

ISU Putnam Practice Set 3

Wednesday, February 10, 2021

1. What is the probability that the sum of two randomly chosen numbers in the interval $[0, 1]$ does not exceed 1 and their product does not exceed $\frac{2}{9}$?
2. If a needle of length 1 is dropped at random on a surface ruled with parallel lines at distance 2 apart, what is the probability that the needle will cross one of the lines?
3. What is the probability that three randomly chosen points on a circle form an acute triangle?
4. Prove that $\frac{\gcd(m,n)}{n} \binom{n}{m}$ is an integer for all pairs of integers $n \geq m \geq 1$.
5. Let f be a twice differentiable real-valued function satisfying

$$f(x) + f''(x) = -xg(x)f'(x)$$

where $g(x) \geq 0$ for all real x . Prove that $|f(x)|$ is bounded.