Computational Framework for Dense Sensor Network Evaluation Based on Model-Assisted Probability of Detection

by Jin Yan*, Simon Laflamme†‡, and Leifur Leifsson§

ABSTRACT

Recent sensing advances empower the deployment of dense sensor networks (DSNs) that can be used in automating the condition assessment process of large-scale structural and mechanical systems. To fully enable DSN technologies, it is critical to develop and implement co-design techniques that allow for the evaluation of their performance to attain the condition assessment targets. In this paper, we propose a technique to validate the design of DSNs. The technique consists of constructing a physical surrogate of the DSN-equipped component based on the sensor configuration, updating the physical surrogate using the sliding mode theory based on generated or collected sensor data, and quantifying the performance of the DSN based on model-assisted probability of detection theory. The proposed technique is numerically verified and validated on a cantilever beam subjected to damage at its root and equipped with a network of soft elastomeric capacitors measuring strain. Various uncertainties are considered in the simulated system. The verification consists of confirming the capability of the adaptive process for the surrogate model at reaching an accurate representation of the full system. The validation consists of ranking the performance of various DSN configuration and benchmarking results against those obtained from the full finite element model. Results show that the proposed technique can be used to evaluate the performance of DSN configurations, but that the damage thresholds used in determining damage need to be standardized for successful field applications.

KEYWORDS: nondestructive evaluation, structural health monitoring, dense sensor network, damage detection, probability of detection, physical surrogate

Introduction

Structural health monitoring (SHM) is the automation of the structural integrity assessment task conducted using sets of measurements from permanently mounted sensors (Lynch et al. 2016). Of interest to this paper are SHM systems based on dense sensor networks (DSNs), which have recently been empowered from advances in smart materials and signal processing. The promise of DSNs lies in their high spatial resolution that yields rich spatio-temporal information about the monitored component (Ubertini et al. 2017). If properly harnessed through strategically tailored signal processing algorithms, such information could dramatically improve structural condition assessment capabilities, directly enabling condition-based maintenance decisions.

Similar to other SHM solutions, linking sensor measurements to decision is a difficult task. This task is ever more complex with DSNs because of the large amount of temporal data acquired over a high spatial resolution. In addition, several complexities in the monitored system may affect the performance of the damage detection capabilities, including weak and/or noisy sensor signal, parametric uncertainties in the modeled system, and nonstationarity of the dynamics. To address these challenges, data-driven signal processing...
techniques (Downey et al. 2018b; Liao et al. 2019) have been proposed to detect damage without relying on physical knowledge, but results often fail to provide a clear link to physical parameters. The injection of physical knowledge in data-based techniques gives rise to hybrid approaches (Downey et al. 2019) that have shown promise at assessing structural conditions in a more useful manner.

In this paper, hybrid procedures are leveraged to provide an integrated approach to evaluating DSNs, therefore empowering condition-based maintenance through the improved efficiency of the SHM system. The concept is based on the probability of detection (POD) theory used in the field of nondestructive evaluation (NDE). POD is generally used to evaluate the capability of an NDE technique to detect particular damage under sources of uncertainties. POD curves in NDE applications were initially based on probabilistic techniques and required the generation of multiple sets of repeated measurements on realistic specimens, which is a very costly procedure (Forsyth 2016). Model-assisted POD (MAPOD) has been proposed to complement limited empirical experiments with physical information (Memmolo et al. 2016; Du et al. 2018a). Some studies (Aldrin et al. 2011; Moriot et al. 2018) have extended to the concept of MAPOD for SHM applications, where a particular challenge is to quantify the performance of multiple sensors monitoring multiple types of damage.

A key difficulty in applying MAPOD is in the creation of a physical representation. A popular technique in SHM is to create a high-fidelity physical representation and update it based on measurements. Many applications of such model-updating techniques are found in vibration-based SHM where the physical representation is updated to match modal parameters extracted from measurements (Cancelli et al. 2019; Li et al. 2018; Cui and Lanza di Scalea 2019). However, this technique is difficult to apply because it requires complex optimization and thus requires large computational time, and it can be ill-conditioned when there are more unknown parameters when compared to available data (Benner et al. 2017; Peherstorfer et al. 2018). In MAPOD applications, it was discussed that these high-fidelity models remain computationally intensive and usually lack robustness with respect to parameter variations (Amsallem and Farhat 2008). A solution is to leverage iterative procedures adopting simplified model representation for near real-time applications, but at the cost of lower accuracy (Benner et al. 2015).

