A

Standard Wavelets

In all listings, p = approximation order, s = Sobolev exponent, and α = Hölder exponent. If α is not listed, $s - 1/2$ is a lower bound for α. In many cases, a common factor for all coefficients in its column is listed separately, for easier readability.

For scalar wavelets, we only list h_k and $\tilde{h}_k$. The coefficients g_k and $\tilde{g}_k$ can be found by reversing $\tilde{h}_k$ and h_k with alternating sign.

All multiwavelets listed here have $m = 2$, $r = 2$.

A.1 Scalar Orthogonal Wavelets

Daubechies Wavelets $\mathbf{D_p}$

Restrictions: $p \geq 1$ integer.

Support $[0, 2p-1]$, approximation order p; values of s, α are given in the tables below (see [49] and [50].)

	h_k	h_k	h_k
p	1	2	3
$k=0$	1	$1+\sqrt{3}$	$1+\sqrt{10}+\sqrt{5+2\sqrt{10}}$
1	1	$3+\sqrt{3}$	$5+\sqrt{10}+3\sqrt{5+2\sqrt{10}}$
2		$3-\sqrt{3}$	$10-2\sqrt{10}+2\sqrt{5+2\sqrt{10}}$
3		$1-\sqrt{3}$	$10-2\sqrt{10}-\sqrt{5+2\sqrt{10}}$
4			$5+\sqrt{10}-3\sqrt{5+2\sqrt{10}}$
5			$1+\sqrt{10}-\sqrt{5+2\sqrt{10}}$
factor	$1/\sqrt{2}$	$1/(4\sqrt{2})$	$1/(16\sqrt{2})$
s	0.500	1.000	1.415
α		0.550	1.088

	h_k	h_k	h_k
p	4	5	6
$k=0$	0.23037781330890	0.16010239797419	0.11154074335011
1	0.71484657055291	0.60382926979719	0.49462389039846
2	0.63088076792986	0.72430852843777	0.75113390802110
3	−0.02798376941686	0.13842814590132	0.31525035170919
4	−0.18703481171909	−0.24229488706638	−0.22626469396544
5	0.03522629188571	−0.03224486958464	−0.12976686756726
6	0.03288301166689	0.07757149384005	0.09750160558733
7	−0.01059740178507	−0.00624149021280	0.02752286553031
8		−0.01258075199908	−0.03158203931749
9		0.00333572528547	0.00055384220116
10			0.00477725751095
11			−0.00107730108531
s	1.775	2.096	2.388
α	1.618	1.596	1.888

Coiflets

We only list the coiflets of length 6 with support $[-2,3]$ (two different coiflets) or $[-1,4]$ (another two). Coiflets on $[-3,2]$ and $[-4,1]$ can be found by reversing coefficients.

$p=2$, $\mu_1=\mu_2=0$; values of s are given in the table below (see [30], [50], and [51].)

	support $[-2,3]$		support $[-1,4]$	
	h_k	h_k	h_k	h_k
$k=-2$	$1+\sqrt{7}$	$1-\sqrt{7}$		
-1	$5-\sqrt{7}$	$5+\sqrt{7}$	$9-\sqrt{15}$	$9+\sqrt{15}$
0	$14-2\sqrt{7}$	$14+2\sqrt{7}$	$13+\sqrt{15}$	$13-\sqrt{15}$
1	$14+2\sqrt{7}$	$14-2\sqrt{7}$	$6+2\sqrt{15}$	$6-2\sqrt{15}$
2	$1+\sqrt{7}$	$1-\sqrt{7}$	$6-2\sqrt{15}$	$6+2\sqrt{15}$
3	$-3-\sqrt{7}$	$-3+\sqrt{7}$	$1-\sqrt{15}$	$1+\sqrt{15}$
4			$-3+\sqrt{15}$	$-3-\sqrt{15}$
factor	$1/(16\sqrt{2})$	$1/(16\sqrt{2})$	$1/(16\sqrt{2})$	$1/(16\sqrt{2})$
s	0.5896	1.0217	1.2321	0.0413

A.2 Scalar Biorthogonal Wavelets

Cohen(p,$\tilde{\mathbf{p}}$) (Cohen–Daubechies–Feauveau)

Restrictions: p, $\tilde{p}$ integers ≥ 1, $p+\tilde{p}$ even.

