ERRATA

This is the errata sheet for the revised edition of "Second Order Parabolic Differential Equations". The errata sheet for the first edition is at http://lieb.public.iastate.edu/book/errata.pdf.

page 11 line -14 "to" should read "To".
line -1 Y should read Y_0.
page 40 line -14 $\overline{\Omega^-[,X_0,r]}$ should read $\overline{\Omega[X_0,r]}$.
page 55 line -6 4.4 should read 4.5
page 65 line 16 G should read Ψ.
line -11 $(F+G)$ should read F.
page 80 line -12 $(1+\alpha f)$ should read $(1+\alpha)f$.
line -1 Mw should read Mw_1.
page 87 In (5.1), b^iu should read b^iD_iu.
page 92 line -7 $Omega$ should read Ω.
page 102 line -13 Du should read Dv.
page 103 line -7 $+f^i$ should read $-f^i$.
line -2 $+f^i$ should read $-f^i$.
page 130 line 12 The definition of m_4 should be

$$m_4 = \inf_{Q(4r)} u.$$

page 138 line -4 b^0u should read b^0z.
line -3 (Λ_1+1) should read $\lambda(\Lambda_1^2+1)$.
page 139 line -11 $Q[2R]$ should read $\Omega[2R]$.
page 153 In Exercise 6.2, the definition of $\|u\|_{p,q}$ should read

$$\|u\|_{p,q} = \left(\int_{I(\Omega)}\left(\int_{\Omega(t)}|u(X)|^p\ dx\right)^{q/p}\ dt\right)^{1/q}.$$

page 154 In Exercise 6.5, "Exercise 6.2(c)" should read "Exercise 6.2(b)".
page 160 line 9 The right hand side should read $\tau|K|$.
page 162 line 2 $C(p,q,)$ should read $C(p,q)$.
line 13 $|t-t_0|$ should read $|t-t_0|^{1/2}$.
page 163 line 3 $|t-t_1|$ should read $|t-t_1|^{1/2}$.
page 168 Proposition 7.14 should read : Let $p\ge 2$, let λ and Λ, let (A^{ij}) be a constant matrix satisfying (7.17) for all $\xi\in\mathbb{R}^n$, and let u be a weak solution of (7.18) with $f\in L^p(Q(X_0,36R))$ for some $X_0\in\Omega$ and $R\in(0,d(X_0)/36)$, and with $u\in L^p(Q(X_0,36R)$. Then $Du\in L^p(Q(X_0,R))$, and there is a constant C, determined only by λ, Λ, and n such that (7.25) holds.
line 13 There should be a factor R^{-1} in front of $\|u\|_{p,Q(X_0,36R)}$.
line -12 replace $R^{(p-2)/(2p)}$ with $|Q(X_0,R)|^{(p-2)/(2p)}$.
page169 line -3 M_{ip} should read M^{ip}.
delete line -1 and add a period at the end of new line -1

page 170 Replace lines 1 through 4 with:
We now note that $M \geq 2$ and hence

$$M^{jp}|\{\mathcal{M}f^2 > M^{2j}\delta^2\theta^2\}| = \frac{1}{M^j - M^{j-1}} \int_{M^{j-1}}^{M^j} M^{jp}|\{\mathcal{M}f^2 > M^{2j}\delta^2\theta^2\}|\, d\sigma$$
$$\leq M^{1-j} \int_{M^{j-1}}^{M^j} M\sigma^p|\{\mathcal{M}f^2 > \sigma^2\delta^2\theta^2\}|\, d\sigma$$
$$\leq M^2 \int_{M^{j-1}}^{M^j} \sigma^{p-1}|\{\mathcal{M}f^2 > \sigma^2\delta^2\theta^2\}|\, d\sigma.$$

