Preface

Wavelets have been around since the late 1980s, and have found many applications in signal processing, numerical analysis, operator theory, and other fields. Classical wavelet theory is based on a refinement equation of the form

$$\phi(x) = \sqrt{2} \sum_k h_k \phi(2x - k)$$

which defines the scaling function ϕ. The scaling function leads to multiresolution approximations (MRAs), wavelets, and fast decomposition and reconstruction algorithms. Generalizations include wavelet packets, multivariate wavelets, ridgelets, curvelets, vaguelettes, slantlets, second generation wavelets, frames, and other constructions.

One such generalization are multiwavelets, which have been around since the early 1990s. We replace the scaling function ϕ by a function vector

$$\boldsymbol{\phi}(x) = \begin{pmatrix} \phi_1(x) \\ \vdots \\ \phi_r(x) \end{pmatrix}$$

called a *multiscaling function*, and the refinement equation by

$$\boldsymbol{\phi}(x) = \sqrt{m} \sum_k H_k \boldsymbol{\phi}(mx - k).$$

The recursion coefficients H_k are now $r \times r$ matrices.

Multiwavelets lead to MRAs and fast algorithms just like scalar wavelets, but they have some advantages: they can have short support coupled with high smoothness and high approximation order, and they can be both symmetric and orthogonal. They also have some disadvantages: the discrete multiwavelet transform requires preprocessing and postprocessing steps. Also, the theory becomes more complicated.

Many of the existing wavelet books have a short section discussing multiwavelets, but there has never been a full exposition of multiwavelet theory in book form.

This book is divided into two main parts. The first part deals with scalar wavelet theory, and can be read by itself. The second part deals with multiwavelet theory and can also be read by itself, assuming the reader is already familiar with scalar wavelets. Most sections of the two parts run in parallel, so it is easy to refer back and forth between the two and check how a scalar result generalizes to the multiwavelet case. In some cases, the generalization is straightforward. In other cases, the multiwavelet results are more complex, in unexpected ways.

I have chosen to use a dilation factor of 2 in the scalar case, the more general $m \geq 2$ in the multiwavelet case. This way, the first part of the book remains simpler for beginners, and the second part of the book contains more general results. The change from 2 to m has virtually no effect on the difficulty of the material; it just requires a slightly more complicated notation.

I have tried to maintain a balance between mathematical rigor and readability to make the book useful to as wide an audience as possible. The most technical material has been concentrated in two separate chapters that can be skipped without affecting the readability of the other chapters. Most concepts are illustrated with examples.

The book as a whole should be accessible to both mathematicians and engineers, and could be used as the basis for an introductory course or a seminar. Some Matlab routines for experimenting with multiwavelets are available from the author (see appendix C).

Both parts of the book contain the following main topics:

- **Basic theory.** Scaling functions, MRAs, wavelets, moments, approximation order, wavelet decomposition and reconstruction.
- **Practical implementation issues.** Fast algorithms for decomposition and reconstruction, preprocessing, modifications at the boundary, computing point values and integrals.
- **Creating wavelets**
- **Applications.** Signal processing, signal compression, denoising, fast numerical algorithms.
- **Advanced theory.** Existence in the distribution, L^1, L^2, or pointwise sense, stability, smoothness estimates.

The appendix contains a list of standard wavelets, a section on mathematical background, and a list of web and software resources.

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