2021 - ISU Putnam Practice Set 5

Wednesday, October 21, 2022

Geometry

1. Prove that the midpoints of the sides of a quadrilateral form a parallelogram.

2. Given any 9 lattice points in space, show that we can find two which have a lattice point on the interior of the segment joining them.

3. Show that the curve $x^3 + 3xy + y^3 = 1$ contains only one set of three distinct points, A, B, and C, which are vertices of an equilateral triangle, and find its area.

4. A convex polygon does not extend outside a square side 1. Prove that the sum of the squares of its sides is at most 4.

5. The vertices of a triangle are lattice points in the plane. Show that the diameter of its circumcircle does not exceed the product of its side lengths.