It then follows that

$$\begin{aligned} I_2 &\leq M \int_0^\infty \sigma^{p-1}|\{\mathcal{M}f^2 > \sigma^2\delta^2\theta^2\}|\, d\sigma \\ &= \frac{M}{2(\delta\theta)^2} \int_0^\infty h^{(p/2)-1}|\{\mathcal{M}f^2 > h\}|\, dh \\ &= \frac{M}{2(\delta\theta)^2} \int_{Q(X_0,6R)} (\mathcal{M}f^2)^{p/2}\, dX. \end{aligned}$$

From Lemma 7.9 with $p/2$ in place of p, we have

$$\int_{Q(X_0,6R)} (\mathcal{M}f^2)^{p/2}\, dX \leq C(n,p) \int_{Q(X_0,6R)} (f^2)^{p/2}\, dX = C(n,p) \int_{Q(X_0,6R)} |f|^p\, dX.$$

line 6 Replace F with F^p.

page 172 line -10 L^∞ should read L^q and $\|Du\|_p$ should read $\|Du\|_p^{p-1}$.

page 173 line -3 Dv_m should read D^2v_m.

page 175 line 4 Ω should read ω.

page 178 Proposition 7.25 should read

Let λ and Λ be positive constants and let (A^{ij}) be a constant matrix satisfying (7.17) for all $\xi \in \mathbb{R}^n$. Let $X_0 \in \mathbb{R}^{n+1}$, let $R > 0$, and set $\Omega = \Sigma^+(X_0, 6R)$. Suppose u is a weak solution of

$$Lu = D_i f^i \text{ in } \Omega, \quad u = 0 \text{ on } \sigma(X_0, 6R), \quad a^{nj}D_ju = f^n \text{ on } \Sigma^0(X_0, 6R), \tag{7.41}$$

and let $X_1 \in \overline{\Omega}$. There there is a positive constant $M(\lambda, \Lambda, n)$ and, for each $\varepsilon > 0$, there is a constant $\delta(\lambda, \Lambda, n, \varepsilon)$ such that, if (7.29) holds for some $\theta > 0$, then (7.30) is valid.

line 16 add "in which $\beta^i = A^{ni}$, so that μ and $\bar{\mu}$ are determined by λ, Λ, and n."

In the proof of Proposition 7.25, delete the second paragraph.

page 179 line 17 7.25 should read 7.23.

line 20 should read

$$v_t + D_j(A^{ij}D_iv) = D_jg^j \text{ in } \Omega, v = 0 \text{ on } \sigma, A^{jn}D_jv = g^n \text{ on } \Sigma^0.$$

page 180 Replace the proof of Proposition 7.28 to read

We argue as in Proposition 7.18 (but with Proposition 7.24 in place of Proposition 7.17) to obtain L^p estimates for $D_{ij}u$ with $i + j < 2n$. With u_m, η, v_m, and $\tilde{f}_m$ as in the proof of Proposition 7.18, we note that we have

$$-v_{m,t} + A^{ij}D_{ij}v_m = \tilde{f}_m$$

on $\Sigma^0(X_0, 2R)$. Since $v_{m,t} = 0$ and $D_{ij}v_m = 0$ (for $i < n$ and $j < n$) on $\Sigma^0(X_0, 2R)$, we infer that

$$\tilde{A}^{nj} D_j v_m = \tilde{f}_m$$

on $\Sigma^0(X_0, 2R)$ with

$$\tilde{A}^{ij} = \begin{cases} A^{ij} & \text{if } i < n, j < n, \\ A^{in} + A^{ni} & \text{if } i < n, j = n, \\ 0 & \text{if } i = n, j < n, \\ A^{nn} & \text{if } i = j = n. \end{cases}$$