In this paper, a MAPOD-based technique for the evaluation and design of DSNs is presented, using the application example of a strain-based DSN developed by the authors (Laflamme et al. 2015). The technique consists of creating a physical surrogate model of the monitored system and using the surrogate to develop a number of scenarios based on uncertainties and damages of interest. Instead of laboratory-produced data, which is common in MAPOD-based techniques, this research uses synthetic signals produced through numerical simulations. This is due to the early development stage of studied sensing technology, which limits laboratory testing capabilities at this stage. In the numerical simulations, different uncertainty scenarios are considered, and the results are assembled in the form of POD curves to quantify and rank the performance of a given DSN configuration. The presented work is an extension to preliminary findings published in conference proceedings (Yan et al. 2018; Du et al. 2018b; Yan et al. 2019), where the authors now validate the proposed technique on a realistic model and formalize the study by using MAPOD to validate the performance of various DSN configurations under different uncertainty scenarios.

**Background**

This section provides a background on the sensors used in this study, followed by an overview of model adaptation and the model-assisted probability of detection techniques.

![Figure 1. Soft elastomeric capacitor (SEC): (a) photo; (b) schematic with key components annotated.](image)
SEC-Based Sensing Skin

In this paper, the DSN of interest consists of densely deployed flexible electronics termed as soft elastomeric capacitors (SECs), previously proposed by the authors. The SEC is a low-cost, flexible sensor constituted from a block copolymer matrix (SEBS) mixed with titania to form the dielectric, and with carbon black to form the electrodes. Details on the fabrication process and derivation of the electromechanical model can be found in the literature (Laflamme et al. 2013; 2015). This sensing skin has the capability to scale to different sizes and to cover large areas, which is suitable for strain sensing over large-scale surfaces. Figure 1a shows a picture of an SEC measuring 76 mm \( \times \) 76 mm. The sensing principle is based on a change in the sensor’s geometry, provoking a measurable change in capacitance. The capacitance \( C \) of an SEC can be estimated from the model of a parallel plate capacitor:

\[
C = \frac{\varepsilon_0 \varepsilon_r A}{h}
\]

where 
- \( \varepsilon_0 \) is the vacuum permittivity,
- \( \varepsilon_r \) is the relative permittivity,
- \( A = d \times l \) is the sensing area with width \( d \) and length \( l \), and 
- \( h \) is the thickness of the dielectric, as illustrated in Figure 1b. 

Assuming small strains, a relative change in capacitance can be obtained by differentiating Equation 1:

\[
\frac{\Delta C}{C_0} = \frac{\Delta d}{d_0} + \frac{\Delta l}{l_0} - \frac{\Delta h}{h_0} = \varepsilon_x + \varepsilon_y - \varepsilon_z
\]

(2)

where 
- \( \nu \) is the Poisson’s ratio,
- \( \lambda = 2 \) is the gauge factor, and 
- \( \varepsilon_x, \varepsilon_y, \) and \( \varepsilon_z \) are the strains along the X, Y, and Z planes, respectively.

It follows from Equation 2 that the SEC measures the average additive in-plane strain of the material located underneath the bottom electrode.

Benchmark Model

The example system used in this study is known as DROPBEAR (dynamic reproduction of projectiles in ballistic environments for advanced research) (Joyce et al. 2018a). It was selected due to the availability of experimental data that can be used to construct an accurate numerical model for simulation purposes. It features a large, rectangular steel cantilever beam of length \( l_x = 505 \) mm, width \( l_y = 51 \) mm, and thickness \( l_h = 6.3 \) mm, as illustrated in Figure 2. Its material properties are taken as follows: Young’s modulus \( E = 190 \) GPa, density \( \rho = 7970 \) kg/m³, and Poisson’s ratio \( \nu = 0.3 \). The study starts with the beam virtually equipped with five SECs with the root of the beam elastically restrained. The boundary element has a width \( l_{bc} = 5 \) mm and

![Figure 2. Degrees of freedom notation for one element: (a) physical surrogate representation; (b) degrees of freedom notation for one element.](image)

![Figure 3. Finite element software model of DROPBEAR.](image)
length \( l_h = 51 \text{ mm} \) (Figure 3). The equation of motion governing the system can be written as

\[
M \ddot{\mathbf{q}}(t) + C \dot{\mathbf{q}}(t) + K \mathbf{q}(t) = B \mathbf{f}(t)
\]

where
- \( t \) denotes time,
- the dot is a time derivative,
- \( \mathbf{q}(t) \in \mathbb{R}^{n \times 1} \) is the displacement vector,
- \( M \in \mathbb{R}^{n \times n} \), \( C \in \mathbb{R}^{n \times n} \), and \( K \in \mathbb{R}^{n \times n} \) are the mass, proportional damping, and stiffness matrices, respectively,
- \( B \in \mathbb{R}^{n \times n} \) is the force application vector, and
- \( \mathbf{f}(t) \in \mathbb{R}^{n \times 1} \) is the vector of external forces for an \( n \) degrees of freedom (nDOF) representation.

