

# 2021 - ISU Putnam Practice Set 7

Friday, November 4, 2022

## Number Theory

1. How many positive integers  $n$  are there such that  $n$  is an exact divisor of at least one of the numbers  $10^{40}, 20^{30}$ .
2. Find the integers  $n$  for which  $(n^3 - 3n^2 + 4)/(2n - 1)$  is an integer.
3. Let  $n$  be an integer greater than 2. Prove that  $n(n - 1)^4 + 1$  is the product of two integers greater than 1.
4. Solve in positive integers the equation

$$x^{x+y} = y^{y-x}.$$

5. Show that each positive integer can be written as the difference of two positive integers having the same number of prime factors.