

# 2021 - ISU Putnam Practice Set 5

Wednesday, October 6, 2021

## Even More Calculus

1. Compute

$$\int_0^{\sqrt{\pi/3}} \sin(x^2) dx + \int_{-\sqrt{\pi/3}}^{\sqrt{\pi/3}} x^2 \cos(x^2) dx.$$

2. Compute

$$I_1 = \int \frac{\sin x}{\sin x + \cos x} dx.$$

3. Evaluate

$$I = \int_2^4 \frac{\sqrt{\ln(9-x)} dx}{\sqrt{\ln(9-x)} + \sqrt{\ln(x+3)}}.$$

4. Let  $f : [0, 1] \rightarrow \mathbb{R}$  be a continuous function such that

$$\int_0^1 f(x) dx = \int_0^1 xf(x) dx = 1.$$

Show that

$$\int_0^1 (f(x))^2 dx \geq 4.$$

5. Compute the integral

$$I = \int_0^\pi \frac{x \sin x}{1 + \sin^2 x} dx.$$