Putnam Practice Set #2

1. Let n be an even positive integer, and let $p(x)$ be an n-degree polynomial such that $p(-k) = p(k)$ for $k = 1, 2, ..., n$. Prove that there is a polynomial $q(x)$ such that $p(x) = q(x^2)$.
2. Let $f(x)$ and $g(x)$ be nonzero polynomials with real coefficients such that $f(x^2 + x + 1) = f(x)g(x)$. Show that $f(x)$ has even degree.
3. The product of two of the four zeros of the quartic equation
$$x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$$
is -32. Find k.
4. A polynomial $f(x) = x^4 + ?x^3 + ?x^2 + ?x + 1$ has three undetermined coefficients denoted by "?". The players A and B move alternately, replacing a question mark by a real number until all question marks are replaced. A wins if all zeros of the polynomial are complex. B wins if at least one zero is real. Show that if B is allowed to pick the coefficient of x^2, then he can win.
5. Let k be a positive integer. Find all polynomials $p(x)$ with real coefficients such that
$$p(p(x)) = p(x)^k.$$