ϕ is the B-spline of order p. ϕ, $\tilde{\phi}$ are symmetric about 0 (p even) or 1/2 (p odd). Approximation orders p and $\tilde{p}$ (see [40] and [50].)

There are too many of them to list here. A table of some of them is in [50, page 277]. Cohen(1,1) = Haar wavelet; Cohen(2,4) is given in example 1.6.

Daubechies(7,9)

Symmetric about 0, $p = \tilde{p} = 4$, $s = 2.1226$, $\tilde{s} = 1.4100$ (see [50].)

	h_k	$\tilde{h}_k$
$k = -4$		0.03782845550700
−3	−0.06453888262894	−0.02384946501938
−2	−0.04068941760956	−0.11062440441842
−1	0.41809227322221	0.37740285561265
0	0.78848561640566	0.85269867900940
1	0.41809227322221	0.37740285561265
2	−0.04068941760956	−0.11062440441842
3	−0.06453888262894	−0.02384946501938
4		0.03782845550700

A.3 Orthogonal Multiwavelets

BAT O1 (Lebrun–Vetterli)

Support $[0, 2]$. ϕ_2 is reflection of ϕ_1 about $x = 1$ and vice versa; wavelet functions are symmetric/antisymmetric about $x = 1$. $p = 2$, balanced of order 1, $s = 0.6406$.

There are also BAT O2 and BAT O3, which are balanced of order 2 and 3 (see [105] for a list of coefficients.)

	H_k	G_k
$k = 0$	$\begin{pmatrix} 0 & 2+\sqrt{7} \\ 0 & 2-\sqrt{7} \end{pmatrix}$	$\begin{pmatrix} 0 & -2 \\ 0 & 1 \end{pmatrix}$
1	$\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$	$\begin{pmatrix} 2 & 2 \\ -\sqrt{7} & \sqrt{7} \end{pmatrix}$
2	$\begin{pmatrix} 2-\sqrt{7} & 0 \\ 2+\sqrt{7} & 0 \end{pmatrix}$	$\begin{pmatrix} -2 & 0 \\ -1 & 0 \end{pmatrix}$
factor	$1/(4\sqrt{2})$	$1/4$

CL2(t) (Chui–Lian)

Restriction: $-1/\sqrt{2} \leq t < -1/2$.

Support $[0, 2]$, symmetric/antisymmetric about $x = 1$, $\alpha = -\log_2 |1/2+2t|$, $p = 1$.

This is a special case of JRZB(s,t,λ,μ).

Special case: CL2 = CL2$(-\sqrt{7}/4)$ is the standard Chui–Lian multiwavelet. $p = 2$, $s = 1.0545$, $\alpha = -\log_2(\sqrt{7}/4) \approx 0.59632$ (see [36] and [90].)

	H_k	G_k
$k = 0$	$\begin{pmatrix} 1 & 1 \\ 2t & 2t \end{pmatrix}$	$\begin{pmatrix} -1 & -1 \\ \mu & \mu \end{pmatrix}$
1	$\begin{pmatrix} 2 & 0 \\ 0 & 2\mu \end{pmatrix}$	$\begin{pmatrix} 2 & 0 \\ 0 & -4t \end{pmatrix}$
2	$\begin{pmatrix} 1 & -1 \\ -2t & 2t \end{pmatrix}$	$\begin{pmatrix} -1 & 1 \\ -\mu & \mu \end{pmatrix}$
factor	$1/(2\sqrt{2})$	$1/(2\sqrt{2})$

where

$$\mu = \sqrt{2 - 4t^2}.$$

CL3 (Chui–Lian)

Support $[0, 3]$, symmetric/antisymmetric about $x = 3/2$, $p = 3$, $s = 1.4408$ (see [36].)

