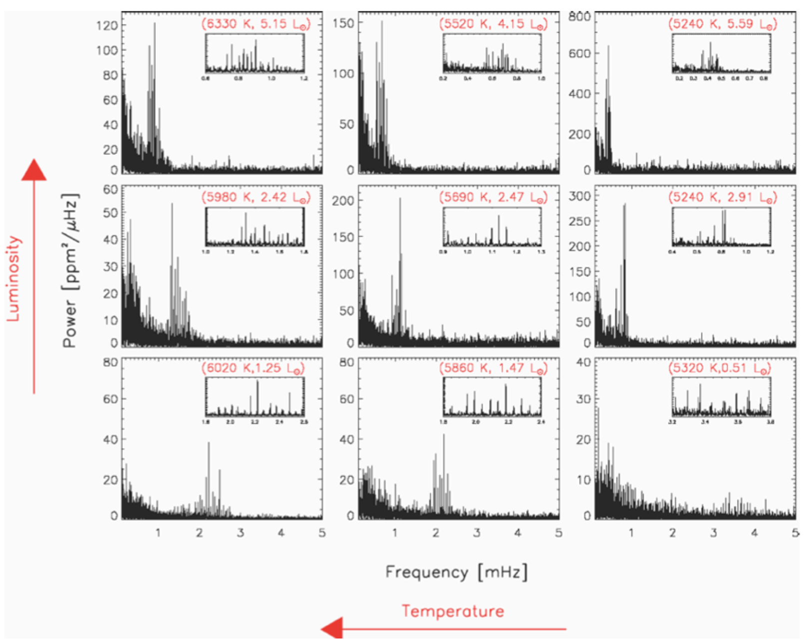
$$\left(\frac{\nu_{max}}{\nu_{max,\odot}}\right) \approx \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-2}\left(\frac{T_{eff}}{T_{eff,\odot}}\right)^{-0.5}$$

- $\Delta\nu$ measures mean density:

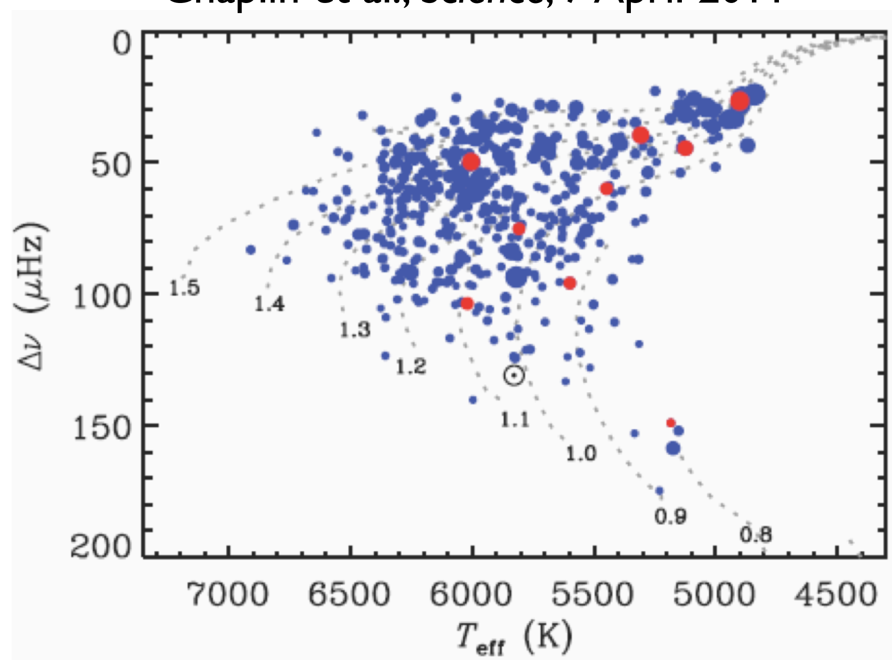
$$\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^2 \approx \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-3}$$



Chaplin et al., *Science*, 7 April 2011



Chaplin et al., *Science*, 7 April 2011



Chaplin et al., *Science*, 7 April 2011

- Δν measures mean density: $\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^2 \approx \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-3}$
- ν_{max} scales with acoustic cutoff frequency ~ $gT_e^{-1/2}$
$$\left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right) \approx \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-2}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{-0.5}$$
- then one can determine M and R:
$$\frac{R}{R_\odot} \approx \left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right)\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-2}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{0.5}$$
$$\frac{M}{M_\odot} \approx \left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right)^{3}\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-4}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{1.5}$$
- and, with T_{eff} from multicolor photometry, one gets L

Chaplin et al., *Science*, 7 April 2011