2021 - ISU Putnam Practice Set 9

Monday, October 26, 2026

Probability

1. What is the probability that 3 points selected at random on a circle lie on some semicircle?

2. Let $n \geq 4$ be given, and suppose that the points $P_1, P_2, \ldots, P_n$ are randomly chosen on a circle. Consider the convex n-gon whose vertices are these points. What is the probability that at least one of the vertex angles of this polygon is acute?

3. The temperatures in Chicago and Detroit are x° and y°, respectively. These temperatures are not assumed to be independent; namely, we are given the following:

   (a) $P(x^\circ = 70^\circ) = a$

   (b) $P(y^\circ = 70^\circ) = b$

   (c) $P(\max\{x^\circ, y^\circ\} = 70^\circ) = c$.

   Determine $P(\min\{x^\circ, y^\circ\} = 70^\circ)$ in terms of a, b, c.

4. What is the probability that a permutation of the first n positive integers has the numbers 1 and 2 within the same cycle?

5. An unbiased coin is tossed n times. Find a formula, in closed form, for the expected value of $|H - T|$, where H is the number of heads, and T is the number of tails.