Math 610: Combinatorics and Commutative Algebra (EGR – Early Graduate Research Course)

Spring 2019 – 3 credits

Instructor	Jason McCullough 452 Carver Hall 4-8150 jmccullo@iastate.edu
Office Hours	MWF 10:00am-11:00am or by appointment
Textbooks	We will be using some combination of "Binomial Ideals" by Herzog-Hibi-Ohsugi, "Monomial Ideals" by Herzog-Hibi, and "Graded Syzygies" by Peeva
Meetings	MWF 11:00 – 11:50am (though groups may meet at other mutually agreeable times later in the semester)

Content

This class is a non-standard graduate class designed to let graduate students get an early taste of research problems in a specific area – in this case, combinatorial commutative algebra. There are several algebraic objects one can associate to a combinatorial object like a graph or a simplicial complex.

In this class we will explore some of the following ideas:

- Graded ideals and modules over polynomial rings
- Graded free resolutions
- Groebner bases
- Toric edge ideals of graphs
- Monomial ideals and Stanley-Reisner complexes
- How to compute examples in the computer algebra system Macaulay2.

Prerequisites

I intend to make this class as accessible as possible and encourage 1st and 2nd year students to enroll. Students should have completed Math 504 or Math 567 at a minimum. Any of the following classes would be helpful: Math 502, Math 505, Math 619.

Assessment

This is an atypical class. There will not be exams. I expect one early homework assignment. Later students will form groups around one of several problems with the goal of producing some original results by the end of the semester. (However, there are no guarantees when researching new problems!) After a few weeks of introductory lectures, I will expect students to present on some of the background material as part of their grade for the course. The rest of the semester will be spent working in groups on specific open problems. Participation (including working with your group outside of class time) makes up the remaining portion of your grade.

Course Websites

Grades and homework will be available via Canvas. There is a tentative schedule below.

Grading scheme

Homework	25%
Presentation	25%
Participation	25%
Final Group Paper	25%

Tentative Schedule

Day 1: Graded rings and modules, Hilbert functions and series (Peeva 1.1-1.2, HHO 2.1)
Day 2: Free resolutions, Betti tables, Hilbert's Syzygy Theorem (Peeva 1.3-1.5, 1.7-1.9, 1.16, HH0 2.2)
Day 3: Simplicial Complexes, homology, posets, lattices (Peeva 1.36, HHO 6.1)
Day 4: Dimension, height, Hilbert Functions (Peeva 1.16, HHO 2.3)
Day 5: Depth, Koszul Complex, Auslander Buchsbaum Theorem (Peeva 1.14-1.15, HHO 2.3)
Day 6: Initial Ideals and Groebner bases (Peeva 1.22, 1.39, HHO 1.1-1.3, 1.5)
Day 7: Monomial ideals, Hochster's formula, multigraded modules (Peeva 1.26, 1.61
Day 8: Toric ideals (Peeva 1.65-1.67, HHO 3.1-3.2,)
Day 9: Cohen-Macaulay, Gorenstein, Koszul rings (Peeva 1.25, 1.34, HHO 2.4)
Day 10: Macaulay Inverse Systems
Day 11: Toric Edge Ideals of Graphs (HHO 5.1-5.5)
Day 12: Hibi rings and join-meet ideals (HHO 6.1-6.5)