2021 - ISU Putnam Practice Set 8

Wednesday, November 3, 2021

Pigeonhole Principle

1. Inside a circle of radius 4 are chosen 61 points. Show that among them there are 2 points at a distance at most $\sqrt{2}$ from each other.

2. Given any five points on a sphere, show that some four of them must lie on a closed hemisphere.

3. Let A be any set of 20 distinct integers chosen from the arithmetic progression $\{1, 4, 7, \ldots, 100\}$. Prove that there must be two distinct integers in A whose sum is 104.

4. During a month with 30 days a baseball team plays at least a game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

5. The points of the plane are colored by finitely many colors. Prove that one can find a rectangle with vertices of the same color.