To create a physical surrogate, a reduced-order finite element model is discretized as a function of the number of sensors and their sizes, plus an additional element linking to the fixity. Each element of the cross-sectional area \( \varepsilon \) (5) and the state space formulation adopted:

\[
\mathbf{X} = \mathbf{A} \mathbf{X} + \mathbf{B} \mathbf{f}
\]

\[
\begin{align*}
\mathbf{A} &= \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \\
\mathbf{B} &= \begin{bmatrix} 0 \\ -M^{-1} \end{bmatrix}, \\
\mathbf{H} &= \begin{bmatrix} 0 \\ \mathbf{H}_e \end{bmatrix}, \\
\mathbf{H}_e &= \frac{l_h}{2} \begin{bmatrix} 0 & 101 \end{bmatrix}^T
\end{align*}
\]

where
- \( A_e \) is the cross-sectional area for each element,
- \( l_h \) is the length,
- \( I \) is the moment of inertia, and
- \( \beta = 3e – 05 \) is the stiffness proportionality term.

The displacement and velocity feedback can be obtained from the measured strain by fitting and integrating spatial strain measurements using a kth order polynomial function, \( \varepsilon = \xi_0 + \sum_{j=1}^{k} \xi_j \alpha^j \). The fitting coefficients \( \xi_j \) are obtained through a least-squares estimator:

\[
\xi = \left(WW^T\right)^{-1} W^T \mathbf{e}
\]

where

\[
W = \begin{bmatrix} 1 & \mathbf{x}_1^2 & \cdots & \mathbf{x}_3^2 \\ 1 & \mathbf{x}_2^2 & \cdots & \mathbf{x}_5^2 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mathbf{x}_5^2 & \cdots & \mathbf{x}_3^2 \end{bmatrix}
\]

Integrating Equation 10 and assigning the proper boundary conditions yield an expression for \( \mathbf{q} \):
**Methodology**

This section presents the model adaptation technique used to parametrize the physical surrogate and ameliorate the quality of the representation. After, the adapted model is used to conduct MAPOD.

**Model Adaptation**

The parametrization of DROPBEAR’s dynamic model is relatively simple given known material geometries and dimensions, except at the root where the beam may not be perfectly fixed but instead partially restrained. Here, the model is adapted by altering the bending rigidity at the first element (at the root) using experimental data. This adaptation is conducted by sequentially updating the model using the sliding mode theory. Note that, while this could easily be conducted heuristically given the simplicity of the model, the technique presented herein extends to higher-dimensional problems. Note that the performance of the model adaptation algorithm is expected to reduce with the increasing number of varying parameters. Taking its time derivative and substituting Equation 9, 12, and 13 yields

\[ V = \frac{1}{2} (s^2 + \Gamma_0 s \dot{\theta}^T) \]

where \( \Gamma_0 = \begin{bmatrix} \Gamma_0 & 0 \\ 0 & \Gamma_0 \end{bmatrix} \) is a diagonal matrix of strictly positive elements representing learning parameters.

In this paper, we assigned \( \Gamma_0 = 1e^{-13} \) and \( c = 1.5e^5 \). The function \( V \) is positive definite and contains all of the time-varying parameters. Taking its time derivative and substituting

\[
V = s^T \dot{P} \dot{A} \dot{X} + \dot{\theta} \dot{\theta}^T
\]

\[
= s^T \dot{P} \dot{A} \dot{X} + \dot{\theta} \Gamma_0 \dot{\theta}^T
\]

showing the stability of the adaptation rule (Equation 18), with expected convergence to the correct model under persistent excitation (Joyce et al. 2018b). In the discrete time form, Equation 12 becomes

\[
\theta_{k+1} = \theta_k - \Gamma \Delta t s^T P \dot{X}_k
\]
Model-Assisted Probability of Detection–Based Evaluation