	H_k	G_k
$k = 0$	$\begin{pmatrix} 10 - 3\sqrt{10} & 5\sqrt{6} - 2\sqrt{15} \\ 5\sqrt{6} - 3\sqrt{15} & 5 - 3\sqrt{10} \end{pmatrix}$	$\begin{pmatrix} 5\sqrt{6} - 2\sqrt{15} & -10 + 3\sqrt{10} \\ -5 + 3\sqrt{10} & 5\sqrt{6} - 3\sqrt{15} \end{pmatrix}$
1	$\begin{pmatrix} 30 + 3\sqrt{10} & 5\sqrt{6} - 2\sqrt{15} \\ -5\sqrt{6} - 7\sqrt{15} & 15 - 3\sqrt{10} \end{pmatrix}$	$\begin{pmatrix} -5\sqrt{6} + 2\sqrt{15} & 30 + 3\sqrt{10} \\ 15 - 3\sqrt{10} & 5\sqrt{6} + 7\sqrt{15} \end{pmatrix}$
2	$\begin{pmatrix} 30 + 3\sqrt{10} & -5\sqrt{6} + 2\sqrt{15} \\ 5\sqrt{6} + 7\sqrt{15} & 15 - 3\sqrt{10} \end{pmatrix}$	$\begin{pmatrix} -5\sqrt{6} + 2\sqrt{15} & -30 - 3\sqrt{10} \\ -15 + 3\sqrt{10} & 5\sqrt{6} + 7\sqrt{15} \end{pmatrix}$
3	$\begin{pmatrix} 10 - 3\sqrt{10} & -5\sqrt{6} + 2\sqrt{15} \\ -5\sqrt{6} + 3\sqrt{15} & 5 - 3\sqrt{10} \end{pmatrix}$	$\begin{pmatrix} 5\sqrt{6} - 2\sqrt{15} & 10 - 3\sqrt{10} \\ 5 - 3\sqrt{10} & 5\sqrt{6} - 3\sqrt{15} \end{pmatrix}$
factor	$1/(40\sqrt{2})$	$1/(40\sqrt{2})$

Balanced Daubechies $\mathbf{D_p}$

Restriction: $p \geq 1$ integer.

Support $[0, 2p]$, balanced of order p, same smoothness properties as scalar Daubechies wavelets D_p.

These multiwavelets use the same coefficients as the scalar Daubechies wavelets, sorted into two rows. ϕ_1 is the Daubechies ϕ compressed by a factor of 2. ϕ_2 is ϕ_1 shifted right by 1/2. The same holds for ψ_1, ψ_2 (see [105].)

	H_k	G_k
$k=0$	$\begin{pmatrix} h_0 & h_1 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} g_0 & g_1 \\ 0 & 0 \end{pmatrix}$
1	$\begin{pmatrix} h_2 & h_3 \\ h_0 & h_1 \end{pmatrix}$	$\begin{pmatrix} g_2 & g_3 \\ g_0 & g_1 \end{pmatrix}$
2	$\begin{pmatrix} h_4 & h_5 \\ h_2 & h_3 \end{pmatrix}$	$\begin{pmatrix} g_4 & g_5 \\ g_2 & g_3 \end{pmatrix}$
$\vdots$		
p	$\begin{pmatrix} 0 & 0 \\ h_{2p-2} & h_{2p-1} \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ g_{2p-2} & g_{2p-1} \end{pmatrix}$

DGHM (Donovan–Geronimo–Hardin–Massopust)

Support $[0,2]$, $p=2$, $\alpha=1$, $s=1.5$. ϕ_1 is symmetric about $x=1/2$, ϕ_2 is symmetric about $x=1$ (see [59].)

