

2023 - ISU Putnam Practice Set 8

Thursday, October 26, 2023

Geometry

1. A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?
2. Let A_0, A_1, \dots, A_{n-1} be the vertices of a regular n -gon inscribed in the unit circle. Prove that

$$A_0A_1 \cdot A_0A_2 \cdots A_0A_{n-1} = n.$$

3. A rectangle, $HOMF$, has sides $HO = 11$ and $OM = 5$. A triangle ABC has H as the intersection of the altitudes, O the center of the circumscribed circle, M the midpoint of BC , and F the foot of the altitude from A . What is the length of BC ?
4. Let s be any arc of the unit circle lying entirely in the first quadrant. Let A be the area of the region lying below s and above the x -axis and let B be the area of the region lying to the right of the y -axis and to the left of s . Prove that $A + B$ depends only on the arc length, and not on the position, of s .
5. Right triangle ABC has right angle at C and $\angle BAC = \theta$; the point D is chosen on AB so that $|AC| = |AD| = 1$; the point E is chosen on BC so that $\angle CDE = \theta$. The perpendicular to BC at E meets AB at F . Evaluate $\lim_{\theta \rightarrow 0} |EF|$.