

2023 - ISU Putnam Practice Set 1

Thursday, September 14, 2023

Polynomials 1

1. Find the zeros of the polynomial

$$P(x) = x^4 - 6x^3 + 18x^2 - 30x + 25$$

given that the sum of two of them is 4.

2. Solve the system of equations

$$x + y + z = 1$$

$$xyz = 1,$$

given that x, y, z are complex numbers of absolute value equal to 1.

3. Find all polynomials whose coefficients are equal to either 1 or -1 and whose zeros are all real.
4. The zeros of the polynomial $P(x) = x^3 - 10x + 11$ are u, v , and w . Determine the value of $\arctan(u) + \arctan(v) + \arctan(w)$.
5. Find all polynomials $P(x)$ with integer coefficients satisfying $P(P'(x)) = P'(P(x))$.