	H_k	G_k
$k=0$	$\begin{pmatrix} 12 & 16\sqrt{2} \\ -\sqrt{2} & -6 \end{pmatrix}$	$\begin{pmatrix} -\sqrt{2} & -6 \\ 2 & 6\sqrt{2} \end{pmatrix}$
1	$\begin{pmatrix} 12 & 0 \\ 9\sqrt{2} & 20 \end{pmatrix}$	$\begin{pmatrix} 9\sqrt{2} & -20 \\ -18 & 0 \end{pmatrix}$
2	$\begin{pmatrix} 0 & 0 \\ 9\sqrt{2} & -6 \end{pmatrix}$	$\begin{pmatrix} 9\sqrt{2} & -6 \\ 18 & -6\sqrt{2} \end{pmatrix}$
3	$\begin{pmatrix} 0 & 0 \\ -\sqrt{2} & 0 \end{pmatrix}$	$\begin{pmatrix} -\sqrt{2} & 0 \\ -2 & 0 \end{pmatrix}$
factor	$1/(20\sqrt{2})$	$1/(20\sqrt{2})$

STT (Shen–Tan–Tam)

Support $[0,3]$, symmetric/antisymmetric about $x=3/2$, $p=1$, $s=0.9919$ (see [130].)

	H_k	G_k
$k=0$	$\begin{pmatrix} 1 & 4+\sqrt{15} \\ 1 & -4-\sqrt{15} \end{pmatrix}$	$\begin{pmatrix} -4-\sqrt{15} & 1 \\ -4-\sqrt{15} & -1 \end{pmatrix}$
1	$\begin{pmatrix} 31+8\sqrt{15} & 4+\sqrt{15} \\ -31-8\sqrt{15} & 4+\sqrt{15} \end{pmatrix}$	$\begin{pmatrix} 4+\sqrt{15} & -31-8\sqrt{15} \\ -4-\sqrt{15} & -31-8\sqrt{15} \end{pmatrix}$
2	$\begin{pmatrix} 31+8\sqrt{15} & -4-\sqrt{15} \\ 31+8\sqrt{15} & 4+\sqrt{15} \end{pmatrix}$	$\begin{pmatrix} 4+\sqrt{15} & 31+8\sqrt{15} \\ 4+\sqrt{15} & -31-8\sqrt{15} \end{pmatrix}$
3	$\begin{pmatrix} 1 & -4-\sqrt{15} \\ -1 & -4-\sqrt{15} \end{pmatrix}$	$\begin{pmatrix} -4-\sqrt{15} & -1 \\ 4+\sqrt{15} & -1 \end{pmatrix}$
factor	$1/(8(4+\sqrt{15}))$	$1/(8(4+\sqrt{15}))$

A.4 Biorthogonal Multiwavelets

HM(s) (Hardin–Marasovich)

Restriction: $-1 < s < 1/7$.

Support $[-1, 1]$; ϕ_1, $\tilde{\phi}_1$ are symmetric about $x = 0$; ϕ_2, $\tilde{\phi}_2$ are symmetric about $x = 1/2$; $p = \tilde{p} = 2$ (see [75].)

	H_k	G_k
$k = -2$	$\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & a \\ 0 & \sqrt{2}a \end{pmatrix}$
-1	$\begin{pmatrix} b & c \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} b & c \\ \sqrt{2}b & \sqrt{2}c \end{pmatrix}$
0	$\begin{pmatrix} 1 & c \\ 0 & d \end{pmatrix}$	$\begin{pmatrix} -1 & c \\ 0 & -\sqrt{2}c \end{pmatrix}$
1	$\begin{pmatrix} b & a \\ e & d \end{pmatrix}$	$\begin{pmatrix} b & a \\ -\sqrt{2}b & -\sqrt{2}a \end{pmatrix}$
factor	$1/\sqrt{2}$	$1/\sqrt{2}$

where

$$
\begin{aligned}
\tilde{s} &= (1 + 2s)/(-2 + 5s) \Leftrightarrow s = (1 + 2\tilde{s})/(-2 + 5\tilde{s}) \\
\alpha &= 3(1 - s)(1 - s\tilde{s})/(4 - s - \tilde{s} - 2s\tilde{s}) \\
\gamma &= \sqrt{6(4 - s - \tilde{s} - 2s\tilde{s})/(7 - 4s - 4\tilde{s} + s\tilde{s})} \\
\delta &= \sqrt{12(-1 + \tilde{s})(-1 + s)(-1 + s\tilde{s})/(-4 + s + \tilde{s} + 2s\tilde{s})} \\
a &= \alpha\gamma(1 - 2\alpha - 2s)/(2\delta) \\
b &= 1/2 - \alpha \\
c &= \alpha\gamma(3 - 2\alpha - 2s)/(2\delta) \\
d &= \alpha + s \\
e &= \delta/\gamma
\end{aligned}
$$

The dual functions have the same form. Exchange tilde and nontilde in all formulas.

