

2023 - ISU Putnam Practice Set 6

Thursday, October 12, 2023

Trigonometry

1. Show that the trigonometric equation

$$\sin(\cos x) = \cos(\sin x)$$

has no solutions.

2. Prove that

$$\frac{1}{\sin 45^\circ \sin 46^\circ} + \frac{1}{\sin 47^\circ \sin 48^\circ} + \cdots + \frac{1}{\sin 133^\circ \sin 134^\circ} = \frac{1}{\sin 1^\circ}.$$

3. Solve the following system of equations in real numbers:

$$\begin{aligned}\frac{3x-y}{x-3y} &= x^2, \\ \frac{3y-z}{y-3z} &= y^2, \\ \frac{3z-x}{z-3x} &= z^2.\end{aligned}$$

4. An ellipse, whose semi-axes have lengths a and b , rolls without slipping on the curve $y = c \sin\left(\frac{x}{a}\right)$. How are a, b, c related, given that the ellipse completes one revolution when it traverses one period of the curve?
5. Compute the indefinite integral

$$\int \sqrt{\frac{1-x}{1+x}} dx, \quad x \in (-1, 1).$$