MAPOD is leveraged to quantify the performance of a given DSN by building the POD curves as a function of the damage of interest under the considered uncertainties. In this study, the damage of interest is loosening of the fixity. The process starts by defining the uncertain model parameters as random variables with specific probability distributions. For the system considered in this study, the uncertainties are arbitrarily selected as arising from the magnitude of the applied load and SEC measurement noise. These uncertainties are introduced in the updated reference model, and several realizations are generated at different damage levels, including the baseline (“healthy” structure). After, damage indicators are computed by extracting user-defined damage-sensitive features. These features are temporarily compared using the baseline strain measurements \( S^0 \) and new measurements \( S \), which are respectively composed of measurements \( S^0_{ik} \) and \( S_{ik} \), with \( i \) corresponding to the sensor location and \( k \) to the discrete time step. Here, three damage-sensitive features are taken from the literature (Lu and Michaels 2009), and their performance is compared as an exercise to demonstrate the promise of the proposed MAPOD-based technique:

Mean-squared error \( J_1 \) is a measure of how close the shape of the measured signal \( S_{ik} \) is to the baseline signal \( S^0_{ik} \):

\[
J_1 = \sum_{i,k} \left( S_{ik} - S^0_{ik} \right)^2
\]

Loss of correlation \( J_2 \) is a measure of the overall match between two signals’ waveform shape:

\[
J_2 = 1 - \frac{\sum_{i,k} \left( S_{ik} - \mu_S \right) \left( S^0_{ik} - \mu_{S^0} \right)}{\sigma_S \sigma_{S^0}}
\]

where \( \mu_S \) and \( \mu_{S^0} \) are the means of \( S_{ik} \) and \( S^0_{ik} \), respectively, and \( \sigma_S \) and \( \sigma_{S^0} \) are their standard deviations.

Differential curve length (DCL) \( J_3 \) is a measure of the signal complexity:

\[
J_3 = \sum_{i,k} |d_k - d_{k-1}|
\]

where \( d_k = S_{ik} - S^0_{ik} \).

The \( J - \alpha \) plots, where \( \alpha \) denotes the damage severity \( (\alpha = 1 - 0) \), as constructed from the realizations and data fitted using a linear regression (Gratiet et al. 2016):

\[
\hat{\alpha} = \beta_0 + \beta_1 \alpha + \epsilon
\]

where coefficients \( \beta_0 \) and \( \beta_1 \) are determined through a least-squares estimator, and error \( \epsilon \) has a normal distribution with zero mean and standard deviation \( \sigma_\epsilon \).

Damage is detected when the damage indicator \( J \) becomes larger than the threshold values \( \hat{\alpha} \). For a given \( \hat{\alpha} \), POD is computed as

\[
POD(\hat{\alpha}) = P(\hat{\alpha} > \bar{a}) = 1 - \Phi \left( \frac{\bar{a} - \beta_0 - \beta_1 \hat{\alpha}}{\sigma_\epsilon} \right)
\]

where

\[
P(\hat{\alpha} > \bar{a}) \text{ is the probability that the degree of damage is higher than the threshold, and}
\]

\( \Phi \) is the standard normal distribution function.

The POD function becomes a cumulative normal distribution function with mean \( (\bar{a} - \beta_0) / \beta_1 \) and standard deviation \( \sigma_\epsilon / \beta_1 \).

Results

Numerical Validation

Synthetic strain data from DROPBEAR was produced in a commercial finite element software. The software model, shown in Figure 3, consists of 1010 S4R shell elements and is taken as the real or “true” system. SEC data measurements were simulated by averaging the strain of all the elements located under a given SEC. The damage location used in the study is at the root of the beam, which is processed by reducing the bending rigidities to \( \theta = 1 - \alpha \) of the elements.

To verify the model adaptation stage, a damage case of \( a = 0.2 \) was generated and the model adapted. An implicit dynamic analysis was utilized with a 100 N magnitude white noise excitation of 100 Hz bandwidth applied at the tip of the beam. A nonsimultaneous harmonic excitation of 20 N at 20 rad/s was also simulated at the tip of the beam to verify the adapted model.

