

2021 - ISU Putnam Practice Set 2

Friday, September 30, 2022

Differential Equations

1. A not uncommon mistake is to believe that the product rule for derivatives says that $(fg)' = f'g'$. If $f(x) = e^{x^2}$, determine whether there exists an open interval (a, b) and a nonzero function g defined on (a, b) such that this wrong product rule is true for f and g on (a, b) .
2. Let f and g be differentiable functions on the real line satisfying the equation

$$(f^2 + g^2)f' + (fg)g' = 0.$$

Prove that f is bounded.

3. Find all twice-differentiable functions defined on the entire real axis that satisfy $f'(x)f''(x) = 0$ for all x .
4. Solve the differential equation

$$(x - 1)y'' + (4x - 5)y' + (4x - 6)y = xe^{-2x}.$$

5. Let f be a twice-differentiable real-valued function satisfying

$$f(x) + f''(x) = -xg(x)f'(x),$$

where $g(x) \geq 0$ for all real x . Prove that $|f(x)|$ is bounded.