1. The triangle ABC has an obtuse angle at B, and angle A is less than angle C. The external angle bisector at A meets the line BC at D, and the external angle bisector at B meets the line AC at E. Also, BA = AD = BE. Find angle A.

2. Let \( \alpha \) and \( \beta \) be positive real numbers such that \( \frac{1}{\alpha} + \frac{1}{\beta} = 1 \). Prove that the line \( mx + ny = 1 \) with \( m, n \) positive reals is tangent to the curve \( x^\alpha + y^\alpha = 1 \) in the first quadrant \((x, y \geq 0)\) iff \( m^\beta + n^\beta = 1 \).

3. Show that, for any positive integer \( n \),

\[
\sum_{r=0}^{\lceil (n-1)/2 \rceil} \left( \frac{n-2r}{n} \binom{n}{r} \right)^2 = \frac{1}{n} \binom{2n-2}{n-1},
\]

where \( \lceil x \rceil \) means the greatest integer not exceeding \( x \), and \( \binom{n}{r} \) is the binomial coefficient "\( n \) choose \( r \)," with the convention \( \binom{n}{0} = 1 \).

4. \( S \) and \( T \) and finite sets. \( U \) is a collection of ordered pairs \((s, t)\) with \( s \in S \) and \( t \in T \). There is no element \( s \in S \) such that all possible pairs \((s, t) \in U\). Every element \( t \in T \) appears in at least one element of \( U \). Prove that we can find distinct \( s_1, s_2 \in S \) and distinct \( t_1, t_2 \in T \) such that \((s_1, t_1), (s_2, t_2) \in U\), but \((s_1, t_2), (s_2, t_1) \notin U\).

5. How many possible bijections \( f \) on \( \{1, 2, \ldots, n\} \) are there such that for each \( i = 2, 3, \ldots, n \) we can find \( j < n \) with \( f(i) - f(j) = \pm 1 \)?