1. What is the probability that the sum of two randomly chosen numbers in the interval \([0, 1]\) does not exceed 1 and their product does not exceed \(\frac{2}{9}\)?

2. If a needle of length 1 is dropped at random on a surface ruled with parallel lines at distance 2 apart, what is the probability that the needle will cross one of the lines?

3. What is the probability that three randomly chosen points on a circle form an acute triangle?

4. Prove that \(\gcd(m,n)\binom{n}{m}\) is an integer for all pairs of integers \(n \geq m \geq 1\).

5. Let \(f\) be a twice differentiable real-valued function satisfying
\[
f(x) + f''(x) = -xg(x)f'(x)
\]
where \(g(x) \geq 0\) for all real \(x\). Prove that \(|f(x)|\) is bounded.