After verifying the model adaptation, the MAPOD-based technique was assessed. This was done by subjecting the cantilever to a harmonic excitation with an amplitude of 100 N at 5 rad/s. Two sources of uncertainties were introduced in the system: one on the force, where its variation is assumed to follow a uniform distribution (Bartel 2005), and one on the SEC measurement noise, where its variation is assumed to follow a gaussian distribution as established in a previous study (Downey et al. 2018a). Three uncertainty cases were considered, each constructed by combining different uncertainty levels on the load and sensor measurements. Case 1 had a \( \pm 0.5 \) N variation on the load and 10% on measurement noise; case 2 had a \( \pm 1 \) N variation on the load and 10% on measurement noise; and case 3 had a \( \pm 1 \) N variation on the load and 5% on measurement noise. Damage cases were generated by reducing the bending rigidities randomly between \( \alpha = 0 \) and \( \alpha = 0.6 \). The analysis was conducted using the adapted simplified physical surrogate model using 1000 realizations of synthetic data sets produced by the Latin hypercube sampling technique (Haddad et al. 2013). This technique was used as an improvement of the traditional Monte Carlo technique to produce more evenly spread sample points from the uncertainty distributions.
Model Adaptation

Figure 4 plots the results for the model adaptation verification task. Figure 4a shows the evolution of the estimated parameter $\theta$ (red dashed line) versus its real value (black solid line), where $\theta$ represents the bending rigidity. The offset between the estimated and real bending rigidities is attributed to the level of noise in the sensor, observable measurement error time histories (Figures 4b and 4c), quality of the least squares estimate used to convert strain into displacements and velocities, and level of simplification of the physical surrogate model. Figure 4b plots the measurement error between the physical surrogate and measured strains (from the finite element software) before the adaptation of the surrogate, while the same measurement error is plotted in Figure 4c after the adaptation of the surrogate. The estimation error is significantly minimized post model adaptation.

Figure 5 plots the first 5 s of results from the harmonic load to further verify the model adaptation technique, before
and after the adaptation of $\theta$. The updated model reproduces strain outputs similar to those from the software, as seen from the error plotted in Figure 5b, with the maximum strain error of SEC 1 reducing from 32.7 $\mu$ε to 16.4 $\mu$ε at the steady state, and the maximum strain error of SEC 5 reducing from 1.3 $\mu$ε to 0.8 $\mu$ε at the steady state. The algorithm was capable of tracking changes in bending rigidity, which can be used as a measure of damage.

Model-Assisted Probability of Detection–Based Evaluation

The performance of damage detection algorithms for a given DSN configuration was assessed using the proposed technique using the harmonic load applied at the tip of the beam. Threshold values $\hat{a}$, listed in Table 1, are taken as the intercept of the 95% upper confidence interval at $a = 0$ for each damage detection algorithm, therefore minimizing false positives. Figures 6a, 6b, and 6c are the $J - a$ plots showing the simulated values, linear regressions, and damage detection thresholds under the first uncertainty combination case for the three damage indicators, respectively. Figure 6d shows the resulting POD curves, with the dashed curve representing the upper 95% confidence intervals of the linear regression. The smoother slope in Figure 6d is from damage indicator $J_3$, which implies a lower performance of the damage indicator under uncertainty case 1. Conversely, damage indicator $J_2$ exhibits the best performance. Two metrics, $a_{50/95}$

<table>
<thead>
<tr>
<th>Uncertainty case</th>
<th>Damage indicator</th>
<th>Detection threshold $\hat{a}$</th>
<th>$a_{50/95}$</th>
<th>$a_{90/95}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$J_1$</td>
<td>1.18e-06</td>
<td>0.24</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>$J_2$</td>
<td>8.38e-04</td>
<td>0.23</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>$J_3$</td>
<td>0.03</td>
<td>0.25</td>
<td>0.46</td>
</tr>
<tr>
<td>Case 2</td>
<td>$J_1$</td>
<td>1.60e-06</td>
<td>0.49</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>$J_2$</td>
<td>1.5e-3</td>
<td>0.47</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>$J_3$</td>
<td>0.04</td>
<td>0.47</td>
<td>0.89</td>
</tr>
<tr>
<td>Case 3</td>
<td>$J_1$</td>
<td>8.66e-7</td>
<td>0.48</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>$J_2$</td>
<td>1.2e-3</td>
<td>0.47</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>$J_3$</td>
<td>0.03</td>
<td>0.45</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Figure 6. Uncertainty case 1: (a) $J_1 - a$ plot; (b) $J_2 - a$ plot; (c) $J_3 - a$ plot; and (d) POD curves and the upper (conservative) 95% confidence interval (CI) of the linear regression for $J_1$, $J_2$, and $J_3$. 

