1. Show that
\[ \sum_{n \in A} \frac{1}{n} < \infty, \]
where \( A \) is the set of positive integers that do not contain a '9' in their decimal expansion.

2. Evaluate
\[ \sum_{n=0}^{\infty} \text{Arccot}(n^2 + n + 1), \]
where \( \text{Arccot}(t) \) for \( t \geq 0 \) denotes the number \( \theta \) in the interval \( 0 < \theta \leq \pi/2 \) with \( \cot \theta = t \).

3. A not uncommon calculus mistake is to believe that the product rule for derivatives says that \((fg)' = f'g'\). If \( f(x) = e^{x^2} \), determine, with proof, whether there exists an open interval \((a, b)\) and a non-zero function \( g \) defined on \((a, b)\) such that the wrong product rule is true for \( x \) in \((a, b)\).

4. Find all real-valued continuously differentiable functions \( f \) on the real line such that for all \( x \),
\[ (f(x))^2 = \int_0^x [(f(t))^2 + (f'(t))^2] \, dt + 1990. \]

5. Let \( f \) be a real function on the real line with continuous third derivative. Prove that there exists a point \( a \) such that
\[ f(a) \cdot f'(a) \cdot f''(a) \cdot f'''(a) \geq 0. \]