In addition, we have

$$-v_{m,t} + D_i(\tilde{A}^{ij} D_j v_m) = \tilde{f}_m$$

in Ω. If we now set $w = D_n v_m$ and $f^i = \delta^{in} \tilde{f}_m$, it follows that

$$-w_t + D_i(\tilde{A}^{ij} D_j w) = D_i f^i \text{ in } \Omega,$$
$$w = 0 \text{ on } \sigma(X_0, 2R), \tilde{A}^{nj} D_j w = f^n \text{ on } \Sigma^0(X_0, 2R),$$

and then Proposition 7.26 implies that

$$\|Dw\|_{p,\Omega} \le C \|\tilde{f}_m\|_{p,\Omega}.$$

Since $|D_{nn} v_m| \le |Dw|$, we infer as before that

$$\|D^2 v_m\|_{p,\Omega} \le C \|\tilde{f}_m\|_{p,\Omega}.$$

The proof is completed as in Proposition 7.18.
The right hand side of line -10 should read

$$C\left(\|f\|_{p,2R,T} + R^{-1}\|Du\|_{p,2R,T}\right)$$

page 183 The proof of Proposition 7.33 should read:
Set

$$\tilde{A}^{ij} = \begin{cases} A^{ij} & \text{if } i, j < n, \\ A^{nn}\beta^j/\beta^n & \text{if } i = n, j \le n, \\ A^{jn} + A^{nj} - A^{nn}\beta^j/\beta^n & \text{if } i < n, j = n. \end{cases}$$

Now, fix $X_1 \in Q^0(X_0, R/2)$ and let η be a $C^{2,1}$ function which vanishes along with its gradient on $\mathcal{P}Q((x_0', -\bar{\mu}R/2, t_0), R/2)$ such that $\eta(X_1) \ge \frac{1}{2}$. It follows that $w = D_k(\eta u)$, $k = 1, \dots, n-1$, is a weak solution of

$$-w_t + D_i(\tilde{A}^{ij} D_j w) = D_i f^i \text{ in } \Sigma^+(X_1, R/2),$$
$$w = 0 \text{ on } \sigma(X_1, R/2), \tilde{A}^{nj} D_i w = 0 \text{ on } \Sigma^0(X_1, R/2)$$

for

$$f^i = \delta^{ik}[\eta f + uL\eta + 2A^{ij} D_i \eta D_j u].$$

Since $f^n = 0$, we now apply Proposition 7.26 to w and then Proposition 7.24 to $\beta \cdot D(\eta u)$.

In this way, we estimate $\|D^2 u\|_{p,\Sigma_1}$, where Σ_1 is the subset of $Q^+(X_0, R/2)$ on which $x^n < \varepsilon R$ for a suitable positive constant ε, determined only by $\bar{\mu}$. Application of Proposition 7.18 gives an estimate of $\|D^2 u\|_{p,\Sigma_2}$ with $\Sigma_2 = Q^+(X_0, R/2) \setminus \Sigma_1$.