Special case: for $s = 0$, V_0 = continuous piecewise linear splines with half-integer knots.

Special case: for $s = 1/4$, V_0 = continuous, piecewise quadratic splines with integer knots.

JRZB(s,t,λ,μ) (Jia–Riemenschneider–Zhou biorthogonal)

Restriction: $|2\lambda + \mu| < 2$.

Support $[0, 2]$, symmetric/antisymmetric about $x = 1$, $p = 1$, $\alpha = 2$ if $|st + 1/4| \le 1/8$, $\alpha = -\log_2 |2st + 1/2|$ if $1/8 < |st + 1/4| < 1/2$ (see [90].)

Special case: $p = 3$ if $t \ne 0$, $\mu = 1/2$, $\lambda = 2st + 1/4$.

Special case: $p = 4$ if $\lambda = -1/8$, $\mu = 1/2$, $st = -3/16$. For $s = 3/2$, $t = -1/8$ we get the cubic Hermite multiwavelet.

Special case: $s = 1$, $\lambda = t$, $\mu = \sqrt{2 - 4t^2}$ is CL2(t).

	H_k
$k = 0$	$\begin{pmatrix} 1 & s \\ 2t & 2\lambda \end{pmatrix}$
1	$\begin{pmatrix} 2 & 0 \\ 0 & 2\mu \end{pmatrix}$
2	$\begin{pmatrix} 1 & -s \\ -2t & 2\lambda \end{pmatrix}$
factor	$1/(2\sqrt{2})$

I could not find multiwavelet or dual multiscaling function coefficients for this multiscaling function published anywhere.

HC (Hermite cubic)

This is a special case of JRZB(s,t,λ,μ).

There are many completions. The one listed here is the smoothest symmetric completion with support length 4 (see [80].)

Support of $\boldsymbol{\phi}$ is $[-1, 1]$, support of $\tilde{\boldsymbol{\phi}}$ is $[-2, 2]$; all functions are symmetric/antisymmetric about $x = 0$; $p = 4$, $\tilde{p} = 2$, $\alpha = 2$, $s = 2.5$, $\tilde{s} = 0.8279$.

	H_k	G_k	$\tilde{H}_k$	$\tilde{G}_k$
$k = -2$			$\begin{pmatrix} -2190 & -1540 \\ 13914 & 9687 \end{pmatrix}$	
-1	$\begin{pmatrix} 4 & 6 \\ -1 & -1 \end{pmatrix}$	$\begin{pmatrix} 5427 & 567 \\ -1900 & -120 \end{pmatrix}$	$\begin{pmatrix} 9720 & 3560 \\ -60588 & -21840 \end{pmatrix}$	
0	$\begin{pmatrix} 8 & 0 \\ 0 & 4 \end{pmatrix}$	$\begin{pmatrix} -19440 & -60588 \\ 7120 & 21840 \end{pmatrix}$	$\begin{pmatrix} 23820 & 0 \\ 0 & 36546 \end{pmatrix}$	$\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$
1	$\begin{pmatrix} 4 & -6 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 28026 & 0 \\ 0 & 56160 \end{pmatrix}$	$\begin{pmatrix} 9720 & -3560 \\ 60588 & -21840 \end{pmatrix}$	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$
2		$\begin{pmatrix} -19440 & 60588 \\ -7120 & 21840 \end{pmatrix}$	$\begin{pmatrix} -2190 & 1540 \\ -13914 & 9687 \end{pmatrix}$	$\begin{pmatrix} -2 & 1 \\ -3 & 1 \end{pmatrix}$
3		$\begin{pmatrix} 5427 & -567 \\ 1900 & -120 \end{pmatrix}$		
factor	$1/(8\sqrt{2})$	$1/(19440\sqrt{2})$	$1/(19440\sqrt{2})$	$1/(8\sqrt{2})$