

# 2021 - ISU Putnam Practice Set 8

Friday, November 11, 2022

## Number Theory II

1. Solve in positive integers the equation

$$2^x \cdot 3^y = 1 + 5^z.$$

2. Let  $(x_n)$  be a sequence of positive integers satisfying the recurrence relation  $x_{n+1} = 5x_n - 6x_{n-1}$ . Prove that infinitely many terms of the sequence are composite.
3. How many primes among the positive integers, written as usual in base 10, are alternating 1's and 0's, beginning and ending with 1?
4. Find all ordered pairs  $(a, b)$  of positive integers for which

$$\frac{1}{a} + \frac{1}{b} = \frac{3}{2018}.$$

5. Find all right triangles whose sides are positive integers and whose perimeter is numerically equal to their area.