page 187	Condition (7.51) should read $$\|\{X \in Q(X_0, r) : \bar{u} < h\}\| < \zeta \|Q(r)\|$$
page 195	In equation (7.68) $Q(R/2)$ should read $Q^+(R/2)$ and Q should read Q^+.
	line -1 Q should read Q^+.
page 219	In the statement of Theorem 9.1, "increasing positive constant k such that $a(X, z, p) + k(M)z$" should read "increasing positive function k such that $a(X, z, p) - k(M)z$".
page 220	line -10 k and b_1 are nonnegative.
page 227	line 3 "Theorem" should read "Theorems".
page 245	line 8 a^{ij}_∞ should read a_∞.
	line 13 $\kappa_0 \le 0$ should read $\kappa_0 \ge 0$.
page 246	line 4 a^{ij} should read a^{ij}_∞.
	line -3 $\kappa_0 \le 0$ should read $\kappa_0 \ge 0$.
page 248	line -3 $p_0\delta$ should read $m_1 + p_0\delta$.
page 249	line 4 Ω_1 should read Ω_2.
	line 13 Ω_1 should read Ω_2.
	line -6 x should read x_0.
	line -5 x should read x_0.
page 250	line 12 R should read $R - \varepsilon$.
page 251	line -7 "Section X.5" should read "Section X.4".
page 252	line 10 $d\tau$ should read ds.
page 259	line -1 $\psi' a^{ij} D_{ij}\bar{u}$ should read $a^{ij} D_{ij}\bar{u}$.
page 270	line -13 "elliptic" should read "parabolic".
page 273	line 13 should read $$\int_0^R \rho^{-n}\bar{\psi}(\rho)\,d\rho < \frac{2}{4^n R}\int_0^R \rho^{-n}\bar{\varphi}(4\rho)\,d\rho \le \frac{S}{2} + \frac{1}{2R}\int_R^\infty s^{-n}\bar{\varphi}(s)\,ds.$$
	line -10 should read $$S \le 2\omega_n\left(2^{-n} + \frac{1}{2^n(n-1)}\right) \le \omega_n.$$
page 279	line 7 There should be a factor of 2 in front of the integral.
	line 13 There should be a factor of 2 in fronot of $\|\zeta\zeta_t\|$.
page 297	In Exercise 11.2, the estimate $$\|u(X) - u(Y)\| \le L_0\|X - Y\|$$ only needs to be satisfied for $X \in \Omega$ and $Y \in \mathcal{P}\Omega$.
page 298	In Exercise 11.6, "Lemma 11.15" should read "Lemma 11.10".
	In Exercise 11.7, "Lemma 11.15" should read "Lemma 11.11".
	In Exercise 11.8, a_1 should satisfy (11.63).
page 306	line -12 C_1 should read C.
page 315	(12.26) should read $$k\|z\|^2 + b_1.$$
	line 10 ((12.25a)′) should read (12.25a)′.
	line -13 $\|Du\|$ should read $\|Du\|^2$.
page 317	line 5 After "If", add "there are nonnegative constants b_0, b_1, and M such that"
	In (12.32), $\eta/(2R_1)$ should read $\|p\|\eta/(2R_1)$.

Add the hypotheses (12.26) and "a^{ij} is continuously differentiable" to Theorem 12.21.

page 318 Add the hypotheses (12.6) and "$a^{11} \in H_\alpha(K)$ for any compact subset K of $\Omega \times \mathbb{R} \times \mathbb{R}$" to Theorem 12.25.

page 336 line -15 In the first integral on the right hand side, dX should read dx.

page 338 In line 4, the correct expression for I_b is $I_b = \int_0^T I_0(t)\,dt$.

page 339 In Example 2.1, the condition $|B| \le \theta_0 v$ is superfluous because $|p \cdot B_p| \le \theta_3/v$ implies that, for each $(X,z) \in \Omega \times \mathbb{R}$ and each unit vector ξ, the function f, defined by $f(s) = B(X,z,s\xi)$ satisfies the inequality $|f'(s)| \le \theta_3/s^2$ so $|f(s)| \le |f(1)| + \theta_3$ and hence $|B|$ is bounded.

page 340 line 15 The region of integration should read

$$\Omega \cap \{|x - x_0| < 2\rho, v_1 \ge \tau\}.$$

page 341 line 9 The correct expression for λ_0 is $\lambda_0 \equiv \max\{1, \theta_0\}$.

page 346 Remove (13.43) from the hypotheses of Lemma 13.14 and Theorem 13.15.

page 354 line 15 Replace "Proposition 13.20" by "Lemma 13.21".

page 381 line 5 should read

$$|Du - Du(0)|^{1+\alpha} \le |u|_2 K_\varepsilon^\alpha R^{1+\alpha\varepsilon}$$

line 6 $R^{\alpha\varepsilon}$ should read $R^{2+\alpha\varepsilon}$.

page 391 line -14 $+L(u - \underline{u})$ should read $-L(u - \underline{u})$

line -12 should read

$$L(u - \underline{u}) \le c_1[u - \underline{u}] - \varepsilon_1[F_\tau + \sum_i F^{ii}] - \delta_0 + \sigma(\varepsilon_1).$$