

2023 - ISU Putnam Practice Set 2

Thursday, September 14, 2023

Inequalities

1. Let $ABCD$ be a convex cyclic quadrilateral. Prove that

$$|AB - CD| + |AD - BC| \geq 2|AC - BD|.$$

2. Show that all real roots of the polynomial $P(x) = x^5 - 10x + 35$ are negative.

3. Let a, b, c be the side lengths of a triangle with perimeter 2. Prove that

$$1 < ab + bc + ca - abc \leq \frac{28}{27}.$$

4. Let a, b, c be the side lengths of a triangle with the property that for any positive integer n , the numbers a^n, b^n, c^n can also be the side lengths of a triangle. Prove that the triangle is necessarily isosceles.

5. Let m and n be positive integers. Show that

$$\frac{(m+n)!}{(m+n)^{m+n}} < \frac{m!}{m^m} \frac{n!}{n^n}.$$