TABLE 1

Probability of detection results under each uncertainty case

<table>
<thead>
<tr>
<th>Uncertainty case</th>
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<td></td>
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<td>0.45</td>
<td>0.85</td>
</tr>
</tbody>
</table>
(bottom dash dotted line) and \( a_{90/95} \) (top dash dotted line), are specified for assessing the particular damage detection algorithms. These two metrics show that there is a 50% probability with a 95% confidence of detecting the change of bending rigidity higher than \( a_{50/95} \) and a 90% probability with a 95% confidence of detecting the change of bending rigidity higher than \( a_{90/95} \), respectively. Metrics \( a_{50/95} \) and \( a_{90/95} \), listed in Table 1, are used for quantifying the performance of a given DSN configuration and its associated damage detection algorithm under different uncertainty cases. Damage indicator \( J_2 \) has a 90% probability with a 95% confidence of detecting damage higher than 43% under uncertainty case 1. However, damage indicator \( J_3 \) exhibits better performance under uncertainty cases 2 and 3 with a 90% probability with a 95% confidence of detecting damage higher than 89% and 85%, respectively. Thus, indicators \( J_2 \) and \( J_3 \) are selected to perform the evaluation of DSN configurations under uncertainty case 1 and uncertainty cases 2 and 3, respectively, in what follows.

Model-Assisted Probability of Detection–Based Evaluation for DSN Design

Here, the proposed MAPOD-based technique is used to evaluate the performance of DSN configurations. The performance of each configuration is evaluated and ranked using the selected damage indicators (\( J_2 \) for uncertainty case 1, and \( J_3 \) for uncertainty cases 2 and 3). Figure 7 shows 10 arbitrarily selected configurations, which include various numbers, sizes, and locations of sensors in both symmetric and asymmetric configurations. These configurations were selected to cover different parts of the beam, with some providing a higher resolution for strain close to damage (such as configurations 1 and 7), a higher resolution for strain much farther away from damage (such as configuration 10), a higher resolution over longer lengths (such as configurations 5 and 6), high-resolution equally spaced sensor measurements (such as configuration 3), a mixture of high-resolution measurement locations (such as configurations 2 and 4), and a lower resolution obtained with larger sensors (such as configurations 8 and 9). SEC measurements from each surrogate model are taken as the averaged strain over the reparametrized elements (shown as dashed lines).

The performance of each configuration is investigated under uncertainty case 1. Configurations illustrated in Figure 7a are ranked by performance using the \( a_{90/95} \) metric established by the surrogate model. Performance results obtained from the surrogate are benchmarked against those obtained using the finite element software. In the software, damage cases at the root of the beam were generated at levels ranging from 0 to 0.6 at 0.05 steps and conducting 100 realizations under each damage case. The POD curves obtained for each DSN configuration from the surrogate model and the finite element software are plotted in Figures 7b and 7c, respectively. While the ranking of configurations produced by the surrogate model and the finite element software are identical, values for \( a_{90/95} \) differ (Figure 8). This is investigated further in what follows.

The capability of the proposed technique at ranking DSN configurations is investigated under different uncertainty cases. Figure 8 plots the results for the \( a_{50/95} \) and \( a_{90/95} \) metrics obtained using the surrogate model and the finite...
Figure 8. Metrics $a_{50/95}$ and $a_{90/95}$ under uncertainty: (a) case 1; (b) case 2; (c) case 3.

Conclusion

In this paper, a methodology for the validation of sensor networks used for structural health monitoring was presented. The methodology consisted of constructing a physical surrogate model based on a given DSN configuration, adapting the surrogate model using field data and sliding mode theory, and using MAPOD to study the performance of various network configurations and damage detection algorithm under various sources of uncertainty.

Numerical simulations were conducted to verify and validate the proposed methodology using a straightforward example consisting of an elastically restrained beam equipped with a DSN measuring strain. The verification of the model adaptation technique showed that the sliding mode theory could be used to update the physical surrogate using measurements, where the measurements were synthetically produced using a high-resolution finite element model constructed in the finite element software. The updated physics surrogate was then used to evaluate the performance of various damage detection algorithms. After, the surrogate was used to investigate the performance of various sensor configurations for detecting damage at the root of the cantilever. Results showed that the proposed technique successfully ranked the performance of various DSN configurations. Such a ranking could be critical in designing efficient DSNs. Results also showed that the damage detection threshold would need to be standardized to enable surrogate-based damage detection and quantification in the field.

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